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Application of ic—open Sets in Topologies Via Ideals

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Abstract

In this paper, new kinds of open sets inside ideal topological spaces are introduced; they are called: ic − I − open, icc − I − open, weakly ic − I − open and weakly icc − I − open. Some properties and relations between these new classes are studied with examples. The concept of continuity in ideal topological spaces is also presented for these new classes. Theorems that provide an equivalence relation between these new classes are proved. Moreover, for ideal topological spaces (Ɲ, Ʈ, Ɩ), we show that all open sets are ic-I-open, icc-I-open, weakly ic-I-open, weakly icc-I-open. Finally, assume $\mathcal{Z} \subset$ Ɲ of ideal topological spaces (Ɲ, Ʈ, Ɩ). Then

1) if $\mathcal{Z}$ is semi − I − open, then $\mathcal{Z}^c$ is ic – I − open.
2) if $\mathcal{Z}$ is open and ic − I − closed, then $\mathcal{Z}$ is semi − I − open.

Keywords:
Ideal topological Space, ic − I − open, icc − I − open.

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1. Introduction

The idea of ideal topological spaces was first included by Kuratowski [6] and Vaidyanathaswamy [7]. Later, this concept was developed by many researchers, including Jankovic and Hamlett [4], Mustafa et al [11], Manoharan and Thangavelu [3], Mohammed and Mohammed [10]. The class ic − open set has lately been presented by Faisal [1]. Given an ideal topological space (Ɲ, Ʈ, Ɩ) we clarify ic − I − open and icc − I − open sets as follows: $\mathcal{Z} \subseteq$ Ɲ is told to be an ic − I − open set if there is E ∈ $Ʈ^c$ \{∅, Ɲ} such that $Cl^*(E \cap \mathcal{Z}) \subseteq int(\mathcal{Z})$. $\mathcal{Z}$ is a subset of ideal topological spaces (Ɲ, Ʈ, Ɩ) is icc − I − open if $\mathcal{Z}$ is ic − I − open set and int($\mathcal{Z}$) = G for some G ∈ Ʈ \{∅, Ɲ}. The most significant finding is that for ideal topological spaces all α − I − open sets are not ic − I − open and not icc − I − open and vice versa. Additionally, all semi − I − open sets are not ic − I − open and not icc − I − open and vice versa. We demonstrate once more that for ideal topological spaces all ic − I − open set are weakly ic − I − open and all icc − I − open set are weakly icc − I − open. Finally, we discuss the ic − I − continuous and weakly ic − I − continuous and we prove if(Ɲ, Ʈ, Ɩ) is an ideal topological space and (ϒ, δ) is topological space, thus every continuous mapping from (Ɲ, Ʈ, Ɩ) into (ϒ, δ) is ic − I – continuous but the opposite is untrue. Throughout this paper, we indicate open sets by (os) and continuous functions by (contm), respectively.

1.1.Definition: [6] The ideal Ɩ of topological space (Ɲ, Ʈ) is a nonempty collection of a subset of Ɲ which fulfills

1. $\mathcal{Z}$ ∈ Ɩ and S ⊆ $\mathcal{Z}$ indicates S ∈ Ɩ.
2. $\mathcal{Z}$ ∈ Ɩ and S ∈ Ɩ indicates $\mathcal{Z} \cup S$ ∈ Ɩ.

1.2.Definition: Let n ∈ Ɲ and let (Ɲ, Ʈ, Ɩ) be ideal topological spaces. Then a mapping from a family of a subsets of Ɲ into itself is thus referred to as a local function and is defined as follows: for $\mathcal{Z}$ ⊂ Ɲ, $(\mathcal{Z})^*$(Ɩ, Ʈ) = {n ∈ Ɲ: $V \cap \mathcal{Z} \notin$ Ɩ ∀ V ∈ Ʈ(n)} where Ʈ(n) = { V ∈ Ʈ: n ∈ V }. The definition of a kuratowski closure operator[6] CL^* *is* $CL^*(\mathcal{Z}) = \mathcal{Z} \cup \mathcal{Z}^*$(Ɩ, Ʈ). We shall simply write $\mathcal{Z}^*$ for $\mathcal{Z}^*$ (Ɩ, Ʈ).

