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d-Index of Graphs

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Abstract

The new distance defined on a connected graph G contains of three terms: The ordinary distance between any two vertices in G, both the sum and the product of the two vertices' degrees, as this distance is more useful than the ordinary distance, especially in chemical structures because of its effect on the number of bonds (edges) on the atoms (vertices) carbon (graph). In this article, were found index with respect to new distance (d- index) of regular graph, in addition, finding the relationships between d-index. Also, The relationships and The graph were found between the diameter and the radius for the ordinary distance and between the diameter and the radius for the new distance, and finally, The d-index was found for the join operation of two and n^{th} graphs.

Keywords:
Wiener index, d-index, diameter and radius of graph.

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1. INTRODUCTION

Wiener, who suggested the Wiener index in 1947, aroused interest in the study of topological indices [1], and the first Zagreb index was proposed in 1972 [2] by Gutman and Trinajstic, followed by the second Zagreb index in 1975 [3] by Randic. Due to their importance to scientific applications, numerous ancient and modern topological indices have attracted the interest of many researchers; see references [4-14]. Jackuline et al., [15] introduced the ***d***-index, which is regarded as one of the most important modern indices that depends on the degrees of the vertices of the two ends of the path and the distance between them in the connected graph.

Let G be a non-trivial undirected connected graph without multiple edges and loops of order p (the elements number of vertex set $V(G)$) and size q (the elements number of edge set $E(G)$), the degree of a vertex v is the number of edges incident to it, denoted as $\deg(v)$. We state the first theorem, is sometimes called The First Theorem of Graph Theory. **Theorem 1.1 [16]:** Let G be a graph of order p and size q, where $V(G)=\{v_1,v_2, \dots ,v_p\}$. Then, $\sum_{i=1}^{p} degv_i = 2q$.

The distance (ordinary distance) in a connected graph G between any two vertices u and v is the length of a shortest $u-v$ path in G, and it's denoted by $d(u,v)$ [16]. The summation of all distances in a connected graph is represented by the Wiener index, which is denoted as $W(G)$ [1], that is,

$$W(G)=\sum_{\{u,v\}\subseteq V(G)} d(u,v) .$$

The length of greatest geodesic in a connected graph G is called the diameter of G, and it is denoted by $diamG$ or $\delta(G)$, that is

$$diam(G)=\max\{d(u,v): u,v \in V(G)\}.$$

The eccentricity of the vertex v is the maximum distance between v and any vertex u of G, denoted by $e(v)$, that is,

$$e(v) = max\left\{d(v,u): u \in V(G)\right\}.$$

The minimum eccentricity among all vertices in the graph G is known as the radius of G, denoted by $rad(G)$, that is,

$$rad(G)=\min\{e(v): v \in V(G)\}.$$

It is clear that

$$diam(G) = \max\{e(v): v \in V(G)\}.$$

It is well known that the radius and the diameter are related by inequality:

$$radG \le diamG \le 2radG$$ [16].

Many concepts are covered in Ref. [16] including definitions of special graphs such as complete, path, cycle, star, and wheel graphs.

2. On d-distance in graphs:

The $d-$distance between vertices u and v in G is represented by $d^d(u,v)$ and defined by Jackuline et al., [15] as follows:

$$d^d(u,v) = d(u,v) + deg(u) + deg(v) + deg(u) \times deg(v). \quad \ldots(2.1)$$

This definition is significant since it includes three indices, distance, the sum and multiply of the degrees of the ends of a shorted path, and it has an impact in chemical applications, such as the distance between carbon atoms, and the degrees of the vertices represent the bonds that lie on the carbon atoms. Due to the significance of the sum or multiply of the degrees at the ends of the shorted path, Schultz and Gutman referred to it in their researchs. [17-19].

