

Numerical Solution for Linear Parabolic Reaction-Double Diffusivity System using the Operational Matrices of the Haar Wavelets Method

Ahmed F. Qasem

College of Computer sciences and Mathematics

University of Mosul, Iraq

Received on: 08/10/2007

Accepted on: 17/12/2007

ABSTRACT

We are using the operational matrices of the Haar wavelets method for solving linear parabolic reaction-diffusion system with double diffusivity. A numerical method based on the Haar wavelets approach which has the property $(H^{-1} = H^T)$, we compared this result with the exact solution for reaction-diffusion system, we found that high accuracy of the results in this method in the solution double diffusivity system even in the case of a small number of grid points is used. However, the computation is simple because consists of the matrices which can be programmed by Matlab language, these matrices which we got of the numerical solution are representing all time steps while the finite difference method and finite elements method need the iteration to get the needed time step, they are complicated and time-consuming.

Keywords: reaction-diffusion system, Haar wavelets approach, finite difference method.

الحل العددي لنظام التفاعل ثنائي الانتشار من نوع القطع المكافئ باستخدام طريقة مصفوفات

العوامل لموجات Haar

احمد فاروق قاسم

كلية علوم الحاسوب والرياضيات، جامعة الموصل

تاريخ القبول: 2007/12/17

تاريخ الاستلام: 2007/10/08

المخلص

تم في هذا البحث استخدام طريقة مصفوفات العوامل لموجات Haar القصيرة لحل نظام التفاعل ثنائي الانتشار الخطي من نوع القطع المكافئ، إن الطريقة العددية المستخدمة في هذا البحث تعتمد على تقريب موجات Haar التي تتميز بأنها مصفوفة مدورها مساو للمعكوس $(H^{-1} = H^T)$ ، و قورنت النتائج مع الحل المضبوط للنظام وقد تبين أن هذه الطريقة ذات دقة عالية في حل نظام ثنائي الانتشار حتى في حالة استخدام عدد صغير من نقاط الشبكة فضلا عن أن الحسابات فيها ايسر وذلك لسهولة التعامل مع المصفوفات وإمكانية برمجتها بلغة Matlab وان المصفوفة الناتجة تمثل كل الخطوات الزمنية المطلوبة بعكس طرائق الفروقات المنتهية والعناصر المنتهية التي تحتاج إلى التكرار للوصول إلى الخطوة الزمنية المطلوبة وهذا يستغرق وقتا أطول.

الكلمات المفتاحية: نظام التفاعل ثنائي الانتشار، تقريب موجات Haar، طرائق الفروقات المنتهية.

1.Introduction:

Partial differential equations are at the heart of many, if not most, computer analysis or simulations of continuous physical systems, such as fluids, electromagnetic fields, the human body, and so on .It is usually classified as parabolic hyperbolic or elliptic according to the form of the equation and the form of the subsidiary conditions which must be assigned to produce a well-posed problem .

In many engineering applications, the numerical solution of partial differential equations is required in the design and simulation of new products. The two most common numerical methods are the finite difference methods and the finite element methods, they are complicated and time-consuming [7].

Haar wavelets have become an increasingly popular tool in the computational sciences. They have had numerous applications in a wide rang of areas such as signal analysis, data compression and many others[8].

Wu and Chen (2003) [7] studied the numerical solution for partial differential equations of first order via operational matrices , they used the Haar wavelets in the solution with constant initial and boundary conditions.

Wu and Chen (2004) [8] are studied the numerical solution for fractional calculus and the fractional differential equation by using the operational matrices of orthogonal functions. The fractional derivatives of the four typical functions and two classical fractional differential equations solved by the new method and they are compared the results with the exact solutions, they are found the solutions by this method is simple and computer oriented.

Lepik and Tamme (2007) [5] are derived the solution of nonlinear Fredholm integral equations via the Haar wavelet method, they are find that the main benefits of the Haar wavelet method are sparse representation, fast transformation, and possibility of implementation of fast algorithms especially if matrix representation is used.

Many authors have studied the reaction-diffusion systems. These systems have many applications in physics, chemistry or biology. For example, chemical reactions, population dynamics or combustion are modeled by reaction-diffusion equations.

Aggarwala and Nasim (1987) [1] derived the solution of reaction-diffusion equations with double diffusivity by laplace technique and Fourier transforms which appear to be simpler and more direct.

Chow Tanya (1996) [4] is studied the derivation of similarity solutions for one-dimensional coupled systems of reaction-diffusion

equations, these solutions are obtained by means of one-parameter group methods.

Antic and Hill (2000) [2] are studied a mathematical model for heat transfer in grain store microclimates, this model is "double diffusivity" such that they are used the heat-balance integral method to transform the coupled partial differential equations to the coupled ordinary differential equations and solved it numerically by using the Fehlberg fourth-fifth order Range-kutta method.

Antic and Hill (2003) [3] are studied a Two-stage heat transfer model for the peripheral layers of a grain store, they are observed that the predictions of the air and grain temperatures of the two-stage model lag behind those of the double-diffusivity heat transfer model, but this lag decreases as time increases.

Polyanin, A. D. (2004) [6] is found the exact solutions of linear and nonlinear reaction-diffusion equations of different kinds parabolic, hyperbolic and elliptic systems, he was used some hypothesis to transform reaction-diffusion systems to equations equivalent to heat equation or wave equation or laplace equation which have exact solutions.

In this paper, we will study the numerical solution for linear reaction-diffusion system with double-diffusivity by the operational matrices of Haar wavelet method and we will compare the results of this method with exact solutions.

2.Mathematical model:

The one-dimensional case of reaction-diffusion equations with double-diffusivity is given by [1,2]:

$$\frac{\partial u}{\partial t} = D_1 \frac{\partial^2 u}{\partial x^2} - b_1 u(x, t) + c_1 v(x, t) \quad \dots(1a)$$

$$\frac{\partial v}{\partial t} = D_2 \frac{\partial^2 v}{\partial x^2} + b_2 u(x, t) - c_2 v(x, t) \quad \dots(1b)$$

where $u(x, t)$ and $v(x, t)$ denote the air-temperature and a grain-temperature respectively, the self-diffusivities D_1, D_2, b_1, b_2, c_1 and c_2 are positive constants.

with initial condition:

$$u(x, 0) = E(x) \quad \dots(2a)$$

$$v(x, 0) = F(x) \quad \dots(2b)$$

such that $E(x)$ and $F(x)$ are prescribed space-dependent for the initial air-temperature and a grain-temperature respectively.

and mixed boundary conditions:

$$u(0, t) = G(t) \quad \dots(3a)$$

$$v(0, t) = I(t) \quad \dots(3b)$$

$$\frac{\partial u(0, t)}{\partial x} = K(t) \quad \dots(4a)$$

$$\frac{\partial v(0, t)}{\partial x} = L(t) \quad \dots(4b)$$

$G(t)$, $K(t)$, $I(t)$ and $L(t)$ are prescribed time-dependent for the boundary air-temperature and a grain-temperature respectively.