1.3.Definition: If $\mathcal{Z}$ is a subset of topological spaces $(\mathfrak{N}, \mathfrak{T})$, then $\mathcal{Z}$ is

a) semi − open [5] if $\mathcal{Z} \subset cl(\text{int}(\mathcal{Z}))$).
b) α− open [8] if $\mathcal{Z} \subset \text{int}(cl(\text{int}(\mathcal{Z}))$.
c) ic − open [1] if there exists $E \in \mathfrak{T}^c \setminus \{\emptyset, \mathfrak{N}\}$ such that $E \cap \mathcal{Z} \subseteq int(\mathcal{Z})$.
For a subset of ideal Topological spaces $(\mathfrak{N}, \mathfrak{T}, \mathfrak{I})$, $\mathcal{Z}$ is
d) semi − I− open [9] if $\mathcal{Z} \subset cl^*(\text{int}(\mathcal{Z}))$.
e) α− I− open [2] if $\mathcal{Z} \subset \text{int}(cl^*(\text{int}(\mathcal{Z}))$.

1.4.Definition : It can be told that a subset $\mathcal{Z}$ of ideal topological spaces $(\mathfrak{N}, \mathfrak{T}, \mathfrak{I})$ is ic − I − open if there is $E \in \mathfrak{T}^c \setminus \{\emptyset, \mathfrak{N}\}$ such that $CL^*(E \cap \mathcal{Z}) \subseteq \text{int}(\mathcal{Z})$. Thus the complete of ic − I − open set is ic − I − closed.

1.5.Definition: $\mathcal{Z}$ is a subset of ideal topological spaces $(\mathfrak{N}, \mathfrak{T}, \mathfrak{I})$ is icc − I − open if $\mathcal{Z}$ is ic − I − open set and $\text{int}(\mathcal{Z}) = G \in \mathfrak{T} \setminus \{\emptyset, \mathfrak{N}\}$. Then the complement of icc – I − open is icc − I – closed.
We will use the following short words:
1)Topological space ≡ (TS).
2)Ideal topological space ≡ (ITS).
3) Open ≡ (O).
4) α − I − open ≡ (α − I − o).
5) Semi − I − open ≡ (s − I − o).
6) ic – I − open ≡ (ic − I − o).
7) icc − I − open ≡ (icc − I − o).

1.1.Theorem: Every (os) in ITS $(\mathfrak{N}, \mathfrak{T}, \mathbf{I})$ is (**ic − I − o**).
Proof. Let G be an open set in $(\mathfrak{N}, \mathfrak{T})$ and by[1, Theorem9.1.1]
$\boldsymbol{G^c}$ is **ic − closed**. Thus, there exists $\boldsymbol{E} \in \mathfrak{T}^c \setminus \{\emptyset, \mathfrak{N}\}$ such that $\textbf{int}(\boldsymbol{G^c}) \subseteq \boldsymbol{E} \cap \boldsymbol{G^c}$. Therefore, $\textbf{int}(\boldsymbol{G^c}) \subseteq \boldsymbol{Cl}(\boldsymbol{E} \cap \boldsymbol{G^c}) \subseteq \boldsymbol{Cl}^*(\boldsymbol{E} \cap \boldsymbol{G^c})$. That is, $\boldsymbol{G^c}$ is **ic − I − closed**. So, G is **ic − I − open**.

