Stability Analysis for Inclined Channel by an Angle 30◦ with The Presence of Magnetic Field

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ABSTRACT

In this research, we study the stability of a system of partial differential equations which represents fluid flow in an inclined channel and under the influence of a magnetic field perpendicular to the plane of the channel and the presence of radiation coefficients and when the channel has an inclination angle $\phi = 30$.

Keywords: Inclination Channel, Magnetic field, and stability**.**

تحليل الثبات للقناة المائلة بزاوية 30 درجة مع وجود المجال المغناطيسي

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الملخص

لقد تم في هذا البحث دراسة استقرارية النظام للمعادلات التفاضلية الجزئية والتي نشأت من التدفق لمائع في قناة مائلة بزاوية $\phi = 30$ وتحت تأثير مجال مغناطيسي عمودي على مستوي القناة وبوجود عامل الإشعاع.

الكلمات المفتاحية: انحناء القناة، حقل المغناطيس، الاستقرارية.

1. Introduction

The flow and heat transfer of electrically conducting fluid in channels and circular pipes under the effect of a transverse magnetic field occurs in magnetohydrodynamic (MHD) generators , pumps and flow meters and have applications in nuclear reactors , filtration , geothermal systems and others .

In 1969, Torrance and Rockett studied the numerical solution of natural convection in enclosures with localized heating from below greeping flow to the onset of laminar instability and obtained solution of Gratshof number from 4×10^4 *to* 4×10^{10} , which agreements with the experimentally observed laminar flows. In 1977, Petry and Busse studied the convection in a rotating cylindrical annulus in the presence of a magnetic field.

In 1990 , Daniess and Ong used A two-dimensional Galerkin formulation of the three dimensional oberbeck – Boussinesq equations to determine the onset of convection in an infinite rigid horizontal channel uniformly heated from bellow .In 1996 , Busse and Clever studied the problem of thermal convection in a fluid layer heated from below and solved it numerically in the case when a strong vertical magnetic field permeates the layer, the stability of two dimensional convection rolls was studied for different values of the Hartmann number within the region (200-400) . In 1997, Hartmann and Busse studied the convection in a rotating cylindrical annulus, they described many features to a good approximation by a system of three coupled amplitude equations.

In 2000, Almakrmi had studied Ekman boundary layer flow, he had studied its steady state solution and stability analysis. At the same year Aljoubory, studied the effect of buoyancy forces on the stability of a fluid flow between two plates one of them is thermally insulated. In 2001, Malashetty, Umavathi and Kumar studied the problem of fully developed free convection of two fluid magnetohydrodynamic (MHD) flow in an inclined channel. In 2009, Singha studied an analytical solution to the problem of magnetohydrodynamic (MHD) free convection flow of an electrically conducting fluid between two heated parallel plates in the presence of an induced magnetic field. In 2011, Akbar[a,*] , Hayat[a,b] , Nadeem[a] and Hendi[b] , studied the effect of slip and heat transfer on the peristaltic flow of a third order fluid in an inclined asymmetric channel.

In this paper , the stability of this system raising from the heat transfer in an inclined channel with the presence of magnetic field and radiation has been investigated , the inclination angle was considered as $\phi = 30$, it's found that the parameters Fs , k and w have a significant effect on the stability of the system.

2. The Model and Governing Equations:

Consider a fully- developed, steady laminar flow of fluid in the inclined channel at angle $(\varphi = 30)$ degree, the distance between the walls of the channel is $2h$ apart. Choosing the coordinate system such that the $x-axis$ in the direction of the flow, $y-axis$ is measured perpendicular to the plane of the channel, whilst the $z-axis$ is in the direction mutually orthogonal to the other two axes, the boundaries of the inclined channel are taken to be perfectly conducting electrodes infinitely far apart so that any dependence upon the $z-coordinate$ vanishes.

In the model under consideration , the electric density has a single component J_z, the magnetic field has a component B_x induced along the inclined channel in the direction of the flow, B_z is zero and the component parallel to the $y-axis$ decomposed into two components $B_0 \sin\varphi$ in the x direction and $B_0 \cos\varphi$ at y direction . The electric field has a single component E_0 which is parallel to the $z-axis$, as illustrated in the figure below :