It is clear that $d^d(u,v) \geq 4$; if $u \neq v$ and $d^d(u,u) \geq 3$, indeed $d^d(u,v) \geq d(u,v) + 3$

Thus, we cannot assume $d^d(u,v) = 0$ in proving that the $d-$distance on $V(G)$ is a metric space, "the first axiom of the metric space", as had been done in reference [16]. Indeed, the triangle inequality does hold for the vertices of any nontrivial connected graph, "the fourth axiom of the metric space". One can easily check that Theorem 2.6 in [16] is not true. Notice that for the vertices u, v and w in the graph depicted in Figure 2.1:

$$d^d(u,w) + d^d(w,v) = 58 < 92 = d^d(u,v).$$

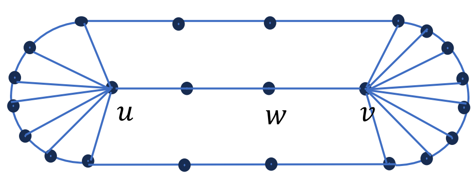

Figure 2.1.

The sum $d-$distances from a certain vertex, $v \in V(G)$ to all the vertices in G which denoted by $\sigma^d(v)$,

$$\sigma^d(v) = \sum_{v \neq u \in V(G)} d^d(v,u) \quad ..(2.2)$$

The $d-$index denoted by $W^d(G)$ is defined as:

$$W^d(G) = \frac{1}{2} \sum_{\substack{v,u \in V(G) \\ v \neq u}} d^d(v,u) \quad \ldots(2.3)$$

It is clear that

$$W^d(G) = \frac{1}{2} \sum_{v \in V(G)} \sigma^d(v). \quad \ldots(2.4)$$

In the following sections, we find $d-$Index, $d-$Diameter and $d-$Radius and the join operation of the graphs respectively.

3. $d-$Index of Graphs

In this section, we give Wiener index with respect to $d-$distance for any connected graph with addition to some special graphs.

Proposition 3.1: For every $v \in V(G)$, then
$\sigma^d(v) = \sigma(v) + 2q + (2q + p - 2)\, deg(v) - (deg(v))^2$.

Proof: From equ. (2.1) and (2.2), we have:

$$\sigma^d(v) = \sum_{v \neq u \in V(G)} d^d(v,u)$$
$$= \sum_{u \in V(G)} d(v,u) + \sum_{u \in V(G)} deg(u) - deg(v) + (p-1)\ \ deg(v) + deg(v) \times \sum_{v \neq u \in V(G)} deg(u).$$
$$= \sigma(v) + 2q + (p-2)\, deg(v) + deg(v)\,(2q - deg(v))$$
$$= \sigma(v) + 2q + (2q + p - 2)\deg(v) - (deg(v))^2 \;. \blacksquare$$

Theorem 3.2: For a non-trivial graph G of order p and size q, we have:

$$W^d(G) = W(G) + 2q(q + p - 1) - \frac{1}{2} \sum_{v \in V} (deg(v))^2 \;.$$

Proof: From equ. (2.4) and Proposition 3.1, we get:

$$W^d(G) = \frac{1}{2} \sum_{v \in V(G)} \sigma^d(v)$$
$$= \frac{1}{2} \left[\sum_{v \in V} \sigma(v) + 2qp + (2q + p - 2) \sum_{v \in V} deg(v) - \sum_{v \in V} (deg(v))^2\right]$$
$$= W(G) + pq + \frac{1}{2}(2q + p - 2).2q - \frac{1}{2} \sum_{v \in V} (deg(v))^2$$
$$= W(G) + 2q(q + p - 1) - \frac{1}{2} \sum_{v \in V} (deg(v))^2. \;\blacksquare$$

Example 3.3:

1. $W^d(P_p) = \sum_{i=1}^{p-1} (p-i)i + 2p(2p-5) + 7$
2. $W^d(S_p) = (p-1)\left(\frac{9}{2}\, p - 5\right)$
3. $W^d(W_p) = (p-1)\left(\frac{25}{2} p - 18\right)$
4. $W^d(K_{n,m}) = \frac{mn}{2}(4mn + 3(m+n) - 2) + m(m-1) + n(n-1)$