1.2.Theorem : Any subset of closed set of ideal topological spaces is **ic − I − closed**.
Proof. Assume that $\boldsymbol{\mathcal{Z}}$ be subset of a closed set $\boldsymbol{E}$ of ideal **topological** space $(\mathfrak{N}, \mathfrak{T}, \mathbf{I})$. Then $\boldsymbol{Cl}^*(\boldsymbol{E} \cap \boldsymbol{\mathcal{Z}}) = \boldsymbol{CL}^*(\boldsymbol{\mathcal{Z}})$. Hence $\boldsymbol{\mathcal{Z}} \subset \boldsymbol{Cl}^*(\boldsymbol{E} \cap \boldsymbol{\mathcal{Z}})$. Since $\textbf{int}(\boldsymbol{\mathcal{Z}}) \subset \boldsymbol{\mathcal{Z}}$ we have $\textbf{int}(\boldsymbol{\mathcal{Z}}) \subset \boldsymbol{Cl}^*(\boldsymbol{E} \cap \boldsymbol{\mathcal{Z}})$. Thus, $\boldsymbol{\mathcal{Z}}$ is **ic − I − closed**.

1.3.Theorem: Each (os) in any ideal topological spaces is (**icc − I − o**).
Proof: For any (os) $\boldsymbol{G} \in \mathfrak{T} \setminus \{\emptyset, \mathfrak{N}\}$ we get, $\boldsymbol{G}$ is (**ic − I − o**) set by theorm1.1. and int $(\boldsymbol{G}) = \boldsymbol{G}$. Therefore, $\boldsymbol{G}$ is (**icc – I– o**).

1.1. Remarks: Every (**α − I − o**) and (**s − I − o**) are not (**ic − I − o**) and not (**icc − I – o**) sets and vice versa as shown in the following **example 1:**
Let $\mathfrak{N} = \{\mathbf{1}, \mathbf{3}, \mathbf{5}\}$. We get,
1) If $\mathfrak{T} = \{\emptyset, \mathfrak{N}, \{\mathbf{5}\}\}$, $\mathbf{I} = \{\emptyset, \{\mathbf{3}\}\}$. Then $\{\mathbf{3}, \mathbf{5}\}$ is (**α − I – o**) and (**s − I – o**) but not (**ic − I − o**) and not (**icc − I − o**).
2) If $\mathfrak{T} = \{\emptyset, \mathfrak{N}, \{\mathbf{1}, \mathbf{3}\}, \{\mathbf{5}\}\}$, $\mathbf{I} = \{\emptyset, \{\mathbf{3}\}\}$. Then $\{\mathbf{3}, \mathbf{5}\}$ is (**ic − I − o**) and (**icc − I − o**) but not (**α − I − o**) and not (**s − I − o**).

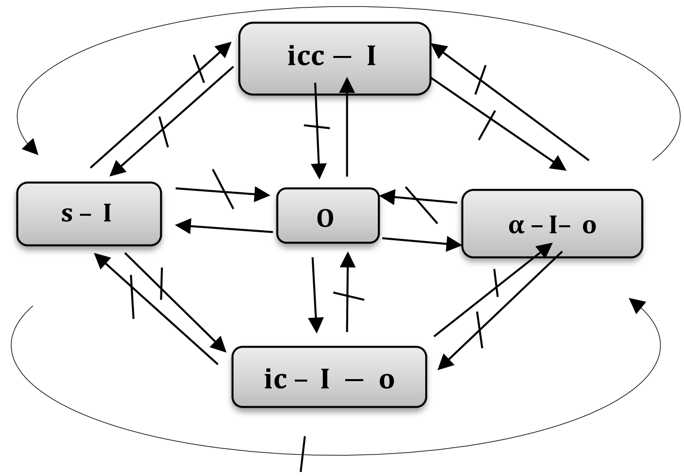

Fig. 1. Relationships of (ic-I-o) with other classes mentioned above.

2. Weakly ic − I− Open and Weakly icc – I− Open.
2.1. Definition : There is a subset $\boldsymbol{\mathcal{Z}}$ of ideal topological space called **Weakly ic − I − open** if there is $\boldsymbol{E} \in \mathfrak{T}^{c} \setminus \{\emptyset, \mathfrak{N}\}$ such that $\boldsymbol{Cl}^*(\boldsymbol{Cl}(\boldsymbol{E} \cap \boldsymbol{\mathcal{Z}})) \subseteq \textbf{int}(\boldsymbol{\mathcal{Z}})$.