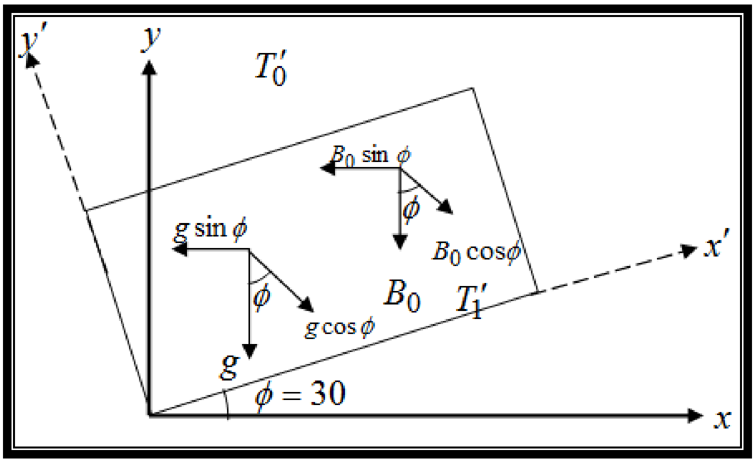

Figure 1 : The Inclined Channel at Angle $\varphi = 30$

Where the velocities u , v *and* w are zero at the edges and T is temperature, T_1 represents the temperature of the lower plate and T_2 represents the temperature of the upper plate.

The governing equations are

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \quad . \qquad \ldots(2.1)$$

$$\frac{\partial u'}{\partial t'}+u'\frac{\partial u'}{\partial x'}+v'\frac{\partial u'}{\partial y'}=-\frac{1}{\rho_1}\frac{\partial P}{\partial x'}+\upsilon\nabla^2 u'-\frac{\sigma B_0^2}{\rho_1}v'\sin\varphi-\rho g\sin\varphi . \qquad \ldots(2.2)$$

$$\frac{\partial v'}{\partial t'}+u'\frac{\partial v'}{\partial x'}+v'\frac{\partial v'}{\partial y'}=-\frac{1}{\rho_1}\frac{\partial P}{\partial y'}+\upsilon\nabla^2 v'+\frac{\sigma B_0^2}{\rho_1}v'\cos\varphi-\rho g\cos\varphi . \qquad \ldots(2.3)$$

$$\frac{\partial T}{\partial t'}+u'\frac{\partial T}{\partial x'}+v'\frac{\partial T}{\partial y'}=\frac{k}{\rho_1 C_v}\nabla^2 T-\frac{1}{\rho_1 C_v}\nabla\vec{q}+\ \ldots+\frac{1}{\rho_1 C_v\sigma\mu_0^2}\left[\frac{\partial B_x}{\partial y'}\sin\varphi-\frac{\partial B_x}{\partial y'}\cos\varphi\right]^2 \quad (2.4)$$

Where , $\rho=\rho_1\{1-\beta(T-T_1)\}$

with boundary conditions:

1− $u=v=0$ *at* $y=\mp h$

2− $T=T_1$, $\frac{\partial T}{\partial y}=0$ *at* $y=-h$

3− $T=T_2$, $\frac{\partial T}{\partial y}=0$ *at* $y=h$

3. Non-dimensional Form:[11]

We define the following non-dimensional variables as follows:

$$\left.\begin{array}{l} t=\frac{ht*}{u_0}\ ,\ \vec{B}=B_0\mu_0\sigma u_0 h\vec{b}\ ,\ T=T_1\theta\ ,\ \vec{q}=T_1^4\sigma\vec{Q} \\ \vec{Q}=\frac{\vec{q}}{T_1^4\sigma}\ ,\ x=hx^*\ ,\ y=hy^*\ ,\ P=P^*\rho_1 u_0^2 \\ u=u_0u^*\ ,\ v=u_0v^*\ , \end{array}\right\} . \qquad \ldots(3.1)$$

After substituting (3.1) in equations (2.1) , (2.2) , (2.3) and (2.4) , we get

$$\frac{\partial u'^*}{\partial x'^*}+\frac{\partial v'^*}{\partial y'^*}=0 . \qquad \ldots(3.2)$$

$$\frac{\partial u'^*}{\partial t'^*}+u'^*\frac{\partial u'^*}{\partial x'^*}+v'^*\frac{\partial u'^*}{\partial y'^*}=-\frac{\partial P^*}{\partial x'^*}+\frac{1}{\mathrm{Re}}\left[\frac{\partial^2 u'^*}{\partial x'^{*2}}+\frac{\partial^2 u'^*}{\partial y'^{*2}}\right]-\frac{M^2}{\mathrm{Re}}v'^*\sin\varphi-\ldots-(E-H\theta)\sin\varphi . \quad (3.3)$$

$$\frac{\partial v'^*}{\partial t'^*}+u^{*\prime}\frac{\partial v'^*}{\partial x'^*}+v'^*\frac{\partial v'^*}{\partial y'^*}=-\frac{\partial P^*}{\partial y'^*}+\frac{1}{\mathrm{Re}}\left[\frac{\partial^2 v'^*}{\partial x'^{*2}}+\frac{\partial^2 v'^*}{\partial y'^{*2}}\right]+\frac{M^2}{\mathrm{Re}}v'^*\cos\varphi-\ldots-(E-H\theta)\cos\varphi \quad (3.4)$$