Polyanin, A. D. [6] is found the exact solution for (1a) and (1b) with $D_1 = D_2 = a$ such that:

$$u = \frac{A}{b_2(\lambda_1 - \lambda_2)} e^{(1+a\lambda_1)\lambda_1 t + \lambda_1 x} - \frac{B}{b_2(\lambda_1 - \lambda_2)} e^{(1+a\lambda_2)\lambda_2 t + \lambda_2 x} \quad \dots(5a)$$

$$v = \frac{1}{(\lambda_1 - \lambda_2)} e^{(1+a\lambda_1)\lambda_1 t + \lambda_1 x} - \frac{1}{(\lambda_1 - \lambda_2)} e^{(1+a\lambda_2)\lambda_2 t + \lambda_2 x} \quad \dots(5b)$$

such that:

$$A = \frac{-(b_1 - c_2) - \sqrt{(b_1 + c_2)^2 - 4(b_1 c_2 - b_2 c_1)}}{2}$$

$$B = \frac{-(b_1 - c_2) + \sqrt{(b_1 + c_2)^2 - 4(b_1 c_2 - b_2 c_1)}}{2}$$

$$\lambda_1 = \frac{-(b_1 + c_2) - \sqrt{(b_1 + c_2)^2 - 4(b_1 c_2 - b_2 c_1)}}{2}$$

$$\lambda_2 = \frac{-(b_1 + c_2) + \sqrt{(b_1 + c_2)^2 - 4(b_1 c_2 - b_2 c_1)}}{2}$$

3. The operational matrices and Haar wavelets:

The main characteristic of the operational method is to convert a differential equation into an algebraic one, and the core is the operational matrix for integration. The integral property of the basic orthonormal matrix, $\phi(t)$. we write the following approximation:

$$\int_0^t \int_0^t \int_0^t \dots \int_0^t \phi(t) (dt)^k \cong Q_\phi^k \phi(t) \quad \dots(6)$$

k -times

where $\phi(t) = [\bar{\varphi}_0(t) \ \bar{\varphi}_1(t) \ \dots \ \bar{\varphi}_{m-1}(t)]^T$ in which the elements $\bar{\varphi}_0(t), \bar{\varphi}_1(t), \dots, \bar{\varphi}_{m-1}(t)$ are the discrete representation of the basis functions which are orthogonal on the interval $[0,1)$ and Q_ϕ is the operational matrix for integration of $\phi(t)$ [7,8].

The operational matrix Q_ϕ of an orthogonal matrix $\phi(t)$ can be expressed by:

$$[Q_\phi] = [\phi] \cdot [Q_B] \cdot [\phi]^{-1} \quad \dots(7)$$

where $[Q_B]$ is the operational matrix of the block pulse function:

$$Q_{B_m} = \frac{1}{m} \begin{bmatrix} 1/2 & 1 & \dots & \dots & 1 \\ 0 & 1/2 & 1 & \dots & 1 \\ 0 & \dots & 1/2 & \dots & 1 \\ 0 & \dots & 0 & 1/2 & 1 \\ 0 & \dots & \dots & 0 & 1/2 \end{bmatrix} \quad \dots(8)$$

If the transform matrix $[\phi]$ is unitary, that is $[\phi]^{-1} = [\phi]^T$, then the equation (7) can be rewritten as [7,8]:

$$[Q_\phi] = [\phi] \cdot [Q_B] \cdot [\phi]^T \quad \dots(9)$$

The Haar functions are an orthogonal family of switched rectangular waveforms where amplitudes can differ from one function to another. They are defined in the interval [0,1] by [7,8]:

$$h_0(t) = \frac{1}{\sqrt{m}} \begin{cases} 2^{\frac{J}{2}} & \frac{k-1}{2^J} \leq t < \frac{k-1/2}{2^J} \\ -2^{\frac{J}{2}} & \frac{k-1/2}{2^J} \leq t < \frac{k}{2^J} \\ 0 & \text{otherwise in } [0,1] \end{cases} \quad \dots(10)$$

where $i=0,1,2,\dots,m-1$, $m=2^\alpha$ and α is a positive integer. J and k represent the integer decomposition of the index i , i.e. $i = 2^J + k - 1$.

Theoretically, this set of functions is complete. $h_0(t)$ is called the scaling function and $h_1(t)$ the mother wavelet, such that from the mother wavelet $h_1(t)$, compression and translation are performed to obtain $h_2(t)$ and $h_3(t)$.

Any function $u(x,t)$ which is square integrable in the interval $0 \leq t < 1$ and $0 \leq x < 1$ can be expanded into Haar series by:

$$u(x,t) = \sum_{i=0}^{m-1} \sum_{J=0}^{m-1} c_{iJ} h_i(x) h_J(t) \quad \dots(11)$$

$$\text{where } c_{iJ} = \int_0^1 u(x,t) h_i(x) dx \cdot \int_0^1 u(x,t) h_J(t) dt \cdot$$

The equation (11) can be written into the discrete form by:

$$u(x,t) = H^T(x) \cdot C \cdot H(t) \quad \dots(12)$$

where

$$[C] = \begin{bmatrix} C_{0,0} & C_{0,1} & \dots & \dots & C_{0,m-1} \\ C_{1,0} & C_{1,1} & \dots & \dots & C_{1,m-1} \\ \vdots & \vdots & \dots & \dots & \vdots \\ \vdots & \vdots & \dots & \dots & \vdots \\ C_{m-1,0} & C_{m-1,1} & \dots & \dots & C_{m-1,m-1} \end{bmatrix}$$

is the coefficient matrix of $u(x,t)$ calculated by:

$$[C] = [H] \cdot [u] \cdot [H]^T \quad \dots(13)$$

For deriving the operational matrix of Haar wavelets, we let $[\phi] = [H]$ in the equation (9), and obtain:

$$[Q_H] = [H] \cdot [Q_B] \cdot [H]^T \quad \dots(14)$$

where $[Q_H]$ is the operational matrix for integration of $[H]$.

For example, the operational matrix of the Haar wavelet in the case of $m=2$ is given by:

$$\begin{aligned} [Q_H] &= [H]_{2 \times 2} \cdot [Q_B] \cdot [H]_{2 \times 2}^T \\ &= \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \cdot \frac{1}{2} \begin{bmatrix} \frac{1}{2} & 1 \\ 0 & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}^T \\ &= \begin{bmatrix} \frac{1}{2} & -\frac{1}{4} \\ \frac{1}{4} & 0 \end{bmatrix} \end{aligned}$$

4.Numerical solution:

We will use the operational matrices of the Haar wavelets to solve the equations (1a) and (1b) numerically.

