



Hybrid Finite Differences Technique for Solving the Nonlinear Fractional Korteweg-De Vries-Burger Equation

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Abstract

This study presents a new algorithm for effectively solving the nonlinear fractional Korteweg-de Vries-Burger equation (NFKDV-B) using a hybrid explicit finite difference technique with the Adomian polynomial (HEFD). The suggested technique addresses the problem of accurately solving the FKDV-B equation with fractional nonlinear space derivatives in numerical solutions. Numerical results are obtained by comparing the exact solution with absolute and mean square errors. The fractional time and space derivatives are estimated using two widely used techniques: the Caputo derivative and the shifted Grünwald-Letnikov (G-L) formulas.

Using a test problem to assess the HEFD method accuracy against the exact solution and the conventional explicit finite difference (EFD) method. The results exhibit excellent agreement between the approximate and exact solutions at different time values. The findings highlight the effectiveness of the proposed method across a range of fractional derivative values when compared to the exact solution and conventional explicit finite difference methods.

Keywords:

Hybrid finite difference; FKDV-B equation; Adomian polynomial ; Caputo sense; Grünwald-Letnikov.

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1. INTRODUCTION

Fractional differential equations (FDEs) and their applications in various branches of science and engineering have attracted increasing attention in recent years [1]. FDEs, including fractional derivatives and integrals, provide a robust mathematical framework for understanding complex physical and chemical processes [2]. Fractional derivatives have applications in diverse areas, such as fractal theory, quantum economics, fluid dynamics, viscoelasticity, and control systems [3].

Our research specifically focuses on the nonlinear fractional Korteweg-de Vries-Burger equation (NFKDV-B), which extends the well-known Korteweg-de Vries-Burger equation by incorporating fractional derivatives [4]. This equation arises from studying various physical phenomena that exhibit nonlinearity and dispersion effects. The introduction of

fractional derivatives allows for the modeling of nonlocal behavior and memory effects, enhancing the accuracy of the representation of complex systems [5].

The FKDV-B equation can be expressed mathematically as:

$$u_t^\alpha + \varepsilon uu_x^\beta + \mu u_{xx}^{2\beta} + \eta u_{xxx}^{3\beta} = 0, 0 < \alpha, \beta \leq 1. \quad (1)$$

where α and β represent the order of fractional derivatives, $u(x, t)$ represents the dependent variable, and ε, μ , and η are coefficients governing the strengths of the nonlinear, dispersive, and dissipative terms, respectively [6].

The FKDV-B equation exhibits intriguing mathematical properties and dynamics, supporting various solution types, including solitary waves, compactons, and localized structures [7]. Due to their utility in characterizing wave phenomena in fluid dynamics, nonlinear optics, and plasma physics, these solutions have attracted great attention [8].

The fractional derivatives make analytical solutions for the

FKDV-B equation difficult. Thus, numerical methods and approximations are necessary for solving the equation[9]. The finite difference[10], spectral, and fractional derivative-specific numerical approximations are used[11]. The Adomian decomposition method (ADM) has become famous for solving fractional differential equations (FDEs). Decomposing the FDE into simpler sub-problems allows this method to approximate solutions efficiently [12].

FDEs model complex phenomena because fractional differentiation orders capture nonlocal and memory effects in system dynamics. Fractional derivatives in the decomposition process allow the ADM to handle these unique characteristics[13]. The ADM relies on fraction differential equation specific Adomian polynomials. These numerical methods reveal the dynamics of the FKDV-B equation, helping us understand its complex behavior and model complex physical systems.

In this research, we propose a new hybrid method that combines the finite difference method with the Adomian polynomial to address the nonlinear term in the FKDV-B equation. This hybrid technique provides an innovative treatment for handling the nonlinearities present in the equation, enhancing the accuracy and efficiency of the solution.

2. Main Concepts

Caputo derivative and the shifted G-L formula are widely utilized for approximating fractional derivatives. In the shifted G-L formula discretizes the derivative operator through finite differences, facilitating numerical approximations. Conversely, the Caputo derivative defines the fractional derivative as an integral of the function's derivative, effectively capturing the behavior associated with fractional orders. These methods are instrumental in accurately modeling nonlocal and memory effects within fractional calculus applications [14].

2.1 Finite Difference Method.

The finite difference technique is a widely used numerical method for approximating derivatives and solving differential equations. It involves discretizing the domain of interest into a set of grid points and replacing the derivatives in the original equation with finite difference approximations. Now, Let H represent the spatial mesh size where $H = (L - a)/n$, and $\tau = t/m$ be the amount of time increase (time step size) write them as: $x_i = a + iH$, $\tau_j = j\tau$, $u_i^j = u(x_i, \tau_j)$ for $i = 0, 1, \dots, n$ and $j = 0, 1, \dots, m$. In Fractional Derivative orders need to use the following formulas:

- The forward finite difference formula with the Caputo derivative is used to approximate the time-fractional derivative [15] :

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = \frac{\tau^{-\alpha}}{\Gamma(2-\alpha)} \sum_{j=0}^k b_j [u(x_i, t_{k+1-j}) - u(x_i, t_{k-j})]. \dots(2)$$
with setting $b_j = (j + 1)^{1-\alpha} - j^{1-\alpha}$.

