i-Open Sets in Bitopological Spaces

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ABSTRACT

In this paper, we defined i-open sets and i-star generalized w-closed sets in bitopological spaces (X,τ_1,τ_2) by using the definition of i-open sets in topological space (X,τ) (see[6]). We present some fundamental properties and relations between these classes of sets, further we give examples to explain these relations.

Keywords: i-open sets, bitopological spaces.

المجاميع المفتوحة من النوع–i في الفضاءات التبولوجية الثنائية

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الملخص

في هذا البحث، عرفنا المجاميع المفتوحة من النوع–i والمجاميع المغلقة من النمط– i* معممة من النوع–wفي الفضاءات التبولوجية الثنائية (X,τ_1,τ_2)باستخدام تعريف المجاميع المفتوحة من النوع–i في الفضاء التبولوجي(X,τ)(انظر[6]) . تم إعطاء بعض الخصائص الأساسية والعلاقات بين هذه الأصناف من المجاميع معززة بالأمثلة والبراهين.

الكلمات المفتاحية: المجاميع المفتوحة من النوع–i ، الفضاءات التبولوجية الثنائية.

0. Introduction

Sheik and Sundaram in 2004 [9], introduced g*-closed sets in bitopological spaces. Kannan and Chandrasekhara in 2006 [4], introduced regular star generalized closed sets in bitopological spaces. Mahdi in 2007 [5], introduced the concept of semi-open and semi-closed sets in bitopological spaces. Benchalli, Patil and Rayanagoudar in 2010 [2], introduced w-locally closed sets in bitopological spaces. Sheik and Maragathavalli in 2010 [8], introduced the concept of strongly $\alpha g*-$ closed sets in bitopological spaces. Nagaveni and Rajarubi in 2012 [7], introduced GRW-closed sets and GRW-continuity in bitopological spaces. Mohammed and Askandar In 2012 [6], introduced the concept of i-open sets as: A subset A of a topological space (X,τ) is said to be i-open set[6] if there exists an open set $G\neq\phi$, X such that $\mathrm{A}\subseteq Cl(\mathrm{A}\cap\mathrm{G})$. The complement of an i-open set is called i-closed set, which could entire them together with many other concepts of generalized open sets. The aim of this paper is to introduce the concept of i-open sets in bitopological spaces (X,τ_1,τ_2). This class of sets may be to enter together with other classes of sets in bitopological spaces which have been mentioned above for comparison and to find the similar properties and characterizations. Throughout this work, τ^i is a family of all i-open sets[6] of X). This work consists of two sections. In the first one, we define i-open sets in bitopological spaces and we give many related examples. In the second section, we define i-star generalized *w-closed* sets, i-star generalized *w-open* sets and study their basic properties in bitopological spaces. (X,τ_1,τ_2) denote a bitopological space, where (X,τ_1) and (X,τ_2) are topological spaces. For any subset $A \subseteq X$, $\tau_i - Int(A)$ and $\tau_i - Cl(A)$ denote the interior and closure of a set A with respect to the topology τ_i. A point $x \in X$ is called a condensation point of A [3] if for each $U \in \tau$ with $x \in U$, the set $U \cap A$ is uncountable. A is called $w - closed$ [3] if it contains all its condensation points. The complement of an $w - closed$ set is called $w - open$. The $w - closure$ [3] and $w - \text{int}\,erior$ [3] of A with respect to the topology τ_i, that can be defined in a manner similar to $\tau_i - Cl(A)$ *and* $\tau_i - \text{int}(A)$, respectively, will be denoted by $\tau_i - Cl_w(A)$ *and* $\tau_i - \text{int}_w(A)$, respectively. A^C denotes the complement of A in X.

1. i-Open Sets in Bitopological Spaces.

In this section, we define i-open sets in bitopological spaces by giving many related examples and we study the properties of these sets. Also we define many concepts of generalized open sets in bitopological spaces and we give many related examples.

Definition 1.1. Let (X,τ_1,τ_2) be a bitopological space, a subset A of X is said to be $(\tau_1\tau_2 - i - open\ set)$ if there exists $\tau_1 - open\,set\ U \neq \phi, X$ s.t. $A \subseteq \tau_2 - Cl(A \cap U)$. The complement of $(\tau_1\tau_2 - i - open\ set)$ is called $(\tau_1\tau_2 - i - closed\ set)$.

Definition 1.2. A bitopological space (X,τ_1,τ_2) is said to be Bi-Topologically Extended for i-open sets ($Bi.T.E.I.$) if $(X,\tau_1\tau_2 - i - open\,sets)$ is a topological space. On the other hand, if $(X,\tau_1\tau_2 - i - open\,sets)$ is not a topological space then, (X,τ_1,τ_2) is called non-Bi-Topologically Extended for i-open sets(not $Bi.T.E.I.$). Where, $\tau_1\tau_2 - i - open\,sets$ denote the family of all i-open sets in the bitopological space (X,τ_1,τ_2).