Corollary 3.4: If G is a $r-$regular graph of order p, then

$$W^d(G) = W(G) + \frac{1}{2} rp(p-1)(r+2).$$

Proof: It is clear that $q = \frac{1}{2} rp$, thus from Theorem 3.2, we obtain

$$W^d(G) = W(G) + rp\left(\frac{1}{2} rp + p - 1\right) - \frac{1}{2} \sum_{v \in V} r^2$$
$$= W(G) + \frac{1}{2} rp(rp + 2p - r - 2).$$

Then,

$$W^d(G) = W(G) + \frac{1}{2} pr(p-1)(r+2). \quad \blacksquare$$

Example 3.5:

1. $W^d(C_p) = \begin{cases} \frac{p}{8}\ (p-1)(p+33), & if\ p\ is\ even \\ \frac{p}{8}\ (p^2 + 32p - 32\ ), & p\ is\ odd \end{cases}$
2. $W^d(K_p) = \frac{1}{2} p^3 (p-1)$
3. $W^d(K_{n,n}) = n(2n^3 + 3n^2 + n - 2)$

4. $d-$Diameter and $d-$Radius of a Graph:

For any connected graph G, the $d-$eccentricity of a vertex v, $v \in V(G)$ is defined as:

$$e^d(v) = max_{u \in V(G)} \{d^d(u,v)\}.$$
$$= e(v) + deg(u) + deg(v) + deg(u) \times deg(v).$$

The maximum $d-$ eccentricity is called the $d-$diameter of a graph G ($diam^d(G)$) is defined as:

$$diam^d(G) = max_{v \in V(G)} \{e^d(v)\}\;.$$

The minimum $d-$ eccentricity is called the $d-$radius of a graph G ($rad^d(G)$) is defined as:

$$rad^d(G) = min_{v \in V(G)} \{e^d(v)\}\;.$$

Hence,

$rad^d(G) \le diam^d(G) \le 2rad^d(G)$.

Proposition 4.1: For every connected graph G of order p, the following inequalities hold:

1. $diam\ (G) + 3 \le diam^d\ (G) \le diam\ G + p^2 - 1$.
2. $rad\ (G) + 3 \le rad^d\ (G) \le rad\ G + p^2 - 1$.

Proof: Let $G = (V, E)$ be a connected graph of order p and u, v are two arbitrary vertices in G then

1. Since $d^d(u,v) = d(u,v) + deg(u) + deg(v) + deg(u) \times deg(v)$, then

$$max_{u,v\in V(G)}\ d^d(u,v) = max_{uv\in V(G)}\ \{d(u,v) + deg(u) + deg(v) + deg(u) \times deg(v)\} \le max_{u,v\in V(G)}\ \{d(u,v)\} + p^2 - 1.$$

Hence, $diam^d\ (G) \le\ diam\ (G) + p^2 - 1$.

On the other side for any connected graph, the minimum degree of vertices is one

$$max_{u,v\in V(G)} d^d(u,v) \ge max_{u,v\in V(G)}\{d(u,v)\} + 3.$$

then

$diam^d\ (G) \ge\ diam\ (G) + 3$.

The equality is satisfied if G $= K_2$.

2. Since $rad^d(G) = min_{v\in V(G)}\{e^d(v)\}$

$$e^d(v) = max_{u\in V(G)}\{d^d(u,v)\}.$$

$$e^d(v) = e(v) + deg(u) + deg(v) + deg(u) \times deg(v)$$

by taking the minimum for both sides we get:

$$min_{v\in V(G)} e^d(v) = min_{v\in V(G)} e(v) + deg(u) + deg(v) + deg(u) \times deg(v)$$