$$\frac{\partial \theta}{\partial t'^*}+u'^*\frac{\partial \theta}{\partial x'^*}+v'^*\frac{\partial \theta}{\partial y'^*}=\frac{\gamma}{\Pr\mathrm{Re}}\left[\frac{\partial^2\theta}{\partial x'^{*2}}+\frac{\partial^2\theta}{\partial y'^{*2}}\right]-\frac{\gamma\, Ec}{B_0}\left[\frac{\partial Q_x}{\partial x'^*}+\frac{\partial Q_y}{\partial y'^*}\right]+\ \ldots$$

$$\ldots\ +\frac{M^2\gamma\, Ec}{\mathrm{Re}}\left[\frac{\partial b_x}{\partial y'^*}\sin\varphi-\frac{\partial b_x}{\partial y'^*}\cos\varphi\right]^2 . \qquad \ldots(3.5)$$

And since $div\vec{Q}=16W\theta-12W$. [11] and by multyping equation (3.5) by $\frac{\Pr\mathrm{Re}}{\gamma}$, we get

$$\Pr L\left[\frac{\partial\theta}{\partial t'^*}+u'^*\frac{\partial\theta}{\partial x'^*}+v'^*\frac{\partial\theta}{\partial y'^*}\right]=\left[\frac{\partial^2\theta}{\partial x'^{*2}}+\frac{\partial^2\theta}{\partial y'^{*2}}\right]-16FsNW\theta+12FsNW+\ \ldots$$

$$\ldots + M^2 Fs\left[\frac{\partial b_x}{\partial y'^*}\sin\varphi - \frac{\partial b_x}{\partial y'^*}\cos\varphi\right]^2 . \qquad \ldots(3.6)$$

Where ,

$\mathrm{Re} = \dfrac{h\rho_1 u_0}{\mu}$ is the Reynolds number .

$\mathrm{Pr} = \dfrac{\mu C_p}{k}$ is the Prandtel number .

$\gamma = \dfrac{C_p}{C_v}$ is the specific heat ratio .

$W = \alpha_0 h$ is Bouger number .

$Ec = \dfrac{u_0^2}{C_p T_1}$ is the Eckert number .

$Fs = \mathrm{Pr}.Ec$ is the new factor .

$M = B_0 h\left(\dfrac{\sigma}{\mu}\right)^{1/2}$ is Hartmann number .

$N = \dfrac{\mathrm{Re}}{B_0}$, $L = \dfrac{NB_0}{\gamma}$

4. Stability Analysis:

We define the functions of velocities, pressure , Maxwell and the temperature as the following convenient form :

$$\left.\begin{aligned} u'^* &= u_1(x,y) + u_2(x,y,t) \\ v'^* &= v_1(x,y) + v_2(x,y,t) \\ p'^* &= p_1(x,y) + p_2(x,y,t) \\ \theta &= \theta_1(x,y) + \theta_2(x,y,t) \\ b_x &= b_{x'} + b_{x''} \end{aligned}\right\} \qquad \ldots(4.1)$$

by substituting equation(4.1) into equations (3.2),(3.3),(3.4) and (3.6),we get

$$\frac{\partial(u_1+u_2)}{\partial x'^*} + \frac{\partial(v_1+v_2)}{\partial y'^*} = 0 . \qquad \ldots(4.2)$$

$$\frac{\partial(u_1+u_2)}{\partial t'^*} + (u_1+u_2)\frac{\partial(u_1+u_2)}{\partial x'^*} + (v_1+v_2)\frac{\partial(u_1+u_2)}{\partial y'^*} = -\frac{\partial(p_1+p_2)}{\partial x'^*} + \ldots$$

$$\ldots + \frac{1}{\mathrm{Re}}\left[\frac{\partial^2(u_1+u_2)}{\partial x'^{*2}} + \frac{\partial^2(u_1+u_2)}{\partial y'^{*2}}\right] - \frac{M^2}{\mathrm{Re}}(v_1+v_2)\sin\varphi - E\sin\varphi + H(\theta_1+\theta_2)\sin\varphi \qquad \ldots(4.3)$$

$$\frac{\partial(v_1+v_2)}{\partial t'^*} + (u_1+u_2)\frac{\partial(v_1+v_2)}{\partial x'^*} + (v_1+v_2)\frac{\partial(v_1+v_2)}{\partial y'^*} = -\frac{\partial(p_1+p_2)}{\partial y'^*} + \ldots$$