By using the equation (6) and the integration of equation (12) with respect to variable t yields [7]:

$$\begin{aligned} \int_0^t u(x,t) dt &= \int_0^t H^T(x) \cdot Cu \cdot H(t) dt = H^T \cdot Cu \cdot \int_0^t H(t) dt \\ &= [H]^T \cdot [Cu] \cdot [Q_H] \cdot [H] \end{aligned} \quad \dots(15)$$

Further integration with respect to variable x gives:

$$\begin{aligned} \int_0^x u(x,t) dx &= \int_0^x H^T(x) \cdot Cu \cdot H(t) dx = \int_0^x H^T(x) dx \cdot Cu \cdot [H] \\ &= [H]^T \cdot [Q_H]^T \cdot [Cu] \cdot [H] \end{aligned} \quad \dots(16)$$

The double integration, we obtain:

$$\begin{aligned} \int_0^t \int_0^x u(x,t) dx dt &= \int_0^t \int_0^x H^T(x) \cdot Cu \cdot H(t) dx dt \\ &= \int_0^x H^T(x) dx \cdot Cu \cdot \int_0^t H(t) dt \\ &= [H]^T \cdot [Q_H]^T \cdot [Cu] \cdot [Q_H] \cdot [H] \end{aligned} \quad \dots(17)$$

also

$$\begin{aligned} \int_0^t \int_0^x \int_0^x u(x,t) dx dx dt &= \int_0^t \int_0^x \int_0^x H^T(x) \cdot Cu \cdot H(t) dx dx dt \\ &= \int_0^x \int_0^x H^T(x) dx dx \cdot Cu \cdot \int_0^t H(t) dt \\ &= [H]^T \cdot [Q_H^2]^T \cdot [Cu] \cdot [Q_H] \cdot [H] \end{aligned} \quad \dots(18)$$

Now integrate equations (1a) and (1b) with respect to t, we get:

$$\int_0^t \frac{\partial u(x,t)}{\partial t} dt = D_1 \int_0^t \frac{\partial^2 u(x,t)}{\partial x^2} dt - b_1 \int_0^t u(x,t) dt + c_1 \int_0^t v(x,t) dt \quad \dots(19a)$$

$$\int_0^t \frac{\partial v(x,t)}{\partial t} dt = D_2 \int_0^t \frac{\partial^2 v(x,t)}{\partial x^2} dt + b_2 \int_0^t u(x,t) dt - c_2 \int_0^t v(x,t) dt \quad \dots(19b)$$

we shall use the initial and boundary conditions which Polyanin was used in find the exact solutions with the case of the diffusion coefficients $D_1 = D_2 = a$ (see [6]).

By using the initial condition (2a) and (2b), we get:

$$u(x,t) - E(x) = a \int_0^t \frac{\partial^2 u(x,t)}{\partial x^2} dt - b_1 \int_0^t u(x,t) dt + c_1 \int_0^t v(x,t) dt \quad \dots(20a)$$

$$v(x,t) - F(x) = a \int_0^t \frac{\partial^2 v(x,t)}{\partial x^2} dt + b_2 \int_0^t u(x,t) dt - c_2 \int_0^t v(x,t) dt \quad \dots(20b)$$

Now, the double integrate for equations (20a) and (20b) with respect to x, we get:

$$\begin{aligned} \int_0^x \int_0^x u(x,t) dx dx - \int_0^x \int_0^x E(x) dx dx &= a \int_0^t \int_0^x \left[\frac{\partial u}{\partial x} - \frac{\partial u(0,t)}{\partial x} \right] dx dt \\ &\quad - b_1 \int_0^t \int_0^x \int_0^x u(x,t) dx dx dt + c_1 \int_0^t \int_0^x \int_0^x v(x,t) dx dx dt \end{aligned} \quad \dots(21a)$$

$$\int_0^x \int_0^x v(x,t) dx dx - \int_0^x \int_0^x F(x) dx dx = a \int_0^t \int_0^x \left[\frac{\partial v}{\partial x} - \frac{\partial v(0,t)}{\partial x} \right] dx dt \quad \dots(21b)$$

$$+ b_2 \int_0^t \int_0^x \int_0^x u(x,t) dx dx dt - c_2 \int_0^t \int_0^x \int_0^x v(x,t) dx dx dt$$

by using the boundary conditions (4a) and (4b), we get:

$$\int_0^x \int_0^x u(x,t) dx dx - \int_0^x \int_0^x E(x) dx dx = a \int_0^t [u(x,t) - u(0,t)] dt - a \int_0^t \int_0^x K(t) dx dt \quad \dots(22a)$$

$$- b_1 \int_0^t \int_0^x \int_0^x u(x,t) dx dx dt + c_1 \int_0^t \int_0^x \int_0^x v(x,t) dx dx dt$$

$$\int_0^x \int_0^x v(x,t) dx dx - \int_0^x \int_0^x F(x) dx dx = a \int_0^t [v(x,t) - v(0,t)] dt - a \int_0^t \int_0^x L(t) dx dt \quad \dots(22b)$$

$$+ b_2 \int_0^t \int_0^x \int_0^x u(x,t) dx dx dt - c_2 \int_0^t \int_0^x \int_0^x v(x,t) dx dx dt$$

by using the boundary conditions (3a) and (3b) and rearranging, we get:

$$\int_0^x \int_0^x u(x,t) dx dx - a \int_0^t u(x,t) dt + b_1 \int_0^t \int_0^x \int_0^x u(x,t) dx dx dt - c_1 \int_0^t \int_0^x \int_0^x v(x,t) dx dx dt \quad \dots(23a)$$

$$= \int_0^x \int_0^x E(x) dx dx - a \int_0^t G(t) dt - a \int_0^t \int_0^x K(t) dx dt$$

$$\int_0^x \int_0^x v(x,t) dx dx - a \int_0^t v(x,t) dt - b_2 \int_0^t \int_0^x \int_0^x u(x,t) dx dx dt + c_2 \int_0^t \int_0^x \int_0^x v(x,t) dx dx dt \quad \dots(23b)$$

$$= \int_0^x \int_0^x F(x) dx dx - a \int_0^t I(t) dt - a \int_0^t \int_0^x L(t) dx dt$$

we transform the equations (23a) and (23b) into the matrices form by using the equation (12), we get:

$$\int_0^x \int_0^x [H(x)]^T \cdot [Cu] \cdot [H(t)] dx dx - a \int_0^t [H(x)]^T \cdot [Cu] \cdot [H(t)] dt \quad \dots(24a)$$

$$+ b_1 \int_0^t \int_0^x \int_0^x [H(x)]^T \cdot [Cu] \cdot [H(t)] dx dx dt - c_1 \int_0^t \int_0^x \int_0^x [H(x)]^T \cdot [Cv] \cdot [H(t)] dx dx dt$$