Example 1.3. Let $X = \{a,b,c\}$, $\tau_1 = \{\phi,\{a\},X\}$, $\tau_2 = \{\phi,\{a\},\{a,b\},X\}$.
$\tau_1 - open\ sets\,are: \phi,\{a\},X$. $\tau_2 - closed\ sets\,are: \phi,\{b,c\},\{c\},X$.
$\{a\} \subset (\tau_2 - Cl(\{a\} \cap \{a\}) = X)$, $\{a,b\} \subset (\tau_2 - Cl(\{a,b\} \cap \{a\}) = X)$
$\{a,c\} \subset (\tau_2 - Cl(\{a,c\} \cap \{a\}) = X)$.
Then, $\{a\},\{a,b\},\{a,c\}$ are $\tau_1\tau_2 - i - open\,sets$.
But, $\{b\},\{c\},\{b,c\}$ are not $\tau_1\tau_2 - i - open\,sets$ because there is no existence $\tau_1 - open\,set\ U$ s.t. $\{b\} \subset (\tau_2 - Cl(\{b\} \cap U))$, $\{c\} \subset (\tau_2 - Cl(\{c\} \cap U))$
$\{b,c\} \subset (\tau_2 - Cl(\{b,c\} \cap U))$ Therefore, $\tau_1\tau_2 - i - open\ sets = \{\phi,\{a\},\{a,b\},\{a,c\},X\}$.
$\tau_1\tau_2 - i - closed\ sets = \phi,\{b,c\},\{c\},\{b\},X$
Where, $(X,\tau_1\tau_2 - i - open\,sets)$ is a topological space. Then, (X,τ_1,τ_2) is a $Bi.T.E.I.$ space.

Example 1.4. Let $X = \{a,b,c,d\}$, $\tau_1 = \{\phi,\{a\},\{b,c,d\}X\}$, $\tau_2 = \{\phi,\{a\},\{c\},\{a,c\},X\}$.

$\tau_1 - open\ sets\ are: \phi, \{a\}, \{b,c,d\}, X$.

$\tau_2 - closed\ sets\ are: \phi, \{b,c,d\}, \{a,b,d\}, \{b,d\}, X$.

By the same way, in Example 1.3, we have:

$\tau_1\tau_2 - i - open\ sets = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{b,c\}, \{b,d\}, \{c,d\},$

$\{b,c,d\}, \{a,b\}, \{a,d\}, \{a,b,d\}, X\}$

$\tau_1\tau_2 - i - closed\ sets = \{\phi, \{b,c,d\}, \{a,c,d\}, \{a,b,d\}, \{a,c,d\}, \{a,d\},$

$\{a,c\}, \{a,b\}, \{a\}, \{c,d\}, \{b,c\}, \{c\}, X\}$.

Where, $(X, \tau_1\tau_2 - i - open\,sets)$ is not a topological space. Then, (X, τ_1, τ_2) is not $Bi.T.E.I.$ space.

Definition 1.5. Let *(X, τ^i)* be a topological space and let *A* be a subset of X . Recall that the intersection of all i-closed sets containing *A* is called i-closure of *A*[6], denoted by *$Cl_i(A)$*: *$Cl_i(A)$*= $\bigcap_{i\in\Lambda} F_i$. $A \subseteq F_i \quad \forall i$ where, F_i is i-closed set $\forall i$ in a topological space *(X, τ^i)*. *$Cl_i(A)$* is the smallest i-closed set containing *A*.

Definition 1.6. Let *(X, τ^i)* be a topological space and let *A* be a subset of X. Recall that the union of all i-open sets contained in *A* is called i-Interior of *A*[6], denoted by *$Int_i(A)$*. *$Int_i(A)$*= $\bigcup_{i\in\Lambda} I_i \quad I_i \subseteq A \quad \forall i$. Where, I_i is i-open set $\forall i$ in a topological space *(X, τ^i)*. *$Int_i(A)$* is the largest i-open set contained in *A*.

Theorem 1.7. Every $\tau_1 - open\ set$ is (X, τ_1, τ_2) in $i - open\,set$

Or ($\tau_1 \subset (\tau_1\tau_2 - i -$ open sets)).

Proof Let X be a finite non empty set. Let $\tau_1 = \{\phi, A_1, A_2, ... A_n, X\}$, $\tau_2 = \{\phi, B_1, B_2, ..., B_n, X\}$.

Where, $A_i \subset X, B_i \subset X \ \forall i$.

$\tau_1 - open\ sets\ are: \phi, A_1, A_2, ... A_n, X$.

$\tau_2 - closed\ sets\ are: \phi, X - B_1, X - B_2, ..., X - B_n, X$.

$\tau_2 - Cl(A_i \cap A_i) = \bigcap_{A_i \cap A_i \subset F} F$, where F is $\tau_2 - closed\ set$.

At least, X is a $\tau_2 - closed\ set$ contains $A_i \cap A_i \ \forall i$.

Hence, $\tau_2 - Cl(A_i \cap A_i) = \bigcap_{A_i \cap A_i \subset F} F = X$.

Therefore, $A_i \subset (\tau_2 - Cl(A_i \cap A_i) = \bigcap_{A_i \cap A_i \subset F} F = X) \ \forall i$.

