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On Positive Solution of a New Class of Nonlocal Fractional Equation with Integral Boundary Conditions

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Abstract

Abstract— We prove that a positive solution to a given boundary problem exists and is unique. This new boundary condition relates the non-local unknown value of unknown function at λ with its influence due to a sup-strip $(\mu,1)$, $0<\lambda<\mu<1$ .Our results are obtainted by using " Banach and Krasnoselskii's theorems"a linked to anywhere. Some classical theorems of fixed points assistance to achieve the greatest results.

Keywords:
"Fractional differential equations"; "Positive solution"; "Nonlocal boundary conditions"; "Fixed point theorems".

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I. INTRODUCTION

For various areas of science and engineering, fractional order differential equations have been employed, such as "physics, mechanics, economy, and biological science," etc... see [5,6,13].

The existence of positive solutions nonlinear fractional order differential equations with multipoint integral boundary conditions has been stutied by several authors using diffrrent methods (see [2,8,10,12] Bashir Ahmad et al. [3] investigated the presence and singularity of three pin integral frontier fractional difference border value solutions of order $q\in(1,2]$

$$^{c}D^{\varsigma}W(\varpi)=\Omega(\varpi,W(\varpi)),\ 0<\varpi<1,\ 1<\varsigma\le 2$$

$$W(0)=0\ ,\ W(1)=a\int_{0}^{\eta}W(\tilde{\ell})\,d\tilde{\ell}\ ,\ 0<\eta<1$$

In paper [7] the authors discussed a problem with the limit value with similar generalized conditions given by

$$^{c}D^{\varsigma}W(\varpi)=\Omega(\varpi,W(\varpi)),\ 0\le\varpi\le 1\ ,\ 1<\varsigma\le 2$$

$$W(0)=0\ ,\ W(\xi)=a\int_{\eta}^{1}W(\tilde{\ell})\,d\tilde{\ell}\ ,\quad \xi\in(0,1)$$

Where Ω is a given function, a is positive constant . The aim of this study is to determine whether positive solutions exist and uniqueness for the following boundary problem

$$^{c}D^{\kappa}\theta(\varpi)=\Omega(\varpi,\theta(\varpi)),\ 2<\kappa\le 3\ ,\ \varpi\in[0,1]$$

$$\theta(0)=\theta'(0)=0\ ,\ \theta(\lambda)=\beta\int_{\mu}^{1}\theta(\tilde{\ell})\,d\tilde{\ell} \qquad ...(1.1)$$

Where $^{c}D^{\kappa}$ the symbolizes: "Caputo fractional derivative" of order k, $\Omega:[0,1]\times\Psi_b\to\Psi_b$ is an ongoing function, and $\beta\in\Re^{+}$ and $\lambda<\mu<1$. Here $(\Psi_b,\|.\|)$ is a "Banach space " and $C=C([0,1],\Psi_b)$ is the "Banach space" of all continuous function from $[0,1]\to\Psi_b$ with norm $\|\theta\|=\sup\{|\theta(\varpi)|,\varpi\in[0,1]\}$

2. Preliminaries

Let's establish some basic fractional calculus definitions [4,1].

Definition 2.1 :For a continuous function $\hbar:[0,\infty)\to\Re$ the derivative of fractional order v is defined as

$$^{c}D^{\hat{\upsilon}}\hbar(\varpi)=\frac{1}{\Gamma(\breve{n}-\hat{\upsilon})}\int_{0}^{\varpi}(\varpi-\tilde{\ell})^{\breve{n}-\hat{\upsilon}-1}\,\hbar^{(\breve{n})}(\tilde{\ell})\,d\tilde{\ell}\,,\ \breve{n}-1<\hat{\upsilon}<n \ \ldots (2.1)$$

$\breve{n}=[\hat{\upsilon}]+1$ and $^{c}D^{\hat{\upsilon}}$ denotes the Caputo derivative.