$$= \int_0^x \int_0^x [H(x)]^T \cdot [J_1] \cdot [H(t)] dx dx - a \int_0^t [H(x)]^T \cdot [J_2] \cdot [H(t)] dt - a \int_0^t \int_0^x [H(x)]^T \cdot [J_3] \cdot [H(t)] dx dx dt$$

$$\begin{aligned}
 & \int_0^x \int_0^x [H(x)]^T \cdot [Cv] \cdot [H(t)] dx dx - a \int_0^t [H(x)]^T \cdot [Cv] \cdot [H(t)] dt \\
 & - b_2 \int_0^t \int_0^x \int_0^x [H(x)]^T \cdot [Cu] \cdot [H(t)] dx dx dt + c_2 \int_0^t \int_0^x \int_0^x [H(x)]^T \cdot [Cv] \cdot [H(t)] dx dx dt \quad \dots(24b) \\
 & = \int_0^x \int_0^x [H(x)]^T \cdot [J_4] \cdot [H(t)] dx dx - a \int_0^t [H(x)]^T \cdot [J_5] \cdot [H(t)] dt - a \int_0^t \int_0^x [H(x)]^T \cdot [J_6] \cdot [H(t)] dx dt
 \end{aligned}$$

such that:

$$[J_1] = [H]_{m \times m} \cdot \begin{bmatrix} E(x_1) & E(x_1) & \dots & \dots & E(x_1) \\ E(x_2) & E(x_2) & \dots & \dots & E(x_2) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ E(x_m) & E(x_m) & \dots & \dots & E(x_m) \end{bmatrix}_{m \times m} \cdot [H]_{m \times m}^T \quad \dots(25)$$

where

$$x_i = \frac{1}{2m} + \frac{i-1}{m} \quad i = 1, 2, 3, \dots$$

m is the dimension of the matrix.

$$[J_2] = [H]_{m \times m} \cdot \begin{bmatrix} G(t_1) & G(t_2) & \dots & \dots & G(t_m) \\ G(t_1) & G(t_2) & \dots & \dots & G(t_m) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ G(t_1) & G(t_2) & \dots & \dots & G(t_m) \end{bmatrix}_{m \times m} \cdot [H]_{m \times m}^T \quad \dots(26)$$

$$[J_3] = [H]_{m \times m} \cdot \begin{bmatrix} K(t_1) & K(t_2) & \dots & \dots & K(t_m) \\ K(t_1) & K(t_2) & \dots & \dots & K(t_m) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ K(t_1) & K(t_2) & \dots & \dots & K(t_m) \end{bmatrix}_{m \times m} \cdot [H]_{m \times m}^T \quad \dots(27)$$

where

$$t_i = \frac{1}{2m} + \frac{i-1}{m} \quad i = 1, 2, 3, \dots$$

and also for the equation (24b):

$$[J_4] = [H]_{m \times m} \cdot \begin{bmatrix} F(x_1) & F(x_1) & \dots & \dots & F(x_1) \\ F(x_2) & F(x_2) & \dots & \dots & F(x_2) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ F(x_m) & F(x_m) & \dots & \dots & F(x_m) \end{bmatrix}_{m \times m} \cdot [H]_{m \times m}^T \quad \dots(28)$$

where

$$x_i = \frac{1}{2m} + \frac{i-1}{m} \quad i = 1,2,3,\dots$$

$$[J_5] = [H]_{m \times m} \cdot \begin{bmatrix} I(t_1) & I(t_2) & \dots & \dots & I(t_m) \\ I(t_1) & I(t_2) & \dots & \dots & I(t_m) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ I(t_1) & I(t_2) & \dots & \dots & I(t_m) \end{bmatrix}_{m \times m} \cdot [H]_{m \times m}^T \quad \dots(29)$$

$$[J_6] = [H]_{m \times m} \cdot \begin{bmatrix} L(t_1) & L(t_2) & \dots & \dots & L(t_m) \\ L(t_1) & L(t_2) & \dots & \dots & L(t_m) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ L(t_1) & L(t_2) & \dots & \dots & L(t_m) \end{bmatrix}_{m \times m} \cdot [H]_{m \times m}^T \quad \dots(30)$$

where

$$t_i = \frac{1}{2m} + \frac{i-1}{m} \quad i = 1,2,3,\dots$$

Now, by using the integrations (15),(16),(17) and (18), the equations (24a) and (24b) becomes:

$$[H]^T \cdot [Q_H^2]^T \cdot [Cu] \cdot [H] - a \cdot [H]^T \cdot [Cu] \cdot [Q_H] \cdot [H] + b_1 \cdot [H]^T \cdot [Q_H^2]^T \cdot [Cu] \cdot [Q_H] \cdot [H] - c_1 \cdot [H]^T \cdot [Q_H^2]^T \cdot [Cv] \cdot [Q_H] \cdot [H] = [H]^T \cdot [Q_H^2]^T \cdot [J_1] \cdot [H] - a \cdot [H]^T \cdot [J_2] \cdot [Q_H] \cdot [H] \dots(31a) - a \cdot [H]^T \cdot [Q_H]^T \cdot [J_3] \cdot [Q_H] \cdot [H]$$

$$[H]^T \cdot [Q_H^2]^T \cdot [Cv] \cdot [H] - a \cdot [H]^T \cdot [Cv] \cdot [Q_H] \cdot [H] - b_2 \cdot [H]^T \cdot [Q_H^2]^T \cdot [Cu] \cdot [Q_H] \cdot [H] + c_2 \cdot [H]^T \cdot [Q_H^2]^T \cdot [Cv] \cdot [Q_H] \cdot [H] = [H]^T \cdot [Q_H^2]^T \cdot [J_4] \cdot [H] - a \cdot [H]^T \cdot [J_5] \cdot [Q_H] \cdot [H] \dots(31b) - a \cdot [H]^T \cdot [Q_H]^T \cdot [J_6] \cdot [Q_H] \cdot [H]$$

such that the dimension for all matrices are $m \times m$, $[H]$ is Haar wavelets matrix, $[Q_H]$ is the operational matrix of the Haar wavelet, $[Cu]$ is the coefficient matrix of $u(x,t)$ and $[Cv]$ is the coefficient matrix of $v(x,t)$:

$$[Cu] = \begin{bmatrix} Cu_{0,0} & Cu_{0,1} & \dots & \dots & Cu_{0,m-1} \\ Cu_{1,0} & Cu_{1,1} & \dots & \dots & Cu_{1,m-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Cu_{m-1,0} & Cu_{m-1,1} & \dots & \dots & Cu_{m-1,m-1} \end{bmatrix}_{m \times m}$$