Then, ($\tau_1 \subset (\tau_1\tau_2 - i -$ open sets)).

The converse of Theorem 1.7 is not true. Indeed, in Example 1.4 $\{b,c\}$ is $\tau_1\tau_2 - i - open\,set$, but is not $\tau_1 -$ open set.■

Definition 1.8. Let (X, τ) be a topological space, recall that extension τ^i [6] is the family of all i-open subsets of space X.

Remark 1.9. [6] (X, τ^i) need not to be a topological space**.**

Definition 1.10. [6] A topological space (X, τ) is said to be Topologically Extended for i-open sets (shortly T.E.I) if and only if
(X, τ^i) is a topological space. Otherwise is called not T.E.I.

Theorem 1.11. [6] Let X be a non-empty finite set and let $\tau = \{\phi, A, X\}$ where, A is a subset of X and containing only one element. Then, *(X, τ)* is T.E.I. (i.e. *(X, τ^i)* is a topological space).

Corollary 1.12. Let (X, τ_1, τ_2) be a bitopological space and let (X, τ_1) be a (T.E.I.) topological space as like as in Theorem 1.11, let $\tau_2 = \tau_1^i$ where, τ_1^i is the family of all i-open sets in a topological space (X, τ_1), then,$. = \tau_2 \ i-open sets \ \tau_1\tau_2 -$

Proof Suppose that $X=\{x_1,x_2,\dots\dots\dots,x_n\}$ and $\tau_1 = \{\phi, \{x_1\}, X\}$.

$\tau_1 - open \ sets \ are: \phi, \{x_1\}, X.$

By definition of i-open sets, we have:

$\tau_1^i = \{\phi, \{x_1\}, \{x_1,x_2\}, \{x_1,x_3\}, \dots\dots\dots, \{x_1,x_n\}, \{x_1,x_2,x_3\}, \{x_1,x_2,x_4\}, \dots\dots,$
$\{x_1,x_2,x_n\}, \dots\dots\dots\dots\dots, \{x_1,x_3,x_4,\dots\dots\dots,x_n\}, \{x_1,x_2,\dots\dots\dots,x_n\} = X\}.$

Since, $\tau_2 = \tau_1^i$ then $\tau_2 - closed \ sets \ are: \{x_1,x_2,\dots,x_n\} = X$,
$\{x_2,x_3,x_4,\dots,x_n\}, \{x_3,x_4,\dots,x_n\}, \{x_2,x_4,\dots,x_n\}, \dots\dots, \{x_2,\dots,x_{n-1}\}, \{x_4,\dots,x_n\},$
$\{x_3,x_5,\dots,x_n\}, \dots, \{x_3,\dots,x_{n-1}\}, \dots, \{x_2\}, \phi$.

Since, $\{x_1\}$ is the alone $\tau_1 - open \ set \neq \phi, X$ and the intersection between $\{x_1\}$ and the sets $\{x_2\}, \{x_3\}, \dots, \{x_n\}, \dots, \{x_2,x_3\}, \dots, \{x_2,x_n\}, \{x_2,x_3,x_4\}, \dots,$
$\{x_2,x_3,x_n\}, \dots, \{x_3,x_4,x_n\}, \dots, \{x_{n-2},x_{n-1},x_n\}$ which does not contain $\{x_1\}$, equal to ϕ and by the same way in Theorem 1.11 we have:

$\tau_1\tau_2 - i - open sets = \tau_2$ where, $\tau_2 = \tau_1^i$. ■

Example 1.13. Let $X = \{a, b, c\}$,

$\tau_1 = \{\phi, \{a\}, X\}$, $\tau_2 = \tau_1^i = \{\phi, \{a\}, \{a,b\}, \{a,c\}, X\}$

$\tau_2 - closed \ sets \ are: \phi, \{b,c\}, \{c\}, \{b\}, X.$

By the same way of the examples mentioned above, we have:

$\tau_1\tau_2 - i - open sets = \tau_2$

Definition 1.14. A set A of a bitopological space (X, τ_1, τ_2) is called:

1. $\tau_1\tau_2 - generalized \ closed \ set$ $(\tau_1\tau_2 - g - closed \ set)$ [3]
   if $\tau_2 - Cl(A) \subseteq U$ where $A \subseteq U$ and $U \subseteq X$ is $\tau_1 - open \ set$.
2. $\tau_1\tau_2 - g - open \ set$ [3] if $X - A$ is $\tau_1\tau_2 - g - closed$.
3. $\tau_1 - closed \ set$. is $\subseteq X$ $F \subseteq A$ where $F \subseteq \tau_2 - Int_i(A)$ if $\tau_1\tau_2 - g \ i - open \ set$
4. $\tau_1\tau_2 - g \ i - open$. is $X - A$ if $\tau_1\tau_2 - g \ i - closed \ set$
5. $\tau_1\tau_2 - i - star \ genralzed \ closed \ set$ $(\tau_1\tau_2 - i^* g - closed \ set)$
   if $\tau_2 - Cl(A) \subseteq U$ where $A \subseteq U$ and $U \subseteq X$ is $i - open set$. $\tau_1 -$
6. $\tau_1\tau_2 - i - star \ genralzed \ open set$ $(\tau_1\tau_2 - i^* g - open \ set)$ if $X - A$ is $\tau_1\tau_2 - i^* g - closed$.
7. $\tau_1\tau_2 - genralzed \ w - closed \ set$ ($\tau_1\tau_2 - gw - closed \ set$)[1]
   if $\tau_2 - Cl_w(A) \subseteq U$ where $A \subseteq U$ and $U \subseteq X$ is $\tau_1 - open \ set$.
8. $\tau_1\tau_2 - genralzed \ w - open \ set$ ($\tau_1\tau_2 - gw - open \ set$)[1] if $X - A$ is $\tau_1\tau_2 - gw - closed$.