$$\ldots + \frac{1}{\mathrm{Re}}\left[\frac{\partial^2(v_1+v_2)}{\partial x'^{*2}} + \frac{\partial^2(v_1+v_2)}{\partial y'^{*2}}\right] + \frac{M^2}{\mathrm{Re}}(v_1+v_2)\cos\varphi - E\cos\varphi + H(\theta_1+\theta_2)\cos\varphi \qquad \ldots(4.4)$$

$$\mathrm{Pr}\,L\left[\frac{\partial(\theta_1+\theta_2)}{\partial t'^*} + (u_1+u_2)\frac{\partial(\theta_1+\theta_2)}{\partial x'^*} + (v_1+v_2)\frac{\partial(\theta_1+\theta_2)}{\partial y'^*}\right] = \frac{\partial^2(\theta_1+\theta_2)}{\partial x'^{*2}} + \ldots$$

$$...+\frac{\partial^2(\theta_1+\theta_2)}{\partial y'^{*2}}-16FsNW(\theta_1+\theta_2)+12FsNW+...$$

$$...+M^2Fs\left[\frac{\partial(b_{x'}+b_{x''})}{\partial y'^{*}}\right]^2(1-2\sin\varphi\cos\varphi) \qquad ...(4.5)$$

And therefore

$$\Pr L\left[\frac{\partial(\theta_1+\theta_2)}{\partial t'^{*}}+(u_1+u_2)\frac{\partial(\theta_1+\theta_2)}{\partial x'^{*}}+(v_1+v_2)\frac{\partial(\theta_1+\theta_2)}{\partial y'^{*}}\right]=\frac{\partial^2(\theta_1+\theta_2)}{\partial x'^{*2}}+...$$

$$...+\frac{\partial^2(\theta_1+\theta_2)}{\partial y'^{*2}}-16FsNW(\theta_1+\theta_2)+12FsNW+M^2Fs\left[\frac{\partial(b_{x'}+b_{x''})}{\partial y'^{*}}\right]^2(1-2\sin\varphi\cos\varphi). \qquad ...(4.6)$$

So

$$\frac{\partial u_1}{\partial x'^*}+\frac{\partial u_2}{\partial x'^*}+\frac{\partial v_1}{\partial y'^*}+\frac{\partial v_2}{\partial y'^*}=0. \qquad ...(4.7)$$

$$\frac{\partial u_1}{\partial t'^*}+\frac{\partial u_2}{\partial t'^*}+u_1\frac{\partial u_1}{\partial x'^*}+u_1\frac{\partial u_2}{\partial x'^*}+u_2\frac{\partial u_2}{\partial x'^*}+u_2\frac{\partial u_1}{\partial x'^*}+v_1\frac{\partial u_1}{\partial y'^*}+v_1\frac{\partial u_2}{\partial y'^*}+ \quad ...$$

$$...+v_2\frac{\partial u_1}{\partial y'^*}+v_2\frac{\partial u_2}{\partial y'^*}=-\frac{\partial p_1}{\partial x'^*}-\frac{\partial p_2}{\partial x'^*}+\frac{1}{\mathrm{Re}}\left[\frac{\partial^2 u_1}{\partial x'^{*2}}+\frac{\partial^2 u_2}{\partial x'^{*2}}+\frac{\partial^2 u_1}{\partial y'^{*2}}+\frac{\partial^2 u_2}{\partial y'^{*2}}\right]$$

$$... \;-\frac{M^2}{\mathrm{Re}}v_1\sin\varphi-\frac{M^2}{\mathrm{Re}}v_2\sin\varphi-E\sin\varphi-H\theta_1\sin\varphi-H\theta_2\sin\varphi \qquad ...(4.8)$$

$$\frac{\partial v_1}{\partial t'^*}+\frac{\partial v_2}{\partial t'^*}+u_1\frac{\partial v_1}{\partial x'^*}+u_1\frac{\partial v_2}{\partial x'^*}+u_2\frac{\partial v_1}{\partial x'^*}+u_2\frac{\partial v_2}{\partial x'^*}+v_1\frac{\partial v_1}{\partial y'^*}+v_1\frac{\partial v_2}{\partial y'^*}+ \quad ...$$

$$...+v_2\frac{\partial v_1}{\partial y'^*}+v_2\frac{\partial v_2}{\partial y'^*}=-\frac{\partial p_1}{\partial y'^*}-\frac{\partial p_2}{\partial y'^*}+\frac{1}{\mathrm{Re}}\left[\frac{\partial^2 v_1}{\partial x'^{*2}}+\frac{\partial^2 v_2}{\partial x'^{*2}}+\frac{\partial^2 v_1}{\partial y'^{*2}}+\frac{\partial^2 v_2}{\partial y'^{*2}}\right]+....$$