$$[Cv] = \begin{bmatrix} Cv_{0,0} & Cv_{0,1} & \dots & \dots & Cv_{0,m-1} \\ Cv_{1,0} & Cv_{1,1} & \dots & \dots & Cv_{1,m-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Cv_{m-1,0} & Cv_{m-1,1} & \dots & \dots & Cv_{m-1,m-1} \end{bmatrix}_{m \times m}$$

by multiplying $[H]^T$ to the right hand side and $[H]$ to the left hand side of each term in equations (31a) and (31b), we get:

$$\begin{aligned} & [Q_H^2]^T \cdot [Cu] - a \cdot [Cu] \cdot [Q_H] + b_1 \cdot [Q_H^2]^T \cdot [Cu] \cdot [Q_H] - c_1 \cdot [Q_H^2]^T \cdot [Cv] \cdot [Q_H] \\ & = [Q_H^2]^T \cdot [J_1] - a \cdot [J_2] \cdot [Q_H] - a \cdot [Q_H]^T \cdot [J_3] \cdot [Q_H] \end{aligned} \quad \dots(32a)$$

$$\begin{aligned} & [Q_H^2]^T \cdot [Cv] - a \cdot [Cv] \cdot [Q_H] - b_2 \cdot [Q_H^2]^T \cdot [Cu] \cdot [Q_H] + c_2 \cdot [Q_H^2]^T \cdot [Cv] \cdot [Q_H] \\ & = [Q_H^2]^T \cdot [J_4] - a \cdot [J_5] \cdot [Q_H] - a \cdot [Q_H]^T \cdot [J_6] \cdot [Q_H] \end{aligned} \quad \dots(32b)$$

To find the coefficient matrices $[Cu]$ and $[Cv]$ which have 2^m of the elements respectively, we solve the system (32a) and (32b) which given linear system of the equations such that the variables number are 2^{m+1} and we can be solved them by Gauss-Jordan method, after this we find the matrices $[u]$ and $[v]$ by using the equation (12) such that:

$$[u] = [H]^T \cdot [Cu] \cdot [H]$$

$$[v] = [H]^T \cdot [Cv] \cdot [H]$$

5. Numerical results:

In this section, we have solved the system (32a) and (32b) with the initial condition for the exact solution (5a) and (5b), such that:

$$u(x,0) = E(x) = \frac{A}{b_2(\lambda_1 - \lambda_2)} e^{\lambda_1 x} - \frac{B}{b_2(\lambda_1 - \lambda_2)} e^{\lambda_2 x}$$

$$v(x,0) = F(x) = \frac{1}{(\lambda_1 - \lambda_2)} e^{\lambda_1 x} - \frac{1}{(\lambda_1 - \lambda_2)} e^{\lambda_2 x}$$

and mixed boundary conditions:

$$u(0,t) = G(t) = \frac{A}{b_2(\lambda_1 - \lambda_2)} e^{(1+a\lambda_1)\lambda_1 t} - \frac{B}{b_2(\lambda_1 - \lambda_2)} e^{(1+a\lambda_2)\lambda_2 t}$$

$$v(0,t) = I(t) = \frac{1}{(\lambda_1 - \lambda_2)} e^{(1+a\lambda_1)\lambda_1 t} - \frac{1}{(\lambda_1 - \lambda_2)} e^{(1+a\lambda_2)\lambda_2 t}$$

$$\frac{\partial u(0,t)}{\partial x} = K(t) = \frac{\lambda_1 A}{b_2(\lambda_1 - \lambda_2)} e^{(1+a\lambda_1)\lambda_1 t} - \frac{\lambda_2 B}{b_2(\lambda_1 - \lambda_2)} e^{(1+a\lambda_2)\lambda_2 t}$$

$$\frac{\partial v(0,t)}{\partial x} = L(t) = \frac{\lambda_1}{(\lambda_1 - \lambda_2)} e^{(1+a\lambda_1)\lambda_1 t} - \frac{\lambda_2}{(\lambda_1 - \lambda_2)} e^{(1+a\lambda_2)\lambda_2 t}$$

with $a = 1, b_1 = 1, c_1 = 1, b_2 = 1$ and $c_2 = 1$ then:

$$\lambda_1 = -2, \lambda_2 = 0, A = -1 \text{ and } B = 1.$$

When $m=4$ then, from the equation (10), we get:

$$[H] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

from the equation (14), we get:

$$[Q_H] = \begin{bmatrix} 0.5 & -0.25 & -0.08838835 & -0.08838835 \\ 0.25 & 0 & -0.08838835 & 0.08838835 \\ 0.08838835 & 0.08838835 & 0 & 0 \\ 0.08838835 & -0.08838835 & 0 & 0 \end{bmatrix}$$

from the equations (25),(26),(27),(28),(29) and (30), we get:

$$[J_1] = [H] \cdot \begin{bmatrix} (1/2)e^{-2(1/8)} + (1/2) & (1/2)e^{-2(1/8)} + (1/2) & (1/2)e^{-2(1/8)} + (1/2) & (1/2)e^{-2(1/8)} + (1/2) \\ (1/2)e^{-2(3/8)} + (1/2) & (1/2)e^{-2(3/8)} + (1/2) & (1/2)e^{-2(3/8)} + (1/2) & (1/2)e^{-2(3/8)} + (1/2) \\ (1/2)e^{-2(5/8)} + (1/2) & (1/2)e^{-2(5/8)} + (1/2) & (1/2)e^{-2(5/8)} + (1/2) & (1/2)e^{-2(5/8)} + (1/2) \\ (1/2)e^{-2(7/8)} + (1/2) & (1/2)e^{-2(7/8)} + (1/2) & (1/2)e^{-2(7/8)} + (1/2) & (1/2)e^{-2(7/8)} + (1/2) \end{bmatrix} \cdot [H]^T$$

where $x_i = \frac{1}{2m} + \frac{i-1}{m}$ then $x_1 = 1/8, x_2 = 3/8, x_3 = 5/8, x_4 = 7/8$.

$$[J_2] = [H] \cdot \begin{bmatrix} (1/2)e^{2(1/8)} + (1/2) & (1/2)e^{2(3/8)} + (1/2) & (1/2)e^{2(5/8)} + (1/2) & (1/2)e^{2(7/8)} + (1/2) \\ (1/2)e^{2(1/8)} + (1/2) & (1/2)e^{2(3/8)} + (1/2) & (1/2)e^{2(5/8)} + (1/2) & (1/2)e^{2(7/8)} + (1/2) \\ (1/2)e^{2(1/8)} + (1/2) & (1/2)e^{2(3/8)} + (1/2) & (1/2)e^{2(5/8)} + (1/2) & (1/2)e^{2(7/8)} + (1/2) \\ (1/2)e^{2(1/8)} + (1/2) & (1/2)e^{2(3/8)} + (1/2) & (1/2)e^{2(5/8)} + (1/2) & (1/2)e^{2(7/8)} + (1/2) \end{bmatrix} \cdot [H]^T$$

where $t_i = \frac{1}{2m} + \frac{i-1}{m}$ then $t_1 = 1/8, t_2 = 3/8, t_3 = 5/8, t_4 = 7/8$.