Thus,

$$rad^d(G) \le min_{v\in V(G)} e(v) + \mathrm{p}^2 - 1 = rad\ G + p^2 - 1$$

the minimum term of inequality can be proved by the same way as the first point in this Proposition. ∎

Corollary 4.2: If G is a $r - regular$ connected graph of order p, then

1. $diam^d(G) = diam(G) + r(r+2)$.
2. $rad^d(G) = rad(G) + r(r+2)$. ∎

Comparing the diameter and the radius of some special graphs according to the normal distance and the $d -$

Special graphs	$diam(G)$	$diam^d(G)$	$rad(G)$	$rad^d(G)$
$K_p, p \ge 2$	1	p^2	1	p^2
$S_p, p \ge 3$	2	$2p$	1	2p
$P_p, p \ge 4$	$p - 1$	$p + 5$	$\lfloor \frac{p}{2} \rfloor$	$\lfloor \frac{p}{2} \rfloor + 7$
$K_{m,n}, m \ge n \ge 2$	2	$m^2 + 2m + 2$	2	(m+1)(n+1)
$W_p, p \ge 5$	2	$4p$	1	4p
$C_p, p \ge 4$	$\lfloor \frac{p}{2} \rfloor$	$\lfloor \frac{p}{2} \rfloor + 8$	$\lfloor \frac{p}{2} \rfloor$	$\lfloor \frac{p}{2} \rfloor + 8$

3. The Join Operation of The Graphs:

In this section, Wiener indices for the d- distance is presented for the operation defined in graph theory, which is the operation of join between two graphs, in addition, we can generalize the results of the join operation to n^{th} from graphs, $G_1, G_2, \dots, G_n$ which are connected graphs disjoint from each other in the following theorem. We denoted to degree of a vertex u in G_i before matching as $deg_{\,G_i}(u)$.

Definition 5.1: The join $G_1 + G_2 + G_2 \dots + G_n$ is a graph has vertices set $V(G_1 + G_2 + G_2 \dots + G_n) = \cup_{i=1}^{n} V(G_I)$ and edges set $E(G_1 + G_2 + G_2 \dots + G_n) = \cup_{i=1}^{n} E(G_i) \cup \mathrm{F}$, $\mathrm{F} = \{uv: u \in V(G_I), v \in V(G_j), \mathrm{I, j} = 1,2,3 \dots, \mathrm{n}, \mathrm{i} \ne \mathrm{j}\}$.It is clearly that the join operation is a commutative operation.

Theorem 5.2: Let G_1 and G_2 are two connected graphs disjoint from each other have the order $|V(G_1)| = p_1$, $|V(G_2)| = p_2$, the size $|E(G_1)| = q_1$ and $|E(G_2)| = q_2$ respectively. Then

$$W^d(G_1 + G_2) = p_1(p_1 + p_2 - 1) + p_2(p_2 - 1) - (q_1 + q_2) + 2(q_1 + q_2 + p_1p_2)(p_1 + p_2 + q_1 + q_2 + p_1p_2 - 1) - \frac{1}{2}\sum_{w\in V(G_1)}(\deg w)^2 - \frac{1}{2}\sum_{w\in V(G_2)}(deg\ w)^2 - 2p_2q_1 - 2p_1q_2 - \frac{1}{2}p_1p_2^2 - \frac{1}{2}p_2p_1^2.$$

Proof: From Definition 5.1, we note that

$deg_{G_1+G_2} w = p_2 + deg_{G_1} w$, if $w \in V(G_1)$,

$deg_{G_1+G_2} w = p_1 + deg_{G_2} w$, if $w \in V(G_2)$,

$p = |V(G_1 + G_2)| = p_1 + p_2$,

$q = |E(G_1 + G_2)| = q_1 + q_2 + p_1\ p_2$,

and

$$d(u,v) = \begin{cases} 1, & if\quad uv \in E(G_1 + G_2), \\ 2, & othere\ wise. \end{cases}$$

Since

$W(G_1 + G_2) = p_1(p_1 + p_2 - 1) + p_2(p_2 - 1) - (q_1 + q_2)$ [20].