2.2. Definition : The subset $\boldsymbol{\mathcal{Z}}$ of ideal topological space is described as Weakly **icc– I − open** if $\boldsymbol{\mathcal{Z}}$ is Weakly **ic – I − open** and $\textbf{int}(\boldsymbol{\mathcal{Z}}) = \boldsymbol{G}$ for some $\boldsymbol{G} \in \mathfrak{T} \setminus \{\emptyset, \mathfrak{N}\}$.

We will use the following short words:
1)**Weakly ic − I − open** ≡ (**w ic − I − o**)
2) **Weakly icc − I − open** ≡ (**w icc − I − o**)

2.1. Theorem : Suppose $(\mathfrak{N}, \mathfrak{T}, \mathbf{I})$ be ITS. Then every (os) is (**w ic − I − o**) but the opposite is untrue**.**
Proof. Let $\boldsymbol{G}$ be an (os) in $(\mathfrak{N}, \mathfrak{T}, \mathbf{I})$ and by theorem1.1. $\boldsymbol{G^c}$ is **ic − I − closed.** This means that $\exists\, \boldsymbol{E} \in \mathfrak{T}^c \setminus \{\emptyset, \mathfrak{N}\}$.
So, $\textbf{int}(\boldsymbol{G^c}) \subseteq \boldsymbol{Cl}^*(\boldsymbol{G^c} \cap \boldsymbol{E}) \subseteq \boldsymbol{Cl}^*(\boldsymbol{Cl}(\boldsymbol{G^c} \cap \boldsymbol{E}))$.
Hence, $\boldsymbol{G^c}$ is **weakly ic − I − closed.** $\boldsymbol{G}$ is (**w ic − I − o**).

2.2. Theorem :Every (ic – I – o) in ITS $(\mathfrak{N}, \mathfrak{T}, \mathbf{I})$ **is** (**w ic − I − o**).
Proof. Let $\boldsymbol{\mathcal{Z}} \subset \mathfrak{N}$ be **ic − I − open** set. $\boldsymbol{\mathcal{Z}^c}$ is **ic − I − closed** this means that, there exists $\boldsymbol{E} \in \mathfrak{T}^{c} \setminus \{\emptyset, \mathfrak{N}\}$ such that $\textbf{int}(\boldsymbol{\mathcal{Z}^c}) \subseteq \boldsymbol{Cl}^*(\boldsymbol{E} \cap \boldsymbol{\mathcal{Z}^c}) \subseteq \boldsymbol{Cl}^*(\boldsymbol{Cl}(\boldsymbol{E} \cap \boldsymbol{\mathcal{Z}^c}))$. Thus, $\boldsymbol{\mathcal{Z}}$ is (**w ic − I − o**).

2.3. Theorem : Assume $(\mathfrak{N}, \mathfrak{T}, \mathbf{I})$ be ITS, then every (**icc − I − o**) is (w **icc − I − o**).
Proof. Clear.

2.1. Corollary : Every ($\mathbf{w\ icc - I - o}$) set in ITS (Ɲ, Ʈ, I) is ($\mathbf{w\ ic - I - o}$) but the opposite is untrue.

Proof. Clear.

Example 2: Let Ɲ $= \{S, D, h\}$, Ʈ $= \{\emptyset$, Ɲ, $\{S, D\}\}$, I $= \{\emptyset, \{D\}\}$, $\mathcal{Z} = \{S\}$, $\{h\} \cap \{S\} = \emptyset \Rightarrow Cl(\emptyset) = \emptyset$, $Cl^*(\emptyset) = \emptyset \subseteq \mathrm{int}(\{S\}) = \emptyset$. $\mathcal{Z}$ is ($\mathbf{w\ ic - I - o}$) set but not ($\mathbf{w\ icc - I - o}$) because $\mathrm{int}(\{S\}) = \emptyset$.