$$[J_3] = [H] \cdot \begin{bmatrix} -e^{2(1/8)} & -e^{2(3/8)} & -e^{2(5/8)} & -e^{2(7/8)} \\ -e^{2(1/8)} & -e^{2(3/8)} & -e^{2(5/8)} & -e^{2(7/8)} \\ -e^{2(1/8)} & -e^{2(3/8)} & -e^{2(5/8)} & -e^{2(7/8)} \\ -e^{2(1/8)} & -e^{2(3/8)} & -e^{2(5/8)} & -e^{2(7/8)} \end{bmatrix} \cdot [H]^T$$

$$[J_4] = [H] \cdot \begin{bmatrix} -(1/2)e^{-2(1/8)} + (1/2) & -(1/2)e^{-2(1/8)} + (1/2) & -(1/2)e^{-2(1/8)} + (1/2) & -(1/2)e^{-2(1/8)} + (1/2) \\ -(1/2)e^{-2(3/8)} + (1/2) & -(1/2)e^{-2(3/8)} + (1/2) & -(1/2)e^{-2(3/8)} + (1/2) & -(1/2)e^{-2(3/8)} + (1/2) \\ -(1/2)e^{-2(5/8)} + (1/2) & -(1/2)e^{-2(5/8)} + (1/2) & -(1/2)e^{-2(5/8)} + (1/2) & -(1/2)e^{-2(5/8)} + (1/2) \\ -(1/2)e^{-2(7/8)} + (1/2) & -(1/2)e^{-2(7/8)} + (1/2) & -(1/2)e^{-2(7/8)} + (1/2) & -(1/2)e^{-2(7/8)} + (1/2) \end{bmatrix} \cdot [H]^T$$

$$[J_5] = [H] \cdot \begin{bmatrix} -(1/2)e^{2(1/8)} + (1/2) & -(1/2)e^{2(3/8)} + (1/2) & -(1/2)e^{2(5/8)} + (1/2) & -(1/2)e^{2(7/8)} + (1/2) \\ -(1/2)e^{2(1/8)} + (1/2) & -(1/2)e^{2(3/8)} + (1/2) & -(1/2)e^{2(5/8)} + (1/2) & -(1/2)e^{2(7/8)} + (1/2) \\ -(1/2)e^{2(1/8)} + (1/2) & -(1/2)e^{2(3/8)} + (1/2) & -(1/2)e^{2(5/8)} + (1/2) & -(1/2)e^{2(7/8)} + (1/2) \\ -(1/2)e^{2(1/8)} + (1/2) & -(1/2)e^{2(3/8)} + (1/2) & -(1/2)e^{2(5/8)} + (1/2) & -(1/2)e^{2(7/8)} + (1/2) \end{bmatrix} \cdot [H]^T$$

$$[J_6] = [H] \cdot \begin{bmatrix} e^{2(1/8)} & e^{2(3/8)} & e^{2(5/8)} & e^{2(7/8)} \\ e^{2(1/8)} & e^{2(3/8)} & e^{2(5/8)} & e^{2(7/8)} \\ e^{2(1/8)} & e^{2(3/8)} & e^{2(5/8)} & e^{2(7/8)} \\ e^{2(1/8)} & e^{2(3/8)} & e^{2(5/8)} & e^{2(7/8)} \end{bmatrix} \cdot [H]^T$$

Now, by substitute the matrices $[Q_H], [J_1], [J_2], [J_3], [J_4], [J_5]$ and $[J_6]$ in the system (32a) and (32b), and solve this system we get to the linear system consist of 32 equations and 32 variables represents the matrices element $[Cu]$ and $[Cv]$ and by solving this system by Gauss-Jordan method, we get:

$$[Cu] = \begin{bmatrix} 4.70440769 & -1.24645913 & -0.25494609 & -0.69254928 \\ 1.25003806 & -0.57992536 & -0.11438408 & -0.30609930 \\ 0.68638424 & -0.31767206 & -0.06437899 & -0.17323139 \\ 0.25182561 & -0.11736944 & -0.0227508 & -0.05562179 \end{bmatrix}$$

$$[Cv] = \begin{bmatrix} -0.70483702 & 1.24746491 & 0.25400126 & 0.69204078 \\ -1.24996062 & 0.57917255 & 0.11517825 & 0.30752743 \\ -0.68633486 & 0.31751944 & 0.06449492 & 0.17333552 \\ -0.25195852 & 0.11702954 & 0.02329369 & 0.05710914 \end{bmatrix}$$

Now, by using the equation (12), we get:

$$[u] = [H]^T \cdot [Cu] \cdot [H]$$

$$\begin{bmatrix} 0.99960733 & 1.32514218 & 1.86050378 & 2.73988636 \\ 0.80326743 & 1.00004431 & 1.32376015 & 1.85667995 \\ 0.68342483 & 0.80556797 & 0.99631441 & 1.32519761 \\ 0.61110075 & 0.68774231 & 0.79087588 & 1.00851549 \end{bmatrix}$$

$$[v] = [H]^T \cdot [Cv] \cdot [H]$$

$$\begin{bmatrix} 0.00033614 & -0.32520812 & -0.86020638 & -1.74034336 \\ 0.19663310 & 0.00007869 & -0.32370972 & -0.85717565 \\ 0.31637759 & 0.19492120 & 0.00325140 & -0.32574974 \\ 0.38849309 & 0.31362409 & 0.20705623 & -0.00772664 \end{bmatrix}$$

Table (1). A comparison between the operational matrix of the Haar wavelets method with exact solution for the air-temperature $u(x,t)$ in the system (1a) and (1b) with: $m=4$

$D_1 = D_2 = a = 1, b_1 = 1, c_1 = 1, b_2 = 1$ and $c_2 = 1$.