$$+\frac{M^2}{\mathrm{Re}}v_1\cos\varphi+\frac{M^2}{\mathrm{Re}}v_2\cos\varphi-E\cos\varphi+H\theta_1\cos\varphi+H\theta_2\cos\varphi. \qquad ...(4.9)$$

$$\Pr L\left[\frac{\partial\theta_1}{\partial t'^*}+\frac{\partial\theta_2}{\partial t'^*}+u_1\frac{\partial\theta_1}{\partial x'^*}+u_2\frac{\partial\theta_1}{\partial x'^*}+u_1\frac{\partial\theta_2}{\partial x'^*}+u_2\frac{\partial\theta_2}{\partial x'^*}\right] + \quad ...$$

$$...+\Pr L\left[v_1\frac{\partial\theta_1}{\partial y'^*}+v_1\frac{\partial\theta_2}{\partial y'^*}+v_2\frac{\partial\theta_1}{\partial y'^*}+v_2\frac{\partial\theta_2}{\partial y'^*}\right]=\frac{\partial^2\theta_1}{\partial x'^{*2}}+\frac{\partial^2\theta_2}{\partial x'^{*2}}+ \quad ...$$

$$... \quad +\frac{\partial^2\theta_1}{\partial y'^{*2}}+\frac{\partial^2\theta_2}{\partial y'^{*2}}-16FsNW\theta_1-16FsNW\theta_2+12FsNW+ \quad ...$$

$$...+M^2Fs\left[\left[\frac{\partial b_{x'}}{\partial y'^*}\right]^2+2\left[\frac{\partial b_{x'}}{\partial y'^*}\right]\left[\frac{\partial b_{x''}}{\partial y'^*}\right]+\left[\frac{\partial b_{x''}}{\partial y'^*}\right]^2\right]*(1-2\sin\varphi\cos\varphi). \qquad ...(4.10)$$

5 . Unsteady State Equations:

This section is devoted to investigate the stability of the disturbance by using an approximated analytical method .From the differential equations (4.7) , (4.8) , (4.9) and (4.10) we get

$$\frac{\partial u_2}{\partial x'^*}+\frac{\partial v_2}{\partial y'^*}=0\ . \qquad ...(5.1)$$

$$\frac{\partial u_2}{\partial t^{'*}}+u_1\frac{\partial u_2}{\partial x^{'*}}+u_2\frac{\partial u_1}{\partial x^{'*}}+u_2\frac{\partial u_2}{\partial x^{'*}}+v_1\frac{\partial u_2}{\partial y^{'*}}+v_2\frac{\partial u_1}{\partial y^{'*}}+v_2\frac{\partial u_2}{\partial y^{'*}}=-\frac{\partial p_2}{\partial x^{'*}}+\quad\dots$$

$$\dots\quad+\frac{1}{\mathrm{Re}}\left[\frac{\partial^2 u_2}{\partial x^{'*2}}+\frac{\partial^2 u_2}{\partial y^{'*2}}\right]-\frac{M^2}{\mathrm{Re}}v_2\sin\varphi+H\theta_2\sin\varphi\,. \qquad \dots(5.2)$$

$$\frac{\partial v_2}{\partial t^{'*}}+u_1\frac{\partial v_2}{\partial x^{'*}}+u_2\frac{\partial v_1}{\partial x^{'*}}+u_2\frac{\partial v_2}{\partial x^{'*}}+v_2\frac{\partial v_1}{\partial y^{'*}}+v_1\frac{\partial v_2}{\partial y^{'*}}+v_2\frac{\partial v_2}{\partial y^{'*}}=-\frac{\partial p_2}{\partial y^{'*}}+\quad\dots$$

$$\dots+\frac{1}{\mathrm{Re}}\left[\frac{\partial^2 v_2}{\partial x^{'*2}}+\frac{\partial^2 v_2}{\partial y^{'*2}}\right]+\frac{M^2}{\mathrm{Re}}v_2\cos\varphi+H\theta_2\cos\varphi\,. \qquad \dots(5.3)$$

$$\Pr L\left[\frac{\partial \theta_2}{\partial t^{'*}}+u_1\frac{\partial \theta_2}{\partial x^{'*}}+u_2\frac{\partial \theta_1}{\partial x^{'*}}+u_2\frac{\partial \theta_2}{\partial x^{'*}}+v_1\frac{\partial \theta_2}{\partial y^{'*}}+v_2\frac{\partial \theta_1}{\partial y^{'*}}+v_2\frac{\partial \theta_2}{\partial y^{'*}}\right]=\frac{\partial^2\theta_2}{\partial x^{'*2}}+\frac{\partial^2\theta_2}{\partial y^{'*2}}-\dots\ \dots-$$

$$16FsNW\theta_2+M^2Fs\left[2\left[\frac{\partial b_{x'}}{\partial y^{'*}}\right]\left[\frac{\partial b_{x''}}{\partial y^{'*}}\right]+\left[\frac{\partial b_{x''}}{\partial y^{'*}}\right]^2\right](1-2\sin\varphi\cos\varphi)\ . \qquad \dots(5.4)$$