2.2. Corollary : Suppose (Ɲ, Ʈ, I) be ITS, then every (os) is ($\mathbf{w\ icc - I - o}$) but the opposite is untrue.

Proof. Clear.

We will use the following short words:

1. **Semi − I − continuous** ≡ (**s – I − contm**).
2. **α – I − continuous** ≡ (**α – I − contm**).
3. **ic − I − continuous** ≡ (**ic − I − contm**).
4. **icc − I − continuous** ≡ (**icc − I − contm**).
5. **Weakly ic − I − continuous** ≡ (**w ic − I − contm**).
6. **Weakly icc − I − continuous** ≡ (**w icc − I − contm**).

2.3. Definition : Let f be a mapping from ITS (Ɲ, Ʈ, I) into TS (ϒ, δ). Then, f is told to be (**contm**) if f^{-1} (W) is open set in (Ɲ, Ʈ, I) for each (os) W of (ϒ, δ).

2.4. Definition : Let f be a mapping from ITS (Ɲ, Ʈ, I) into TS (ϒ, δ). Then, f is told to be (**ic − I − contm**) if f^{-1} (W) is (**ic − I − o**) in (Ɲ, Ʈ, I) for each (os) W of (ϒ, δ).

2.4. Theorem : Suppose (Ɲ, Ʈ, I) ITS and (ϒ, δ) TS. Thus every (contm) from (Ɲ, Ʈ, I) into (ϒ, δ) is (**ic − I − contm**) but the opposite is untrue.

proof. Let f be a continuous mapping from (Ɲ, Ʈ, I) into (ϒ, δ) and let W ∈ δ. Then f^{-1} (W) is an (os) in Ɲ. Since all (os) is (**ic − I − o**), thus f^{-1} (W) is (**ic – I − o**) in Ɲ. Hence, f is (**ic − I – contm**).

Example 3: Assume Ɲ $= \{o, g, w\}$, Ʈ $= \{\emptyset$, Ɲ, $\{o\}, \{g, w\}\}$, I $= \{\emptyset, \{g\}\}$, $\Upsilon = \{1, 2, 3\}$, $\delta = \{\emptyset, \Upsilon, \{1\}, \{1, 3\}\}$, f: (Ɲ, Ʈ, I) ⟶ (ϒ, δ) *where* $f(o) = 1, f(g) = 2, f(w) = 3$. Thus f is (**ic − I − contm**). Nevertheless, f is not a (contm).

2.5. Definition : Let f be a mapping from ideal **topological** spaces(Ɲ, Ʈ, I) into **topological** space(ϒ, δ). Then, f is named (**w ic– I − contm**) if f^{-1}(W) is (**w ic − I − o**)in (Ɲ, Ʈ, I) for each (os) W of (ϒ, δ).

2.5. Theorem : Let (Ɲ, Ʈ, I) ITS and (ϒ, δ) TS. Then every (contm) from (Ɲ, Ʈ, I) into (ϒ, δ) is (**w ic − I − contm**) but the opposite is untrue.

Proof. Let f be (contm) mapping from (Ɲ, Ʈ, I) into (ϒ, δ) and let W ∈ δ. Then, f^{-1} (W) is (os) in Ɲ. As all (os) is (**w ic − I − o**). We obtain, f is (**w ic – I − contm**).

Example 4: Ɲ $= \{k, n, r\}$, Ʈ $= \{\emptyset$, Ɲ, $\{n\}, \{k, r\}\}$, I $= \{\emptyset, \{r\}\}$, ϒ$= \{1, 2, 3\}$, δ= { ∅, Y, {2}, {2,3}},
f: (Ɲ, Ʈ, I) ⟶(ϒ, δ) $s.t.$ $f(k) = 1$, $f(n) = 2$, $f(r) = 3$, $f^{-1}(\{2, 3\}) = \{n, r\} \notin$ Ʈ. Then f is not continuous but $\{n, r\} \cap \{n\} = \{n\}$ **and** $cl(\{n\}) = \{n\}$, $cl^*(\{n\}) = \{n\} \subseteq$ **int** $(\{n, r\}) = \{n\}$. Then f is (**w ic − I − contm).**

2.6. Theorem : Assume that (Ɲ, Ʈ, I) **ITS and** (ϒ, δ) TS. Then all (**ic − I − contm**) from (Ɲ, Ʈ, I) into (ϒ,δ) is (**w ic – I – contm**).