| The value of (x) | The value of (t) | The numerical solution of $u(x,t)$ | The exact solution of $u(x,t)$ |
|------------------|------------------|------------------------------------|--------------------------------|
| 0.125 | 0.125 | 0.99960733 | 1.00000000 |
| 0.125 | 0.375 | 1.32514218 | 1.32436064 |
| 0.125 | 0.625 | 1.86050378 | 1.85914091 |
| 0.125 | 0.875 | 2.73988636 | 2.74084454 |
| 0.375 | 0.125 | 0.80326743 | 0.80326533 |
| 0.375 | 0.375 | 1.00004431 | 1.00000000 |
| 0.375 | 0.625 | 1.32376015 | 1.32436064 |
| 0.375 | 0.875 | 1.85667995 | 1.85914091 |
| 0.625 | 0.125 | 0.68342483 | 0.68393972 |
| 0.625 | 0.375 | 0.80556797 | 0.80326533 |
| 0.625 | 0.625 | 0.99631441 | 1.00000000 |
| 0.625 | 0.875 | 1.32519761 | 1.32436064 |
| 0.875 | 0.125 | 0.61110075 | 0.61156508 |
| 0.875 | 0.375 | 0.68774231 | 0.68393972 |
| 0.875 | 0.625 | 0.79087588 | 0.80326533 |
| 0.875 | 0.875 | 1.00851549 | 1.00000000 |

Table (2). A comparison between the operational matrix of the Haar wavelets method with exact solution for the grain-temperature $v(x,t)$ in the system (1a) and (1b) with: $m=4$

$D_1 = D_2 = a = 1, b_1 = 1, c_1 = 1, b_2 = 1$ and $c_2 = 1$.

| The value of (x) | The value of (t) | The numerical solution of $v(x,t)$ | The exact solution of $v(x,t)$ |
|------------------|------------------|------------------------------------|--------------------------------|
| 0.125 | 0.125 | 0.00033614 | 0.00000000 |
| 0.125 | 0.375 | -0.32520812 | -0.32436064 |
| 0.125 | 0.625 | -0.86020638 | -0.85914091 |
| 0.125 | 0.875 | -1.74034336 | -1.74084454 |
| 0.375 | 0.125 | 0.19663310 | 0.19673467 |
| 0.375 | 0.375 | 0.00007869 | 0.00000000 |
| 0.375 | 0.625 | -0.32370972 | -0.32436064 |
| 0.375 | 0.875 | -0.85717565 | -0.85914091 |
| 0.625 | 0.125 | 0.31637759 | 0.31606028 |
| 0.625 | 0.375 | 0.19492120 | 0.19673467 |
| 0.625 | 0.625 | 0.00325140 | 0.00000000 |
| 0.625 | 0.875 | -0.32574974 | -0.32436064 |
| 0.875 | 0.125 | 0.38849309 | 0.38843492 |
| 0.875 | 0.375 | 0.31362409 | 0.31606028 |
| 0.875 | 0.625 | 0.20705623 | 0.19673467 |
| 0.875 | 0.875 | -0.00772664 | 0.00000000 |

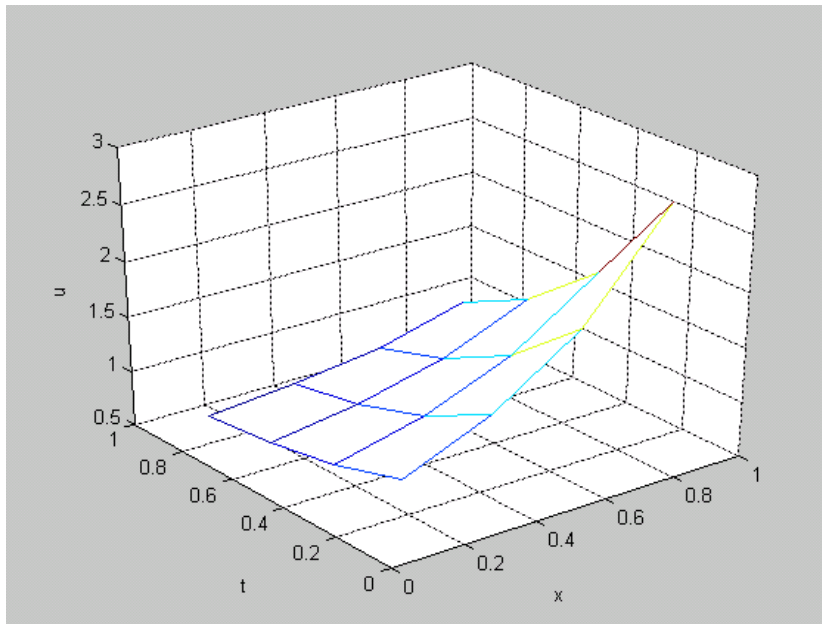


Figure (1). An illustration the numerical solution for the air-temperature $u(x,t)$ in the system (1a) and (1b) by the operational matrix of the Haar wavelets method with: $m=4$ $D_1 = D_2 = a = 1, b_1 = 1, c_1 = 1, b_2 = 1$ and $c_2 = 1$.

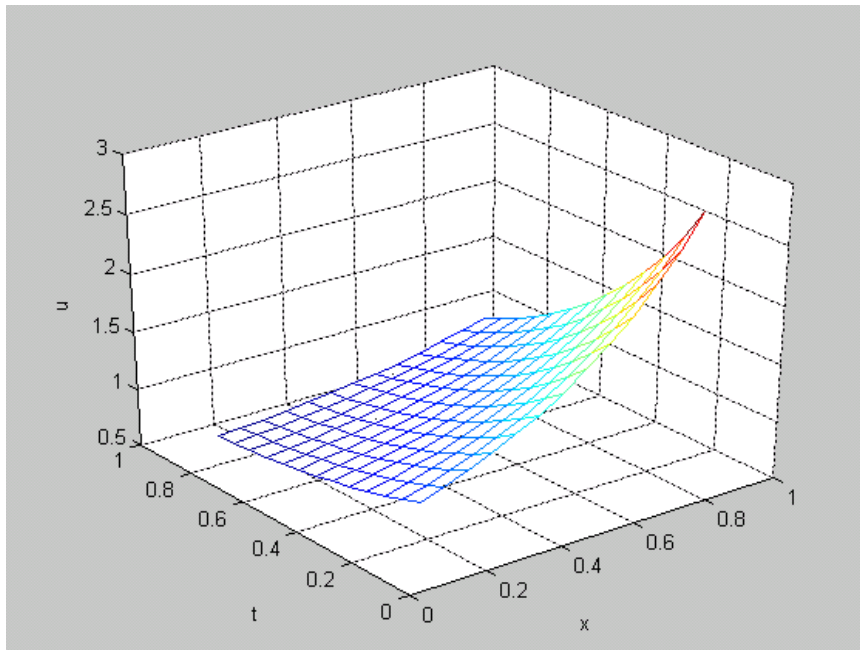


Figure (2). An illustration the exact solution for the air-temperature $u(x,t)$ in the system (1a) and (1b) with: $D_1 = D_2 = a = 1, b_1 = 1, c_1 = 1, b_2 = 1$ and $c_2 = 1$.