In the following example X is a finite set.

Example 1.15. Let $X = \{a,b,c\}$, $\tau_1 = \{\phi,\{a\},X\}$ $\tau_2 = \{\phi,\{a\},X\}$.

From definitions mentioned above we have:

$\tau_1 - open\,sets : \phi,\{a\},X$, $\tau_1 - closed\ sets : \phi,\{b,c\},X$.

$\tau_1 - w - closed\ sets : \phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},X$.

$\tau_1 - w - open\,sets : \phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},X$.

$\tau_1 - i - open\,sets : \phi,\{a\},\{a,b\},\{a,c\},X$.

$\tau_1 - i - closed\ sets : \phi,\{b,c\},\{c\},\{b\},X$.

$\tau_2 - open\,sets : \phi,\{a\},X$, $\tau_2 - closed\ sets : \phi,\{b,c\},X$.

$\tau_2 - w - closed\ sets : \phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},X$.

$\tau_2 - w - open\,sets : \phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},X$.

$\tau_2 - i - open\,sets : \phi,\{a\},\{a,b\},\{a,c\},X$.

$\tau_2 - i - closed\ sets : \phi,\{b,c\},\{c\},\{b\},X$.

$\tau_1\tau_2 - g - closed\ sets : \phi,\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},X$.

$\{a\}\,is\ not\ \ \tau_1\tau_2 - g - closed\ set\ \ because\ \tau_2 - Cl(\{a\}) = X \subseteq X$

$but\,\tau_2 - Cl(\{a\}) = X \not\subset \{a\}\ (definition(1.14)(1))$.

$\tau_1\tau_2 - g - open\,sets : \phi,\{a,c\},\{a,b\},\{c\},\{b\},\{a\},X$.

$but\ \{b,c\}\,is\,not\ \tau_1\tau_2 - g - open\,set\,because\ \{a\}^C = \{b,c\}$

$and\ \{a\}\ is\ not\ \tau_1\tau_2 - g - closed\ set(difinition1.14(2))$

$\tau_1\tau_2 - gi - open\,sets : \phi,X$, $\tau_1\tau_2 - gi - closed\ sets : \phi,X$.

$\tau_1\tau_2 - i * g - closed\ sets : \phi,\{b,c\},X$, $\tau_1\tau_2 - i * g - open\,sets : \phi,\{a\},X$.

$\tau_1\tau_2 - gw - closed\ sets : \phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},X$.

$\tau_1\tau_2 - gw - open\,sets : \phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},X$.

In the following example X is an infinite set.

Example 1.16. Let $X = R$, $\tau_1 = \{\phi, R-Q, R\}$ $\tau_2 = \{\phi, Q, R\}$. Where, R is the set of real numbers, Q is the set of rational numbers and $R-Q$ is the set of irrational numbers.

From definitions mentioned above, we have:

$\tau_1 - open\,sets : \phi, R-Q, R$, $\tau_1 - closed\ sets : \phi, Q, R.$

$\tau_1 - w - closed\ sets : \phi, R-Q, Q, R,\ other\ sets \subseteq R\,which\ \ satisfies\ \ the$
$definition\,of\ w - closed\ sets.$

$\tau_1 - w - open\,sets : \phi, R-Q, Q, R,\ other\ sets \subseteq R\,which\ \ are\ \ the$
$complements\ \ of\ \tau_1 - w - closed\ sets.$

$\tau_1 - i - open\,sets : \phi, R-Q, R,\ other\ sets \subseteq R\,which\ \ it\ \ satisfies\,the$
$definition\ \ of\ i - open\,sets. Q\ \ is\ \ not\,\tau_1 - i - open\,set.$

$\tau_1 - i - closed\ sets : \phi, Q, R,\ other\ sets \subseteq R\,which\ \ are\ \ the$
$complements\,of\ \tau_1 - i - open\,sets. R-Q\ \ is\ \ not\,\tau_1 - i - closed\ set.$

$\tau_2 - open\,sets : \phi, Q, R$, $\tau_2 - closed\ sets : \phi, R-Q, R.$

$\tau_2 - w - closed\,sets : \phi, R-Q, Q, R,\ other\,sets \subseteq R\,which\ \ it\ \ satisfies\ \ the$
$definition\ \ of\ w - closed\,sets.$