Definition 2.2 : The fractional integral of order ς is define as

$$\mathrm{I}^{\varsigma}\lambda(\varpi)=\frac{1}{\Gamma(\varsigma)}\int_{0}^{\varpi}\frac{\lambda(\ell)}{(\varpi-\tilde{\ell})^{1-\varsigma}}\,d\tilde{\ell},\ \varsigma>0 \qquad \ldots (2.2)$$

Which is called Riemann- Liouville integral, where there is an integral exists.

Definition 2.3 : The derivative p "Riemann-Liouville" for continuous function $\hat{g}(\varpi)$ is given by

$$D^{p}\,\hat{g}(\varpi)=\frac{1}{\Gamma(\breve{n}-p)}\,\left(\frac{d}{d\varpi}\right)^{\breve{n}}\int_{0}^{\varpi}(\varpi-\tilde{\ell})^{\breve{n}-p-1}\,\hat{g}(\tilde{\ell})\,d\tilde{\ell} \ldots (2.3)$$

If point specified on the right side on $(0,\infty)$, $\breve{n}=[p]+1$.
These definitions sings to the nonlocal fractional derivative which is different from the local fractional derivative which is defined in [11] .

Lemma 2.4 : (see [9]) The overall answer for the equation $^{c}D^{\delta}\,\theta(\varpi)=0$ is provided by

$$\theta(\varpi)=c_1\varpi^{\delta-1}+c_2\varpi^{\delta-2}+\ldots+c_{\mathcal{N}}\varpi^{\delta-\mathcal{N}} \qquad \ldots(2.4)$$

Where $c_i\in\Re$, $i=0,1,2,\ldots,\mathcal{N}-1(\mathcal{N}=[\delta]+1)$ where $\mathcal{N}$ is smallest integer grater than or equal to δ $(\delta>0)$.

Lemma 2.5 : A unique solution of the boundary problem (1.1) is given by

$$\theta(\varpi)=\frac{1}{\Gamma(\kappa)}\int_{0}^{\varpi}(\varpi-\tau)^{\kappa-1}\Omega(\tau,\theta(\tau))\,d\tau$$
$$-\frac{\gamma\varpi^{\kappa-1}}{\Gamma(\kappa)}\int_{0}^{\lambda}(\lambda-\tau)^{\kappa-1}\Omega(\tau,\theta(\tau))\,d\tau$$
$$+\frac{\beta\gamma\varpi^{\kappa-1}}{\Gamma(\kappa)}\int_{\mu}^{1}\left(\int_{0}^{\tau}(\tau-\breve{n})^{\kappa-1}\Omega(\breve{n},\theta(\breve{n}))\,d\breve{n}\right)d\tau \quad \ldots(2.5)$$

Proof : For certain constants $c_1,c_2,c_3\in\Re$ We've get:

$$\theta(\varpi)=\frac{1}{\Gamma(\kappa)}\int_{0}^{\varpi}(\varpi-\tau)^{\kappa-1}\Omega(\tau)\,d\tau+c_1\varpi^{\kappa-1}+c_2\varpi^{\kappa-2}+c_3\varpi^{\kappa-3}\ldots(2.6)$$

$From\ \theta(0)=\theta'(0)=0, we\, have\, c_2=c_3=0$

By applying the second condition to (1.1)

$$\beta\int_{\mu}^{1}\theta(\tau)\,d\tau=\beta\int_{\mu}^{1}\left(\int_{0}^{\tau}\frac{(\tau-\breve{n})^{\kappa-1}}{\Gamma(\kappa)}\Omega(\breve{n})\,d\breve{n}\right)d\tau$$
$$=\beta\int_{\mu}^{1}\left(\int_{0}^{\tau}\frac{(\tau-\breve{n})^{\kappa-1}}{\Gamma(\kappa)}\Omega(\breve{n})\,d\breve{n}\right)d\tau+\beta c_1\frac{1-\mu^{\kappa}}{\kappa} \ \ldots (2.7)$$