Proof. Let f be an (**ic − I − contm**) from (Ɲ, Ʈ, I) into (ϒ, δ) and let W ∈ δ. Then f^{-1} (W) is (**ic − I − o**). By theorem 2.2. f^{-1} (W) is (**w ic − I − o**). Hence f is (**w ic − I – contm**).

2.7. Theorem : [10] Let (Ɲ, Ʈ, I) be ITS and (ϒ, δ) be TS. A map f: (Ɲ, Ʈ, I) ⟶ (ϒ, δ) is (**α − I – contm**), thus f is (**s – I − contm**) but the opposite is untrue.

Proof. Clear.

2.1. Remarks : Each (**α − I – contm**) and each (**s – I – contm**) are not (**ic– I – contm**) and not (**icc − I − contm**) and vice versa as signify in the following **example5:**

Suppose that Ɲ $= \{u, m, g, j\}$. We get,

1) If Ʈ $= \{\emptyset$, Ɲ, $\{u\}, \{u, m\}\}$, I $= \{\emptyset, \{m\}\}$, $\Upsilon = \{p, q\}$, δ$=\{\Upsilon, \emptyset, \{q\}\}$, f: (Ɲ, Ʈ, I) ⟶(ϒ, δ). And $f(u) = f(g) = q$, $f(m) = f(j) = p$,
$f^{-1}(\{q\}) = \{u, g\} \subseteq int\,(Cl^*(\mathrm{int}(\{u, g\}))) =$ Ɲ.
Therefore, $f^{-1}(\{q\})$ are (**α – I – contm**) and (**s – I – contm**) but not (**ic – I – contm**) and not (**icc– I – contm**).

2)If Ʈ$= \{\emptyset$, Ɲ, $\{u, m, g\}, \{j\}\}$, I $= \{\emptyset, \{j\}\}$, $\Upsilon = \{p, q\}$, δ$= \{\Upsilon, \emptyset, \{p\}\}$ and $f(u) = f(j) = p$, $f(m) = f(g) = q$, $f^{-1}(\{p\}) = \{u, j\} \notin$ Ʈ, $\{u, j\} \cap \{j\} = \{j\}$, $Cl^*(\{j\}) = \{j\} \subseteq \mathrm{int}(\{u, j\}) = \{j\}$, since $\mathrm{int}(\{u, j\}) = \{j\}$. That is $f^{-1}(\{p\})$ are (**ic − I − contm**) and (**icc − I − contm**) but not (**α − I – contm**) and not (**s − I − contm**).