$\tau_2 - w - open\, sets: \phi, R-Q, Q, R,\ other\, sets \subseteq R$ *which are the complements of* $\tau_2 - w - closed\ sets.$

$\tau_2 - i - open\, sets: \phi, Q, R,\ other\, sets \subseteq R$ *which it satisfies the definition of* $i - open\, sets.$

$\tau_2 - i - closed\ sets: \phi, R-Q, R,\ other\, sets \subseteq R$ *which are the complements of* $\tau_2 - i - open\, sets.$

$\tau_1\tau_2 - g - closed\ sets: \phi, R-Q, Q, R, other\ sets \subseteq R$ *which it satisfies the definition of* $\tau_1\tau_2 - g - closed\ sets.$

$\tau_1\tau_2 - g - open\, sets: \phi, R-Q, Q, R, other\ sets \subseteq R$ *which are the complements of* $\tau_1\tau_2 - g - closed\ sets.$

$\tau_1\tau_2 - gi - open\, sets: \phi, Q, R, other\ sets \subseteq R$ *which it satisfies the definition of* $\tau_1\tau_2 - gi - open\, sets. R-Q$ *is not* $\tau_1\tau_2 - gi - open\, set.$

$\tau_1\tau_2 - gi - closed\ sets: \phi, R-Q, R, other\ sets \subseteq R$ *which are the complements of* $\tau_1\tau_2 - gi - open\, sets.$ Q *is not* $\tau_1\tau_2 - gi - closed\ set.$

$\tau_1\tau_2 - i^*g - closed\ sets: \phi, R-Q, Q, R, other\ sets \subseteq R$ *which it satisfies the definition of* $\tau_1\tau_2 - i^*g - closed\ sets.$

$\tau_1\tau_2 - i^*g - open\, sets: \phi, R-Q, Q, R, other\ sets \subseteq R$ *which are the complements of* $\tau_1\tau_2 - i^*g - closed\ sets.$

$\tau_1\tau_2 - gw - closed\ sets: \phi, R-Q, Q, R, other\ sets \subseteq R$ *which it satisfies the definition of* $\tau_1\tau_2 - gw - closed\ sets.$

$\tau_1\tau_2 - gw - open\, sets: \phi, R-Q, Q, R, other\ sets \subseteq R$ *which are the complements of* $\tau_1\tau_2 - gw - closed\ sets.$

2. i-Star Generalized w-Closed and i-Star Generalized w-Open Sets in Bitopological Spaces.

Throughout this section, we define i-star generalized w-closed, i-star generalized w-open sets and study their basic properties in bitopological spaces.

Definition 2.1. A set A of a bitopological space (X, τ_1, τ_2) is said to be $\tau_1\tau_2 - i - star\ genralized\ w - closed\ set$ ($\tau_1\tau_2 - i^*g\,w - closed\ set$), if $\tau_2 - Cl_w(A) \subseteq U$ where, $A \subseteq U$ and $U \subseteq X$ is a $\tau_1 - i - open\, set.$

In Example 1.15, we have:

$\tau_1\tau_2 - i^*gw - closed\ sets: \phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, X.$

in Example 1.16, we have:

$\tau_1\tau_2 - i^*gw - closed\ sets: \phi, R-Q, Q, R, other\ sets \subseteq R$ *which it satisfies the definition of* $\tau_1\tau_2 - i^*gw - closed\ sets.$

Remark 2.2. [6] Every open set in a topological space (X, τ) is i-open .

Theorem 2.3. Let (X, τ_1, τ_2) be a bitopological space and $A \subseteq X$ then, the followings are true:

1. If A is $\tau_2 - w - closed$ then, A is $\tau_1\tau_2 - i^*gw - closed$.

2. If A is $\tau_1 - i - open$ and $\tau_1\tau_2 - i * gw - closed$ then, A is $\tau_2 - w - closed$.
3. If A is $\tau_1\tau_2 - i * gw - closed$ then, A is $\tau_1\tau_2 - gw - closed$.

Proof

1. Suppose that A is $i - open.$ $\tau_1 -$ are $U \subseteq X$ and $A \subseteq U$. Let $\tau_2 - w - closed$ then $\tau_2 - Cl_w(A) = A \subseteq U$.

Therefore, A is $\tau_1\tau_2 - i * gw - closed$.

2. Suppose that A is $\tau_1 - i - open$ and $\tau_1\tau_2 - i * gw - closed$. Let $A \subseteq A$ and A is $\tau_1 - i - open$. Then, $\tau_2 - Cl_w(A) \subseteq A$. Therefore, $\tau_2 - Cl_w(A) = A$. Then, A is $\tau_2 - w - closed$.

3. Suppose that A is $\tau_1\tau_2 - i * gw - closed$. Let $A \subseteq U$ and. Since, $\tau_1 - open$ is $U \subseteq X$ is A. Then, $\tau_2 - Cl_w(A) \subseteq U$ (Remark 2.2), we have X in $i - open$ $\tau_1 -$ is U . ■ $\tau_1\tau_2 - gw - closed$

Theorem 2.4. Let (X, τ_1, τ_2) be a bitopological space, then every $\tau_1\tau_2 - i * g - closed\ set$ in X is $\tau_1\tau_2 - i * gw - closed$.