and

$$\theta(\lambda)=\frac{1}{\Gamma(\kappa)}\int_{0}^{\lambda}(\lambda-\tau)^{\kappa-1}\Omega(\tau)\,d\tau\ +c_1\lambda^{\kappa-1}$$

which imply that

$$c_1=\frac{-\gamma}{\Gamma(\kappa)}\int_{0}^{\lambda}(\lambda-\tau)^{\kappa-1}\Omega(\tau)\,d\tau$$
$$+\frac{\beta\gamma}{\Gamma(\kappa)}\int_{\mu}^{1}\left(\int_{0}^{\tau}(\tau-\breve{n})^{\kappa-1}\Omega(\breve{n})\,d\breve{n}\right)d\tau\ldots(2.8)$$

where

$$\gamma=[\lambda^{\kappa-1}-\frac{\beta}{\kappa}(1-\mu^{\kappa})]^{-1}$$

Replacing the values of $c_1,c_2\ and\, c_3\ in (2.6)$ we have obtained the solution (2.5), the proof is complete.

In view of lemma 2.5, An operator $\aleph:\wp\to\wp$ is given by

$$(\aleph\theta)(\varpi)=\frac{1}{\Gamma(\kappa)}\int_{0}^{\varpi}(\varpi-\tau)^{\kappa-1}\Omega(\tau,\theta(\tau))\,d\tau$$
$$-\frac{\gamma\varpi^{\kappa-1}}{\Gamma(\kappa)}\int_{0}^{\lambda}(\lambda-\tau)^{\kappa-1}\Omega(\tau,\theta(\tau))\,d\tau$$
$$+\frac{\beta\gamma\varpi^{\kappa-1}}{\Gamma(\kappa)}\int_{\mu}^{1}\left(\int_{0}^{\tau}(\tau-\breve{n})^{\kappa-1}\Omega(\breve{n},\theta(\breve{n}))\,d\breve{n}\right)d\tau \quad \ldots(2.9)$$

3- Existence Results in a "Banach space"

Theorem 3.1 : Let $\Omega:[0,1]\times\Psi_b\to\Psi_b$ be a continuous function and assume that

$(Z1)\ \|\Omega(\varpi,\theta)-\Omega(\varpi,\vartheta)\|\le L\|\theta-\vartheta\|$,
$\forall\varpi\in[0,1],\quad L>0,\quad \theta,\vartheta\in\Psi_b$.

$with\ L<\frac{1}{\Delta},\ where\ \Delta\ is\ given by$

$$\Delta=\frac{1}{\Gamma(\kappa+1)}\left(1+\frac{|\gamma|\left[\lambda^{\kappa}(\kappa+1)+|\beta|(1-\mu^{\kappa+1})\right]}{\kappa+1}\right) \quad \ldots (3.1)$$

Then the limit value of the problem (1.1) has a unique solution.

Proof : Let $\sup\limits_{\varpi\in[0,1]}|\Omega(\varpi,0)|=H$ and choosing $z\ge\Delta H(1-L\Delta)^{-1}$, we show that $\aleph B_z\subset B_z$ where

$B_z = \{\theta \in \wp : \|\theta\| \le z\}.\ for\, \theta \in B_z$ we have

$$\|(\aleph\theta)(\varpi)\| \le \Bigg\{ \int_0^{\varpi} \frac{(\varpi-\tau)^{\kappa-1}}{\Gamma(\kappa)} |\Omega(\tau,\theta(\tau)) - \Omega(\tau,0) + \Omega(\tau,0)| d\tau$$

$$+|\gamma|\varpi^{\kappa-1}\int_0^{\lambda} \frac{(\lambda-\tau)^{\kappa-1}}{\Gamma(\kappa)} |\Omega(\tau,\theta(\tau)) - \Omega(\tau,0) + \Omega(\tau,0)| d\tau$$