With boundary conditions and from (4.1) , we have :

$$1-\ u_2=v_2=0\quad,\ \theta_2=0\ ,\ \frac{\partial\theta_2}{\partial y^{'*}}=0\qquad at\qquad y^{'*}=-1$$

$$2-u_2=\ v_2=0\quad,\ \theta_2=0\ ,\ \frac{\partial\theta_2}{\partial y'*}=0\qquad at\qquad y'*=1$$

6 . Disturbance in x and y Directions:

To solve the linearized system (or to analyze the stability) and because the coefficient in the differential equations are independent of $t\ and\ x$ we attempt to find the solution of the form [5]

$$\left.\begin{aligned} u_2&=u(y)e^{ikx}e^{at}\\ v_2&=v(y)e^{ikx}e^{at}\\ p_2&=p(y)e^{ikx}e^{at}\\ \theta_2&=\theta(y)e^{ikx}e^{at}\\ b_{x''}&=b_x(y)e^{ikx}e^{at}\end{aligned}\right\} \qquad \dots(6.1)$$

Where k is wave number in the direction x , and a is the complex number which has the form $a=a_1+ia_2$, a_1 , $a_2\in R$ is the speed number. When $a_1>0$,the system is unstable while when $a_1<0$,the system is stable. Also $u(y),v(y),p(y),\theta(y),b_x(y)$ are the amplitude functions.

After substitute (6.1) in the equations (5.1) , (5.2) , (5.3) and (5.4) , we get:

$$(iku+v')e^{ikx}e^{at}=0$$

$$(au+u_1(iku)+\frac{\partial u_1}{\partial x'*}u+v_1u'+\frac{\partial u_1}{\partial y'*}v+ikp-\frac{1}{\mathrm{Re}}(-k^2u+u'')-\frac{M^2}{\mathrm{Re}}v\sin\varphi+..$$

$$\dots+H\theta\sin\varphi)e^{ikx}e^{at}=0.$$

$$(av+u_1(ikv)+\frac{\partial v_1}{\partial x^{'*}}u+v_1v'+\frac{\partial v_1}{\partial y^{'*}}v+p'-\frac{1}{\mathrm{Re}}(-k^2v+v'')-\frac{M^2}{\mathrm{Re}}v\cos\Phi-...$$

$$...-H\theta\cos\Phi)e^{ikx}e^{at}=0$$

$$\Pr L((a\theta+u_1(ik\theta)+\frac{\partial\theta_1}{\partial x'^*}u+v_1\theta'+\frac{\partial\theta_1}{\partial y'^*}v)-k^2\theta-\theta''+16FsNW\theta- \quad \dots$$

$$\dots \quad -M^2Fs\left[2(\frac{\partial b_{x'}}{\partial y})(b_x^1)+(b_x^1)^2\right](1-2\sin\varphi\cos\varphi))e^{ikx}e^{at}=0$$

Since $e^{ikx}e^{at}\neq 0$, then

$$\left.\begin{aligned}
&iku+v'=0\\
&au+u_1(iku)+\frac{\partial u_1}{\partial x'^*}u+\frac{\partial u_1}{\partial y'^*}v+ikp+\frac{1}{\text{Re}}(-k^2u+u'')+v_1u'-....\\
&\qquad ......-\frac{M^2}{\text{Re}}\sin\varphi+H\theta\sin\varphi=0\\
&av+u_1(ikv)+\frac{\partial v_1}{\partial x'^*}u+v_1v'+\frac{\partial v_1}{\partial y'^*}v+p'-\frac{1}{\text{Re}}(-k^2v+v'')-........\\
&\qquad ......-\frac{M^2}{\text{Re}}v\cos\varphi-H\theta\cos\varphi\\
&a\Pr L((a\theta+u_1(ik\theta)+\frac{\partial\theta_1}{\partial x'^*}u+v_1\theta'+\frac{\partial\theta_1}{\partial y'^*}v)-k^2\theta-\theta''+16FsNW-...\\
&\qquad ....-M^2Fs\left[(2b_{x'})\left[\frac{\partial bx'}{\partial y'^*}\right]+(b_{x'})^2\right](1-2\sin\varphi\cos\varphi))=0
\end{aligned}\right\} \quad \dots(6.2)$$