Proof Suppose that A is $\tau_1\tau_2 - i * g - closed\ set$, we have $\tau_2 - Cl(A) \subseteq U$, where $A \subseteq U$ and $U \subseteq X$ are $\tau_1 - i - open\ set$.

Since, $\tau_2 - Cl_w(A) \subseteq \tau_2 - Cl(A)$,

we have $\tau_2 - Cl_w(A) \subseteq \tau_2 - Cl(A) \subseteq U$.

Therefore, A is $\tau_1\tau_2 - i * gw - closed$.■

Remark 2.5. The converse of Theorem 2.4 is not true. Indeed, in Example 1.15, A = {a, b} is $\tau_1\tau_2 - i * gw - closed$ set, but is not $\tau_1\tau_2 - i * g - closed$.

$$\tau_1\tau_2 - i * g - closed\ set \;\underset{\not\Leftarrow}{\Rightarrow}\; \tau_1\tau_2 - i * gw - closed$$

Theorem 2.6. If A is $\tau_1\tau_2 - i * gw - closed\ set$ in X and $A \subseteq B \subseteq \tau_2 - Cl_w(A)$, then B is $\tau_1\tau_2 - i * gw - closed\ set$.

Proof Suppose that A is $\tau_1\tau_2 - i * gw - closed\ set$ in X and $A \subseteq B \subseteq \tau_2 - Cl_w(A)$. Let $B \subseteq U$ and U is $\tau_1 - i - open\ set$. Then, $A \subseteq U$ .Since, A is $\tau_1\tau_2 - i * gw - closed\ set$,we have $\tau_2 - Cl_w(A) \subseteq U$. Since, $B \subseteq \tau_2 - Cl_w(A), \tau_2 - Cl_w(B) \subseteq \tau_2 - Cl_w(A) \subseteq U$. Hence, B is $\tau_1\tau_2 - i * gw - closed$.■

Theorem 2.7. If A and B are $\tau_1\tau_2 - i * gw - closed\ sets$ then, so is $A \cup B$.

Proof Suppose that A and B are $\tau_1\tau_2 - i * gw - closed\ sets$. Let $U \subseteq X$ be $\tau_1 - i - open\ set$ and $(A \cup B) \subseteq U$. Then, $A \subseteq U$ and $B \subseteq U$. Since, A and $\boldsymbol{B}$ are $\tau_1\tau_2 - i * gw - closed\ sets$, we have $\tau_2 - Cl_w(A) \subseteq U$ and $\tau_2 - Cl_w(B) \subseteq U$. Then, $\tau_2 - Cl_w(A \cup B) \subseteq U$. Therefore, $A \cup B$ is $\tau_1\tau_2 - i * gw - closed\ set$.■

Theorem 2.8. Let (X, τ_1, τ_2) be a bitopological space and $A \subseteq X$ then, the following are true:

1. If A is $\tau_2-closed$ then, A is $\tau_2-w-closed$.
2. If A is $\tau_1\tau_2-i*g-closed$ then,. $\tau_1\tau_2-g-closed$ is A
3. If A is $\tau_1\tau_2-g-closed$ then, A is $\tau_1\tau_2-gw-closed$.

Proof

1. Suppose that A is $\tau_2-closed$. Then $\tau_2-Cl\ (A)=A$. Since, $\tau_2-Cl_w(A)\subseteq\tau_2-Cl\ (A)=A$, we have $\tau_2-Cl_w(A)=A$. Therefore, A is $\tau_2-w-closed$.

2. Suppose that A is $A\subseteq U$. Let $\tau_1\tau_2-i*g-closed$ and. τ_1-open is $U\subseteq X$ Therefore, $\tau_2-Cl\ (A)\subseteq U$. Then, A is $\tau_1\tau_2-g-closed$.

3. Suppose that A is $\tau_1\tau_2-g-closed$. Let $A\subseteq U$ and. τ_1-open are $U\subseteq X$ Therefore, $\tau_2-Cl\ (A)\subseteq U$.

Since $\tau_2-Cl_w(A)\subseteq\tau_2-Cl\ (A)\subseteq U$, we have $\tau_2-Cl_w(A)\subseteq U$. Then, A is $\tau_1\tau_2-gw-closed$. ■

Remark 2.9. The converses of Theorem 2.8 are not true. Indeed, In Example 1.15, *A = {a, c}* is $\tau_2-w-closed\ set$, but is not $\tau_2-closed$, *A={a,c}* is $\tau_1\tau_2-g-closed\ set$, but is not $\tau_1\tau_2-i*g-closed\ set$. Also {a} is $\tau_1\tau_2-gw-closed$ set but, is not $\tau_1\tau_2-g-closed$.

$$\tau_2-closed\ set \;\underset{\nLeftarrow}{\Rightarrow}\; \tau_2-w-closed$$

$$\tau_1\tau_2-i*g-closed\ set \;\underset{\nLeftarrow}{\Rightarrow}\; \tau_1\tau_2-g-closed$$

$$\tau_1\tau_2-g-closed\ set \;\underset{\nLeftarrow}{\Rightarrow}\; \tau_1\tau_2-gw-closed$$