$$+|\gamma|\beta\varpi^{\kappa-1}\int_\mu^1 \left(\int_0^{\tau} \frac{(\tau-\breve{n})^{\kappa-1}}{\Gamma(\kappa)} |\Omega(\breve{n},\theta(\breve{n})) - \Omega(\breve{n},0) + \Omega(\breve{n},0)| d\breve{n} \right) d\tau \Bigg\}$$

$$\le \sup_{\varpi\in[0,1]} \Bigg\{ \int_0^{\varpi} \frac{(\varpi-\tau)^{\kappa-1}}{\Gamma(\kappa)} \big[|\Omega(\tau,\theta(\tau)) - \Omega(\tau,0)\ | + |\ \Omega(\tau,0)| \big] d\tau$$

$$+|\gamma|\varpi^{\kappa-1}\int_0^{\lambda} \frac{(\lambda-\tau)^{\kappa-1}}{\Gamma(\kappa)} \big[|\Omega(\tau,\theta(\tau)) - \Omega(\tau,0)\ | + |\ \Omega(\tau,0)| \big] d\tau$$

$$+|\gamma|\beta\varpi^{\kappa-1}\int_\mu^1 \left(\int_0^{\tau} \frac{(\tau-\breve{n})^{\kappa-1}}{\Gamma(\kappa)} \big[|\Omega(\tau,\theta(\tau)) - \Omega(\tau,0)\ | + |\ \Omega(\tau,0)| \big] d\breve{n} \right) d\tau \Bigg\}$$

$$\le (Lz+H) \Bigg[\int_0^{\varpi} \frac{(\varpi-\tau)^{\kappa-1}}{\Gamma(\kappa)} d\tau + |\gamma|\varpi^{\kappa-1}\int_0^{\lambda} \frac{(\lambda-\tau)^{\kappa-1}}{\Gamma(\kappa)} d\tau$$

$$+|\gamma||\beta|\varpi^{\kappa-1}\int_\mu^1 \left(\int_0^{\tau} \frac{(\tau-\breve{n})^{\kappa-1}}{\Gamma(\kappa)} \right) d\breve{n}\, d\tau \Bigg]$$

$$\le (Lz+H) \left[\frac{1}{\Gamma(\kappa+1)} + \frac{|\gamma|\lambda^{\kappa}}{\Gamma(\kappa+1)} + \frac{|\gamma||\beta|(1-\mu^{\kappa+1})}{\Gamma(\kappa+2)} \right]$$

$$\le (Lz+H)\frac{1}{\Gamma(\kappa+1)} \left[1 + \frac{|\gamma|\left[\lambda^{\kappa}(\kappa+1) + |\beta|(1-\mu^{\kappa+1})\right]}{\kappa+1} \right]$$

$$\le (Lz+H)\Delta \le z \qquad ...(3.2)$$

Now, for $\theta, \vartheta \in \wp$ and for each $\varpi \in [0,1]$, we get

$$\|(\aleph\theta)(\varpi) - (\aleph\vartheta)(\varpi)\| \le \frac{1}{\Gamma(\kappa)} \int_0^{\varpi} (\varpi-\tau)^{\kappa-1} \|\Omega(\tau,\theta(\tau)) - \Omega(\tau,\vartheta(\tau))\| d\tau$$

$$+\frac{|\gamma|\varpi^{\kappa-1}}{\Gamma(\kappa)} \int_0^{\lambda} (\lambda-\tau)^{\kappa-1} \|\Omega(\tau,\theta(\tau)) - \Omega(\tau,\vartheta(\tau))\| d\tau$$

$$+\frac{|\gamma||\beta|\varpi^{\kappa-1}}{\Gamma(\kappa)} \int_\mu^1 \left(\int_0^{\tau} (\tau-\breve{n})^{\kappa-1} \|\Omega(\breve{n},\theta(\breve{n})) - \Omega(\breve{n},\vartheta(\breve{n}))\| d\breve{n} \right) d\tau$$