And after linearization equations of (6.2) we get

$$\left.\begin{aligned}
&iku+v'=0\\
&(a+\frac{K^2}{\text{Re}})u+ikp-\frac{1}{\text{Re}}u''-\frac{M^2}{\text{Re}}v\sin\varphi-H\theta\sin\varphi=0\\
&(a+\frac{K^2}{\text{Re}}-\frac{M^2}{\text{Re}}\cos\varphi)v+p'-\frac{1}{\text{Re}}v''-H\theta\cos\varphi)=0\\
&(a\Pr L+k^2+16FsNW)\theta-\theta''=0
\end{aligned}\right\} \quad \dots(6.3)$$

From (6.3), let $\phi=30$, we have

$$\left.\begin{aligned}
&u'=h\\
&h'=(ik\,\text{Re})p+(a\,\text{Re}+k^2)u+(\frac{M^2}{2})v-(\frac{1}{2}H\,\text{Re})\theta\\
&p'=-(\frac{ik}{\text{Re}})h+(\frac{\sqrt{3}}{2}H)\theta-(a+\frac{k^2}{\text{Re}}-\frac{\sqrt{3}}{2}\frac{M^2}{\text{Re}})v\\
&v'=-(ik)u\\
&\theta'=s\\
&s'=(a\Pr L+k^2+16FsNW)\theta
\end{aligned}\right\} \quad \dots(6.4)$$

Where $X(y)=[u(y),h(y),p(y),v(y),\theta(y),s(y)]^T$, and

$$\frac{dX}{dy}=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ (a\,\mathrm{Re}+k^2) & 0 & ik\,\mathrm{Re} & -\frac{M^2}{2} & -\frac{H\,\mathrm{Re}}{2} & 0 \\ 0 & \frac{-ik}{\mathrm{Re}} & 0 & -(a+\frac{k^2}{\mathrm{Re}}-\frac{\sqrt{3}}{2}\frac{M^2}{\mathrm{Re}}) & \frac{\sqrt{3}}{2}H & 0 \\ -ik & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & (a\Pr L+k^2 +16FsNW) & 0 \end{bmatrix}\begin{bmatrix} u \\ h \\ p \\ v \\ \theta \\ s \end{bmatrix}$$

And by using $|\Omega-\lambda I|=0$, where

$$\Omega=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ A & 0 & B & W & G & 0 \\ 0 & C & 0 & J & T & 0 \\ E & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & F & 0 \end{bmatrix}$$

Where $A=(a\,\mathrm{Re}+k^2)$, $B=(ik\,\mathrm{Re})$, $E=-(ik)$, $J=(a+\frac{k^2}{\mathrm{Re}}-\frac{\sqrt{3}}{2}\frac{M^2}{\mathrm{Re}})$, $C=-(\frac{ik}{\mathrm{Re}})$,

$F=(a\Pr L+k^2+16FsNW)$, $G=-\frac{H\,\mathrm{Re}}{2}$, $W=-\frac{M^2}{2}$, $T=\frac{\sqrt{3}}{2}H$

we get

$$\begin{vmatrix} -\lambda & 1 & 0 & 0 & 0 & 0 \\ A & -\lambda & B & W & G & 0 \\ 0 & C & -\lambda & J & T & 0 \\ E & 0 & 0 & -\lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & -\lambda & 1 \\ 0 & 0 & 0 & 0 & F & -\lambda \end{vmatrix}=0$$

or

$$f(\lambda)=\lambda^6-(F+A+BC)\lambda^4-EW\lambda^3+(BCF+AF-BEJ)\lambda^2+EFW\lambda+EJBF=0 \qquad \ldots(6.5)$$

Now, we solve equation (6.5) numerically by using (Maple 11) [7] to find the roots of this equation .

7. Conclusions

In this paper, we study the stability of the system under the effect of an inclination to the horizontal channel with the presence of magnetic field and radiation, when the system has inclined by angle $\phi=30$, it's noticed that the wave number k has a negative effect on the velocity function and depends it towards unstable and this is clear from figure (3). From figure (2), the increase of the factor Fs causes the increase to unstable probability of the system. Finally it is found that the increasing of Bouger number has an effect on the system towards stable, it's noticed from figure (4).