Definition 2.10. A set A of a bitopological space (X,τ_1,τ_2) is said to be $\tau_1\tau_2-i-star\ genralzed\ \ w-open\,set\ (\tau_1\tau_2-i*gw-open\,set)$, if $X-A$ is $\tau_1\tau_2-i*gw-closed\ set.$

In Example 1.15. we have:

$\tau_1\tau_2-i*gw-open\,sets:\phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{b,c\},X.$

Also, in Example 1.16 we have:

$\tau_1\tau_2-i*gw-open\,sets:\phi,R-Q,Q,R,other\ sets\subseteq R\ which\ \ it\ satisfies$

$the\ \ definition\ \ \ of\ \tau_1\tau_2-i*gw-open\,sets.$

Theorem 2.11. A set A is $\tau_1\tau_2-i*gw-open\,set$ if and only if $F\subseteq\tau_2-Int_w(A)$, where $F\subseteq A$ and $F\subseteq X$ is. $i-closed\ set\ \tau_1-$

Proof Suppose that A is $\tau_1\tau_2-i*gw-open\,set$. Suppose that $F\subseteq X$ is τ_1- F^C. Then $F\subseteq A$ and $i-closed\,set$ is is A^C. Since, $A^C\subseteq F^C$ and $i-open\ \tau_1-$

. Since, $\tau_2 - Cl_w(A^C) \subseteq F^C$, we have $\tau_1\tau_2 - i*gw - closed\ set$. $F \subseteq \tau_2 - Int_w(A)$, we have $\tau_2 - Cl_w(A^C) = [\tau_2 - Int_w(A)]^C$

Conversely, suppose that $F \subseteq \tau_2 - Int_w(A)$ where $F \subseteq A$ and $F \subseteq X$ is $\tau_1 -$ F^C and $A^C \subseteq F^C$. Then, $i - closed\ set$ is and $F \subseteq \tau_2 - Int_w(A)$. Since, $i - open\ \tau_1 -$ is A^C. Then, $\tau_2 - Cl_w(A^C) \subseteq F^C$, we have $\tau_2 - Cl_w(A^C) = [\tau_2 - Int_w(A)]^C$ $\tau_1\tau_2 - i*gw - closed\ set$. Therefore,. ■ $\tau_1\tau_2 - i*gw - open\ set$ is A

Theorem 2.12. If A and B are separated $\tau_1\tau_2 - i*gw - open\ sets$, then so is $A \bigcup B$.

Proof Suppose that A and B are $\tau_1\tau_2 - i*gw - open\ sets$. Let $F \subseteq X$ be $\tau_1 - i - closed\ set$ and $F \subseteq (A \bigcup B)$. Since A and B are separated sets, we have $\tau_1 - Cl(A) \bigcap B = A \bigcap \tau_1 - Cl(B) = \phi$.

Also, $\tau_2 - Cl(A) \bigcap B = A \bigcap \tau_2 - Cl(B) = \phi$. Then, $F \bigcap \tau_2 - Cl(A) \subseteq (A \bigcup B) \bigcap \tau_2 - Cl(A) = A$. By the same way, we have $F \bigcap \tau_2 - Cl(B) \subseteq B$. Since, $F \subseteq X$ is $\tau_1 - i - closed\ set$, we have $F \bigcap \tau_1 - Cl(A)$ and $F \bigcap \tau_1 - Cl(B)$ are $\tau_1 - i - closed\ sets$. Since, A and B are $\tau_1\tau_2 - i*gw - open\ sets$, we have $F \bigcap \tau_2 - Cl(A) \subseteq \tau_2 - Int_w(A)$ and $F \bigcap \tau_2 - Cl(B) \subseteq \tau_2 - Int_w(B)$. Now

$$F = F \bigcap (A \bigcup B) \subseteq (F \bigcap \tau_2 - Cl(A)) \cup (F \bigcap \tau_2 - Cl(B))$$
$$\subseteq \tau_2 - Int_w(A \bigcup B).$$

Therefore, $A \bigcup B$ is $\tau_1\tau_2 - i*gw - open\ set$. ■

Theorem 2.13. If A and B are $\tau_1\tau_2 - i*gw - open\ sets$ then so is $A \bigcap B$.

Proof Suppose that A and B are $\tau_1\tau_2 - i*gw - open\ sets$. Let $F \subseteq X$ be $\tau_1 - i - closed\ set$ and $F \subseteq (A \bigcap B)$, we have $F \subseteq A$ and $F \subseteq B$. Since, A and B are $\tau_1\tau_2 - i*gw - open\ sets$, we have $F \subseteq \tau_2 - Int_w(A)$ and $F \subseteq \tau_2 - Int_w(B)$. Then $F \subseteq \tau_2 - Int_w(A \bigcap B)$.