$$\le L\|\theta-\vartheta\| \Bigg[\frac{1}{\Gamma(\kappa)} \int_0^{\varpi} (\varpi-\tau)^{\kappa-1} d\tau + \frac{|\gamma|\varpi^{\kappa-1}}{\Gamma(\kappa)} \int_0^{\lambda} (\lambda-\tau)^{\kappa-1} d\tau$$

$$+\frac{|\gamma||\beta|\varpi^{\kappa-1}}{\Gamma(\kappa)} \int_\mu^1 \left(\int_0^{\tau} (\tau-\breve{n})^{\kappa-1} d\breve{n} \right) d\tau \Bigg]$$

$$\le \frac{L}{\Gamma(\kappa-1)} \left(1 + \frac{|\gamma|\left[\lambda^{\kappa}(\kappa-1) + |\beta|(1-\mu^{\kappa-1})\right]}{\kappa+1} \right) \|\theta-\vartheta\| \qquad \text{where}$$

$$\le L\Delta\|\theta-\vartheta\| \qquad ...(3.3)$$

for Δ shall be provided by (3.1). Note that Δ just one of the issue parameters depends on.

As $L < \frac{1}{\Delta}$, so $\aleph$ is a contraction.

Next, we argue that (1.1) solutions exist by the use "fixed point theorem" [9] of Krasnoselskii.

Theorem 3.2 : ("The fixed point theorem of Krasnoselskii"). Let S be a closed convex and not void subset of a "Banach space" Ψ_b. Let A_c, B_c to be the operators
That's it.

(a) $A_c\theta + B_c\vartheta \in S$ *whenever* $\theta, \vartheta \in S$;

(b) A_c is compact and continuous ;

(c) B_c is a contraction. Then it is available $z \in S$
That's it. $z = \ddot{A}z + \ddot{B}z$;

Theorem 3.3 : Let $\Omega : [0,1] \times \Psi_b \to \Psi_b$ be a continual common mapping of function limited sub sets of $[0,1] \times \Psi_b$ into comparatively built-in subsets of Ψ_b and assume that

(Z2) $\|\Omega(\varpi,\theta)\| \le \delta(\varpi),\ for\ all\,(\varpi,\theta) \in [0,1] \times \Psi_b$ and $\delta \in L^1([0,1], \Re^+)$ and (Z1) holds with

$$\frac{L}{\Gamma(\kappa+1)} \left(\frac{|\gamma|\left[\lambda^{\kappa}(\kappa+1) + |\beta|(1-\mu^{\kappa+1})\right]}{\kappa+1} \right) < 1 \qquad ...(3.4)$$

Then the problem (1.1) has at least one solution on [0,1].

Proof: Setting $\sup_{\varpi\in[0,1]} |\delta(\varpi)| = \|\delta\|,\ we\ fix$

$$\bar{z} \ge \frac{\|\delta\|}{\Gamma(\kappa+1)} \left(1 + \frac{|\gamma|\left[\lambda^{\kappa}(\kappa+1) + |\beta|(1-\mu^{\kappa+1})\right]}{\kappa+1} \right) \qquad ...(3.5)$$

and consider $\ddot{B}_{\bar{z}} = \{\theta \in \wp : \|\theta\| \le \bar{z}\}$. We define operators $I\ and\ J\ on\ \ddot{B}_{\bar{z}}$ as

$$(I\theta)(\varpi) = \frac{1}{\Gamma(\kappa)} \int_0^{\varpi} (\varpi-\tau)^{\kappa-1} \Omega(\tau,\theta(\tau))\, d\tau \quad ...(3,6)$$

$$(J\theta)(\varpi) = -\frac{\gamma\varpi^{\kappa-1}}{\Gamma(\kappa)} \int_0^{\lambda} (\lambda-\tau)^{\kappa-1} \Omega(\tau,\theta(\tau))\, d\tau$$

$$+\frac{\beta\gamma\varpi^{\kappa-1}}{\Gamma(\kappa)} \int_\mu^1 \left(\int_0^{\tau} (\tau-\breve{n})^{\kappa-1} \Omega(\breve{n},\theta(\breve{n}))\, d\breve{n} \right) d\tau$$