Then, from Theorem 2.3, we obtain

$$W^d(G_1 + G_2) = W(G_1 + G_2) + 2q(q + p - 1) - \frac{1}{2}\sum_{w\in V(G_1+G_2)}(deg(w))^2 .$$
$$= p_1(p_1 + p_2 - 1) + p_2(p_2 - 1) - (q_1 + q_2) + 2(q_1 + q_2 + p_1p_2)(q_1 + q_2 + p_1p_2 + p_1 + p_2 - 1) - \frac{1}{2}\sum_{w\in V(G_1)}(deg\ w + p_2)^2 - \frac{1}{2}\sum_{w\in V(G_2)}(deg\ w + p_1)^2$$

$$= p_1(p_1 + p_2 - 1) + p_2(p_2 - 1) - (q_1 + q_2) + 2(q_1 + q_2 + p_1p_2)(p_1 + p_2 + q_1 + q_2 + p_1p_2 - 1)$$

$$-\frac{1}{2}\sum_{w\in V(G_1)}(\deg w)^2 - \frac{1}{2}\sum_{w\in V(G_2)}(\deg u)^2 - 2p_2q_1 - 2p_1q_2 - \frac{1}{2}p_1p_2^2 - \frac{1}{2}p_2p_1^2 .$$ ∎

Corollary 5.3: If $G = G_1 = G_2$ of order p and size q. Then

$$W^d(G_1 + G_2) = 2p^4 + 3p^3 + p^2 - 2p - 6q + 8q^2 + 4pq + 8qp^2 - \sum_{w\in V(G_1)}(\deg w)^2.$$ ∎

Theorem 5.4: Let G_i be a connected graph of order p_i and size q_i for

$i = 1,2,\dots,n,\ n \geq 2$, the Wiener index and the $d-$index of the joining on $n^{th}-$graphs $G^+ = G_1 + G_2 + G_3 \dots + G_n$ have the following forms:

1. $W(G^+) = \frac{1}{2}\sum_{i=1}^{n} p_i^2 + p\left(\frac{1}{2}p - 1\right) - \sum_{i=1}^{n} q_i$.

2. $W^d(G^+) = \frac{1}{2}\sum_{i=1}^{n} p_i^2 + p\left(\frac{1}{2}p - 1\right) - \sum_{i=1}^{n} q_i + 2q(p + q - 1) - \frac{1}{2}\sum_{i=1}^{n}\sum_{u\in V(G_i)}\left(deg_{G_i}u + (p - p_i)\right)^2$.

Where $p = \sum_{i=1}^{n} p_i$, $q = \sum_{i=1}^{n} q_i + \frac{1}{2}\sum_{i=1}^{n}(p - p_i)\, p_i$

Proof:

1. Let $G_1, G_2, G_3, \dots, G_n$ are n^{th} connected graphs with orders $p_1, p_2, \dots, p_n$ and sizes $q_1, q_2, \dots, q_n$ respectively such that:

$p = |V(G^+)| = \sum_{i=1}^{n}|V(G_i)|$ …(5.1)

$q = |E(G^+)| = \sum_{i=1}^{n}|E(G_i)| + |F|, F = \{uv: u \in V(G_i)\,, v \in V(G_j)\}, i,j = 1,2,\dots n, \mathrm{i} \neq j$ …(5.2)

the ordinary distance between two vertices u and v in the joining graph G^+has two values:

$$d(u,v) = \begin{cases} 1, & if uv \in \mathrm{E}(G^+), \\ 2\,, & othere\ wise. \end{cases} \quad \dots(5.3)$$

Since the number of adjacent vertices is equal to

$q = \sum_{i=1}^{n}|E(G_i)| + |F|$

$= \sum_{i=1}^{n} q_i + \frac{1}{2}\sum_{i=1}^{n}(p - p_i)p_i$ ... (5.4)