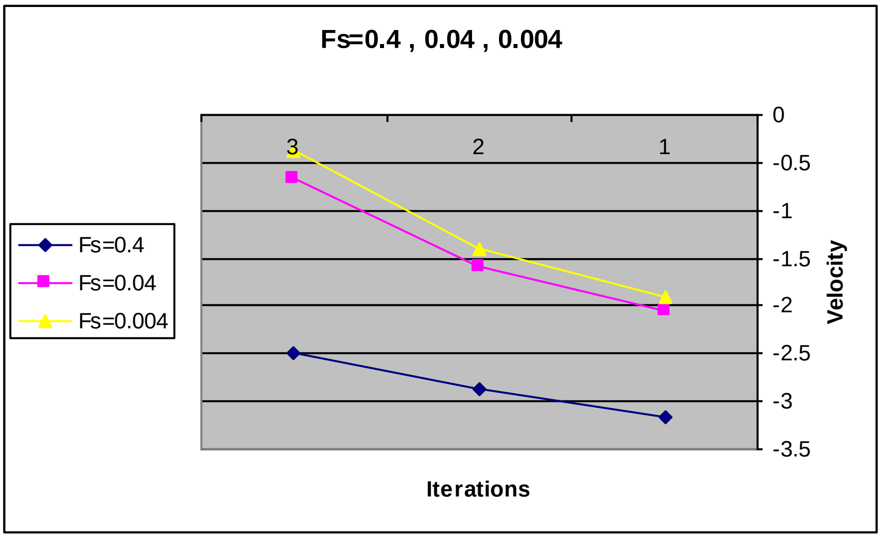

Figure (2): Effect of The Factor Fs =0.4 , 0.04 , 0.004 at Angle 30

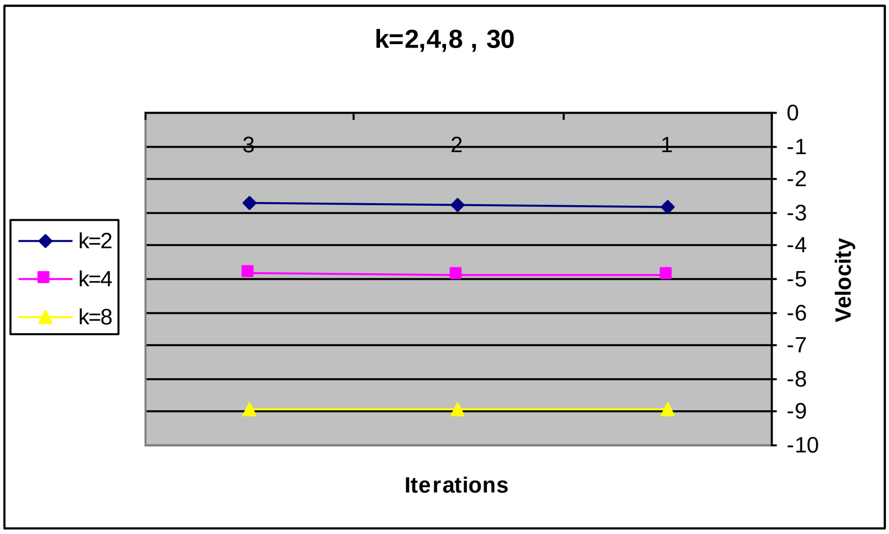

Figure (3): Effect of The Wave Number k =2, 4 , 8 at Angle 30

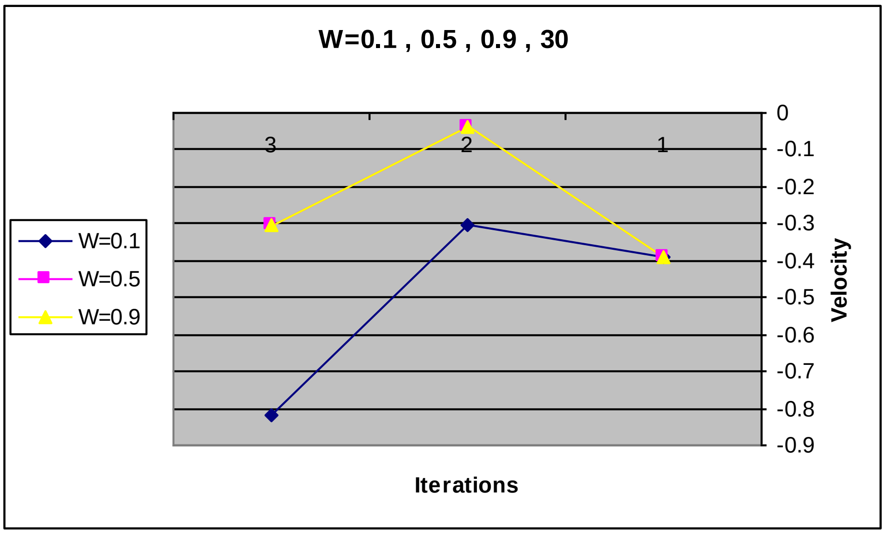

Figure (4): Effect of The Bouger Number W =0.1, 0.5 , 0.9 at Angle 30

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