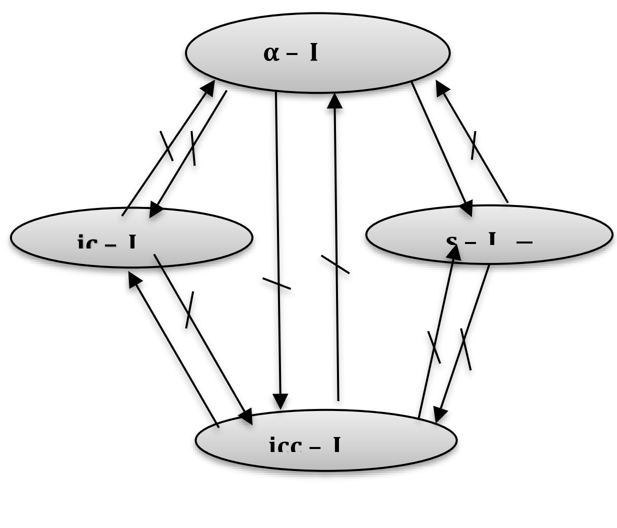

Fig. 2. Relationships of (ic-I-contm) with other (contm) mentioned above**.**

2.8. Theorem : Assume $\mathcal{Z} \subset \mathrm{N}$ of ideal Topological spaces $(\mathrm{N}, \mathrm{T}, \mathbf{I})$. Then
1) if $\mathcal{Z}$ is $\mathbf{semi-I-open}$, then $\mathcal{Z}^c$ is $\mathbf{ic-I-open}$.
2) if $\mathcal{Z}$ is open and $\mathbf{ic-I-closed}$, then $\mathcal{Z}$ is $\mathbf{semi-I-open}$.

Proof. Part[1] Assume that $\mathcal{Z}$ be an $(\mathbf{s-I-o})$ this means $\mathcal{Z} \subset \boldsymbol{Cl}^*(\mathbf{int}(\mathcal{Z}))$. If $\mathbf{int}(\{\mathcal{Z}\}) \not\subset \boldsymbol{E}$, for all $\boldsymbol{E} \in \mathrm{T}^c \setminus \{\emptyset, \mathrm{N}\}$ leads to $\mathbf{int}(\mathcal{Z}) \subseteq \boldsymbol{E}^c$, for all $\boldsymbol{E}^c \in \mathrm{T} \setminus \{\emptyset, \mathrm{N}\}$ and this kind of topology is not our interest. So, $\mathbf{int}(\mathcal{Z}) \subseteq \boldsymbol{E}$, for some $\boldsymbol{E} \in \mathrm{T}^c \setminus \{\emptyset, \mathrm{N}\}$,since $\mathbf{int}(\mathcal{Z}) \subset \mathcal{Z}$ always, we have $\mathbf{int}(\mathcal{Z}) \subseteq (\mathcal{Z} \cap \boldsymbol{E})$ for some $\boldsymbol{E} \in \mathrm{T}^c \setminus \{\emptyset, \mathrm{N}\}$. Now, since $\mathcal{Z}$ is $(\mathbf{s-I\ o})$ we get $\mathbf{int}(\mathcal{Z}) \subseteq \mathcal{Z} \subseteq \boldsymbol{Cl}^*(\mathcal{Z} \cap \boldsymbol{E})$ for some $\boldsymbol{E} \in \mathrm{T}^c \setminus \{\emptyset, \mathrm{N}\}$. That is $\mathcal{Z}$ is $\mathbf{ic-I-closed}$. So, $\mathcal{Z}^c$ is $(\mathbf{ic-I-o})$.

Part[2] Let $\mathcal{Z}$ is $\mathbf{ic-I-closed}$. We have $\mathbf{int}(\mathcal{Z}) = \mathcal{Z} \subseteq \boldsymbol{Cl}^*(\mathcal{Z} \cap \boldsymbol{E}) \subseteq \boldsymbol{Cl}^*(\mathcal{Z}) = \boldsymbol{Cl}^*(\mathbf{int}(\mathcal{Z}))$, we get $\mathcal{Z}$ is $(\mathbf{s-I-o})$.

An application example of above theorem is the following:
Let $\mathrm{N} = \{\mathbf{1}, \mathbf{3}, \mathbf{5}\}$, $\mathrm{T} = \{\mathrm{N}, \emptyset, \{\mathbf{5}\}, \{\mathbf{1}, \mathbf{3}\}\}$, $\mathbf{I} = \{\emptyset, \{\mathbf{1}\}\}$, $\mathcal{Z} = \{\mathbf{5}\}$. By theorem 2.8. (part 1) $\mathcal{Z}^{\mathbf{c}}$ is $\mathbf{ic-I-open}$. By theorem 2.8. (part 2), $\mathcal{Z}$ is $\mathbf{semi-I-open}$.

2. Conclusions

This work concludes that all α-I-open and all semi-I-open are not ic-I-open and not icc-I-open set. Furthermore, all weakly icc-I-continuous mapping is weakly ic-I-continuous mapping.

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