The number of vertices which are not adjacent is equal to$\sum_{i=1}^{n}\left\{\frac{1}{2}p_i(p_i - 1) - q_i\right\}$ …(5.5)

Thus from the relations (1.1)and (5.1)-(5.5) we get:

$$W(G^+) = \sum_{i=1}^{n} q_i + \frac{1}{2}\sum_{i=1}^{n}(p - p_i)p_i + 2\sum_{i=1}^{n}\left\{\frac{1}{2}p_i(p_i - 1) - q_i\right\}$$

$= \frac{1}{2}\sum_{i=1}^{n} p_i^2 + \sum_{i=1}^{n} p_i\left(\frac{1}{2}\sum_{i=1}^{n} p_i - 1\right) - \sum_{i=1}^{n} q_i$ ...(5.6)

We can prove the second part of this theorem by depending on the first part of it and the theorem (3.2) specially that the d-distance has also two values as we note:

$$d^d(u,v) = \begin{cases} 1 + deg(u) + deg(v) + deg(u) \times deg(v)\,, if\ uv \in \mathrm{E}(G^+), \\ 2 + deg(u) + deg(v) + deg(u) \times deg(v)\ \ , othere\ wise. \end{cases}$$

any vertex $u \in G_i \subseteq G^+$has the degree:

$deg_{\,\mathrm{G}^+}(u) = deg_{\,G_i}(u) + (p - p_i)$ …(5.7)

From the equations (5.1) , (5.2) ,(5.6),(5.7) and theorem (3.2) the $d-$index of G^+ $(W^d(G^+))$ is defined as:

$$W^d(G^+) = W(G^+) + 2q(p + q - 1) - \frac{1}{2}\sum_{u\in V(G^+)}\left(deg_{\,\mathrm{G}^+}(u)\right)^2$$

$= \frac{1}{2}\sum_{i=1}^{n} p_i^2 + p\left(\frac{1}{2}p - 1\right) - \sum_{i=1}^{n} q_i + 2q(p + q - 1) - \frac{1}{2}\sum_{i=1}^{n}\sum_{u\in V(G_i)}\left(deg_{G_i}u + (p - p_i)\right)^2$,where

$p = \sum_{i=1}^{n} p_i$, $q = \sum_{i=1}^{n} q_i + \frac{1}{2}\sum_{i=1}^{n}(p - p_i)\, p_i$. ■

Corollary 5.5: Let $G = G_1 + G_2 + \cdots + G_{\mathrm{n}}$ be a joining graph of order p and size q. If $G^* = G_1 = G_2 = \cdots = G_{\mathrm{n}}$. of order p^* and size q^*, then

$$W^d(G) = \frac{1}{2}[p(4q + p + p^* - 2) + 4q(q - 1) - 2nq^*] - \frac{1}{2}\,n\sum_{u\in V(G^*)}(deg_{G^*}(u) + (n - 1)p^*)^2 \text{ , where}$$

$$p = np^*, q = nq^* + \frac{n}{2}(n - 1)(p^*)^2$$

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للبيانات d – الدليل

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الملخص

المسافة الجديدة في بيان متصل G تضم ثلاثة حدود و هي على التوالي : المسافة الاعتيادية بين أي رأسين في البيان G ومجموع درجات هذين الرأسين وحاصل ضرب درجات الرأسين . وان هذه المسافة اكثر فائدة من المسافة الاعتيادية وخاصة في دراسة التراكيب الكيميائية. في هذا البحث تم إيجاد الدليل d- لبيان منتظم بالإضافة الى إيجاد علاقات بين القطر ونصف القطر بالنسبة للمسافة الاعتيادية والمسافة d- وأخيرا تم إيجاد الدليل d – لاتصال بيانين واتصال n من البيانات.

الكلمات المفتاحية: دليل وينر، دليل – d القطر ونصف القطر للبيانات .