For $\theta, \vartheta \in \ddot{B}_{\bar{z}}$, we find that

$$\|I\theta + J\vartheta\| = \Bigg\| \frac{1}{\Gamma(\kappa)} \int_0^{\varpi} (\varpi-\tau)^{\kappa-1} \Omega(\tau,\theta(\tau))\, d\tau$$

$$-\frac{\gamma\varpi^{\kappa-1}}{\Gamma(\kappa)} \int_0^{\lambda} (\lambda-\tau)^{\kappa-1} \Omega(\tau,\vartheta(\tau))\, d\tau$$

$$+\frac{\beta\gamma\varpi^{\kappa-1}}{\Gamma(\kappa)}\int_{\mu}^{1}\left(\int_{0}^{\tau}(\tau-\breve{n})^{\kappa-1}\Omega(\breve{n},\vartheta(\breve{n}))d\breve{n}\right)d\tau\Bigg\|$$

$$\leq\frac{\|\delta\|}{\Gamma(\kappa+1)}\left(1+\frac{|\gamma|\left[\lambda^{\kappa}(\kappa+1)+\left|\beta(1-\mu^{(\kappa+1)})\right|\right]}{\kappa+1}\right)\leq\bar{z}\quad \text{... (3.7)}$$

Hence $I\theta+J\vartheta\in\ddot{B}_{\bar{z}}$ so J is contraction mapping by (Z1) together with (3.4). Continuity of Ω means that the operator I is contiuous. Also is uniformly bounded on $B_{c\bar{z}}$ *as*

$$\|I\theta\|\leq\frac{\|\delta\|}{\Gamma(\kappa+1)}\quad \text{... (3.8)}$$

We verify next the compactness of I.

Bu using (Z1) we know $\sup_{(\varpi,\theta)\in[0,1]\times B_{c\bar{z}}}|\Omega(\varpi,\theta)|=M<\infty$

then we have

$$\|I\theta(\varpi_2)-I\theta(\varpi_1)\|=\Bigg\|\frac{1}{\Gamma(\kappa)}\Bigg\{\int_{0}^{\varpi}\left[(\varpi_2-\tau)^{\kappa-1}-(\varpi_1-\tau)^{\kappa-1}\right]\Omega(\tau,\theta(\tau))d\tau+$$

$$\int_{\varpi_1}^{\varpi_2}(\varpi_2-\tau)^{\kappa-1}\Omega(\tau,\theta(\tau))d\tau\Bigg\}\Bigg\|\leq\frac{M}{\Gamma(\kappa+1)}\left|2(\varpi_2-\varpi_1)^{\kappa}+(\varpi_1^{\kappa}-\varpi_2^{\kappa})\right|\text{...(3.9)}$$

That is unrelated to θ. So I is relatively compact on $B_{c\bar{z}}$. Hence, by "Arzela – Ascoli's" theorem, I is compact on $B_{c\bar{z}}$ So all the assumptions of theorem 3.2. are satisfied, which implies that the boundary problem (1.1) has at least one solution on [0,1].

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الحل الموجب لصنف جديد من المعادلات الكسرية الغير محلية ذات شروط حدودية تكاملية

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الخلاصة:

في هذا البحث تم اثبات وجود ووحدانية الحل الموجب لمسألة القيم الحدودية λ حيث ترتبط الشروط الحدودية المقترحة هنا بين قيمة الدالة الغير معرفة والقيم اللامحلية وتاثيرها عند $0<\lambda<\mu<1$ و $(\mu,1)$ والتي ساعدتنا بعض النظريات الاساسية للنقطة الثابتة في تحقيق افضل النتائج. Banach and Krasnosilskii' تم الحصول على نتائجنا باستخدام مبرهنتي

الكلمات المفتاحية: المعادلات التفاضلية الكسرية- الحل الموجب- الشروط الحدودية الغير محلية- نظرية النقطة الثابتة.