Therefore, $A \bigcap B$ is $\tau_1\tau_2 - i*gw - open\ set$. ■

Theorem 2.14. If A is $\tau_1\tau_2 - i*gw - open\ set$ in X and $\tau_2 - Int_w(A) \subseteq B \subseteq A$, then ***B*** is $\tau_1\tau_2 - i*gw - open\ set$.

Proof Suppose that A is $\tau_1\tau_2 - i*gw - open\ set$ in X and $\tau_2 - Int_w(A) \subseteq B \subseteq A$. Let $F \subseteq X$ be $\tau_1 - i - closed\ set$ and $F \subseteq B$. Since, $F \subseteq B$ and $B \subseteq A$, we have $F \subseteq A$. Since, A is $\tau_1\tau_2 - i*gw - open\ set$, we have $F \subseteq \tau_2 - Int_w(A)$ and Since, $\tau_2 - Int_w(A) \subseteq B$, we have $\tau_2 - Int_w(A) \subseteq \tau_2 - Int_w(B)$. Then, $F \subseteq \tau_2 - Int_w(B)$. Therefore, ***B*** is $\tau_1\tau_2 - i*gw - open$. ■

Theorem 2.15. Let (X, τ_1, τ_2) be a bitopological space and $A \subseteq X$ then the followings are true:

1. If A is $\tau_2 - w - open$, then A is $\tau_1\tau_2 - i*gw - open$.
2. If A is $\tau_1 - i - closed$ and $\tau_1\tau_2 - i*gw - open$, then A is $\tau_2 - w - open$.
3. If A is $\tau_1\tau_2 - i*gw - open$ then A is $\tau_1\tau_2 - gw - open$.

4. If A is $\tau_1\tau_2 - i*g - open$ then A is $\tau_1\tau_2 - i*gw - open$.
5. If A is $\tau_2 - open$ then A is $\tau_2 - w - open$. [3]
6. If A is $\tau_1\tau_2 - i*g - open$ then A is $\tau_1\tau_2 - g - open$.
7. If A is $\tau_1\tau_2 - g - open$ then A is $\tau_1\tau_2 - gw - open$.

Proof

1. Suppose that A is $\tau_2 - w - open$. We have A^C is $\tau_2 - w - closed$. Then,is A^C (Theorem2.3(1)) . $\tau_1\tau_2 - i*gw - closed$
   Therefore, A is $\tau_1\tau_2 - i*gw - open$.
2. Suppose that A is $\tau_1 - i - closed$ and $\tau_1\tau_2 - i*gw - open$. Then, A^C is $\tau_1 - i - open$ and $\tau_1\tau_2 - i*gw - closed$.
   Then,. $\tau_2 - w - open$ is A (Theorem2.3(2)). Therefore , $\tau_2 - w\ closed$ is A^C
3. Suppose that A is $\tau_1\tau_2 - i*gw - open$. Then, A^C is $\tau_1\tau_2 - i*gw - closed$, hence A^C is $\tau_1\tau_2 - gw - closed$ (Theorem2.3(3)). Therefore, A is $\tau_1\tau_2 - gw - open$.
4. Suppose that A is $\tau_1\tau_2 - i*g - open$. Then, A^C is $\tau_1\tau_2 - i*g - closed$, hence A^C is $\tau_1\tau_2 - i*gw - closed$ (Theorem 2.4). Therefore, A is $\tau_1\tau_2 - i*gw - open$.
5. (see [3]).
6. Suppose that A isis A^C , hence $\tau_1\tau_2 - i*g - closed$ is A^C . Then, $\tau_1\tau_2 - i*g - open$ $\tau_1\tau_2 - g - closed$ (Theorem 2.8(2))
   Therefore, A is $\tau_1\tau_2 - g - open$.
7. Suppose that A is $\tau_1\tau_2 - g - open$. Then, A^C is $\tau_1\tau_2 - g - closed$, hence A^C is $\tau_1\tau_2 - gw - closed$ (Theorem 2.8(3)).
   Therefore, A is $\tau_1\tau_2 - gw - open$. ■

Remark 2.16. The converses of Theorem 2.15(4)(5)(6)(7) are not true. Indeed, In Example 1.15 , *A = {b}* is $\tau_1\tau_2 - i*gw - open$, but it is not= $A$and $\tau_1\tau_2 - i*g - open$ *{b}* is $\tau_2 - w - open\ set$, but it is not $\tau_2 - open$. Also, *A*={b} is $\tau_1\tau_2 - g - open\ set$, but it is not $\tau_1\tau_2 - i*g - open\ set$.
{b, c} is $\tau_1\tau_2 - gw - open$ set but it is not $\tau_1\tau_2 - g - open$.

$\tau_1\tau_2 - i*g - open\ set \ \rightleftarrows\ \tau_1\tau_2 - i*gw - open$ (converse fails)

$\tau_2 - open\ set \ \rightleftarrows\ \tau_2 - w - open$ (converse fails)

$\tau_1\tau_2 - i*g - open\ set \ \rightleftarrows\ \tau_1\tau_2 - g - open$ (converse fails)

$\tau_1\tau_2 - g - open\ set \ \rightleftarrows\ \tau_1\tau_2 - gw - open$ (converse fails)

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