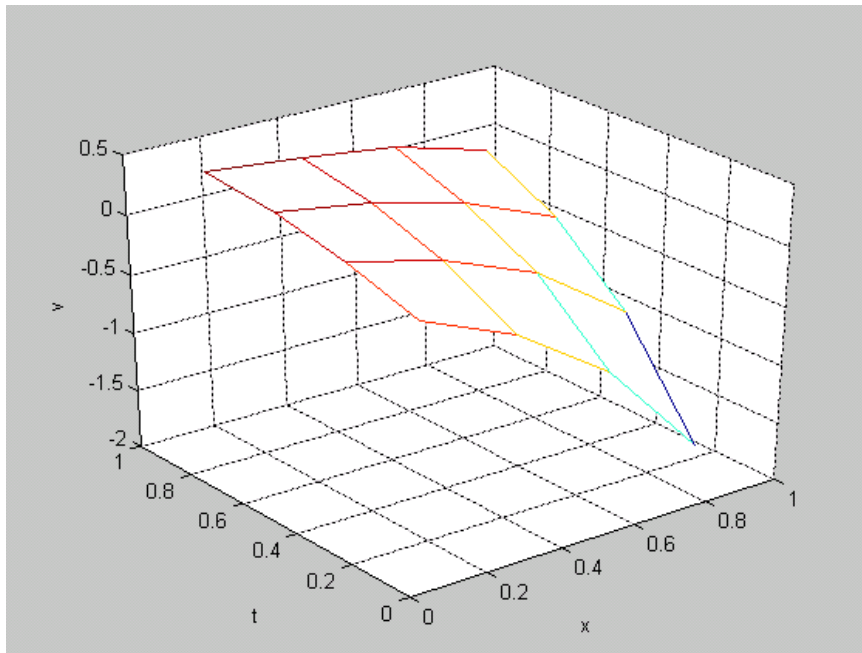


Figure (3). An illustration the numerical solution for the grain-temperature $v(x,t)$ in the system (1a) and (1b) by the operational matrix of the Haar wavelets method with: $m=4$ $D_1 = D_2 = a = 1, b_1 = 1, c_1 = 1, b_2 = 1$ and $c_2 = 1$

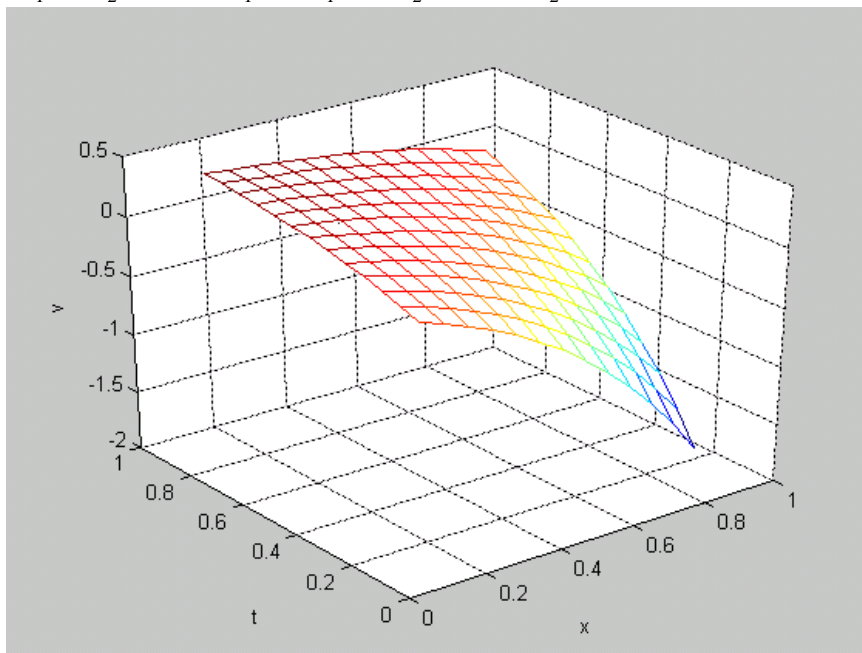


Figure (4). An illustration the exact solution for the grain-temperature $v(x,t)$ in the system (1a) and (1b) with: $D_1 = D_2 = a = 1, b_1 = 1, c_1 = 1, b_2 = 1$ and $c_2 = 1$

6. Conclusions:

In this paper, We are using the operational matrices of the Haar wavelets method for solving linear parabolic reaction-diffusion system with double diffusivity, A numerical method based on the Haar wavelets approach which have the property $(H^{-1} = H^T)$, we compared this results with the exact solution for this system, we found that the operational matrices of the Haar wavelets method is simple in the computation, However, we note that high accuracy of the results in this method in the solution double diffusivity system even in the case of a small number of grid points is using, as shown in the table (1) and (2) and figures (1),(2),(3) and (4) such that the number of grid points (the dimensions of the matrices) are 4×4 , but when the dimension of the matrices are increase then the numerical solution converges towards the exact solution.

The matrices which we got it of the numerical solution are representing of all time steps in the interval $[0,1)$, while in the finite difference method and finite elements method are need to the iteration to get the needed time step, they are complicated and time-consuming. Matlab language is using in find the results and figures draw, it's characteristic of high accuracy and large speed.

REFERENCES

- [1] Aggarwala, B. D. and Nasim, C., (1987), "on the solution of reaction-diffusion equations with double diffusivity", *Internet. J.. Sci Math.*, Vol. 10, No. 1, Pp. 163-172.
- [2] Antic, A. and Hill, J. M., (2000), "A mathematical model for heat transfer grain store microclimates", *Anziam J.* 42(E), Pp. 117-133.
- [3] Antic, A. and Hill, J. M., (2003), "A two stage heat transfer model for the peripheral layers of a grain store", *Journal of applied math. And decision sciences*, Vol. 7, No. 3, Pp. 147-164.
- [4] Chow, T. L. M., (1996), "System of partial differential equations and group methods", Thesis, University of Western Sydney.
- [5] Lepik, U. and Tamme, E., (2007), "Solution of nonlinear Fredholm integral equations via the Haar wavelet method", *Proc. Estonian Acad. Sci. Phys. Math.*, Vol. 56, No. 1, Pp.17-27.
- [6] Polyanin, A. D., (2004), "Exact solutions of nonlinear systems of reaction-diffusion equations and mathematical biology equations", *Theory. Found. Chem. Eng.*, Vol. 37, No. 6.
- [7] Wu, J. L. and Chen, C. H., (2003), "A novel numerical method for solving partial differential equations via operational matrices", *Proceeding of the 7th world multiconference on systemic, Cybernetics and Informatics, Orlando, USA*, Vol. 5, Pp. 276-281.
- [8] Wu, J. L. and Chen, C. H., (2004), "A new operational approach for solving fractional calculus and fractional differential equations numerically", *IEICE Trans. On Fundamentals of Electronics, Communications and Computer Sciences, Special Section on Discrete Mathematics and It's Applications*, Vol. E87-A, No. 5, Pp. 1077-1082.