- For fraction space derivatives use the standard G-L formula as following[14] :

$$(u_x^\beta)_i^j = \frac{1}{H^\beta} \sum_{k=0}^{i+1} g_k^\beta u_{i-k+1}^j. \dots(3)$$

2.2 The Adomian Polynomials:

The Adomian polynomials can be determined for all nonlinearities [13]. In this nonlinearity, the general formula of Adomian polynomials A_i yields:

$$A_i = \frac{1}{i!} \left[\frac{d^i}{d\lambda^i} \phi \left(\sum_{i=0}^{\infty} \lambda^i u_i \right) \right]_{\lambda=0} = \frac{1}{i!} \left[\frac{d^i}{d\lambda^i} \left(\sum_{i=0}^{\infty} \lambda^i u_i(x, t) \right) D_x^\beta \left(\sum_{i=0}^{\infty} \lambda^i u_i(x, t) \right) \right], i \geq 0. \dots(4)$$

where D_x^β represents the β^{th} order derivative . For easier computations using the Maple software to get as many polynomials as we need :

$$\begin{aligned} A_0 &= u_0 D_x^\beta u_0 \\ A_1 &= u_1 D_x^\beta u_0 + u_0 D_x^\beta u_1 \\ A_2 &= u_2 D_x^\beta u_0 + u_1 D_x^\beta u_1 + u_0 D_x^\beta u_2 \\ A_3 &= u_3 D_x^\beta u_0 + u_2 D_x^\beta u_1 + u_1 D_x^\beta u_2 + u_0 D_x^\beta u_3 \\ &\vdots \end{aligned}$$

In generals :

$$A_n = \left(\sum_{m=j}^j u_i^{j-1+m} D_x^\beta u_i^{j-1+n} \right). \dots(5)$$

3. Methodology

In this section, we proposed the HEFD to get numerical approximation to Equation (1). The HEFD method which has a tretment for the fraction nonlinear term in FKDV-B equation as will derive its formula and an algorithm for it.

3.1 Mathematical Formulation of the Proposed Method:

The FKDV-B Equation (1) approximated at the mesh point (x_i, τ_{j+1}) as follows:

First replacing the time-fractional derivative in Equation (1) by Caputo derivative (2) in the forward finite difference formula. In the second and third terms the fractional space derivatives are approximated by the standard G-L formula (3) as follows:

$$(u_{xx}^{2\beta})_i^j = \frac{1}{H^{2\beta}} \sum_{k=0}^{i+1} g_k^{2\beta} u_{i-k+1}^j. \dots(6)$$

$$(u_{xxx}^{3\beta})_i^j = \frac{1}{H^{3\beta}} \sum_{k=0}^{i+1} g_k^{3\beta} u_{i-k+1}^j. \dots(7)$$

The nonlinear term is approximated using the Adomian polynomials formula (5) by replacing the derivative in the term by the standard G-L formula (3) as follows :

$$\text{For } j=0, A_0 = (u_0)_i^j \left(\frac{1}{H^\beta} \sum_{k=0}^{i+1} g_k^\beta (u_0)_{i-k+1}^j \right)$$

$$\text{For } j=1, A_1 = (u_1)_i^j \left(\frac{1}{H^\beta} \sum_{k=0}^{i+1} g_k^\beta (u_0)_{i-k+1}^j \right) +$$

$$(u_0)_i^j \left(\frac{1}{H^\beta} \sum_{k=0}^{i+1} g_k^\beta (u_1)_{i-k+1}^j \right)$$

$$\text{For } j=2, A_2 = (u_2)_i^j \left(\frac{1}{H^\beta} \sum_{k=0}^{i+1} g_k^\beta (u_0)_{i-k+1}^j \right) +$$

$$(u_1)_i^j \left(\frac{1}{H^\beta} \sum_{k=0}^{i+1} g_k^\beta (u_1)_{i-k+1}^j \right) + (u_0)_i^j \left(\frac{1}{H^\beta} \sum_{k=0}^{i+1} g_k^\beta (u_2)_{i-k+1}^j \right).$$

And so on. In general

$$A_j = \sum_{i=0}^m (u_i)^{j-m+1} \left(\frac{1}{H^\beta} \sum_{k=0}^{i+1} g_k^\beta (u)_{i-k+1}^{j-m} \right). \quad \dots(8)$$

Now by compensation equations (2,6,7) and (8) in equation (1) Produce :

$$\begin{aligned} & \frac{\tau^{-\alpha}}{\Gamma(2-\alpha)} \sum_{k=0}^j b_k [u(x_i, \tau_{j+1-k}) - u(x_i, \tau_{j-k})] + \\ & \varepsilon \sum_{j=0}^m (u_i)^{j-m+1} \left(\frac{1}{H^\beta} \sum_{k=0}^{i+1} g_k^\beta (u)_{i-k+1}^{j-m} \right) + \\ & \mu \frac{1}{H^{2\beta}} \sum_{k=0}^{i+1} g_k^{2\beta} u_{i-k+1}^j + \eta \frac{1}{H^{3\beta}} \sum_{k=0}^{i+1} g_k^{3\beta} u_{i-k+1}^j. \quad \dots(9) \end{aligned}$$

By simple calculations and let $\vartheta = \Gamma(2 - \alpha) \tau^\alpha$:

$$\begin{aligned} u(x_i, \tau_{j+1}) &= u(x_i, \tau_j) - \sum_{k=1}^j b_k [u(x_i, \tau_{j+1-k}) - u(x_i, \tau_{j-k})] - \\ & \varepsilon \vartheta \sum_{j=0}^m (u_i)^{j-m+1} \left(\frac{1}{H^\beta} \sum_{k=0}^{i+1} g_k^\beta (u)_{i-k+1}^{j-m} \right) - \\ & \mu \frac{\vartheta}{H^{2\beta}} \sum_{k=0}^{i+1} g_k^{2\beta} u_{i-k+1}^j - \eta \frac{\vartheta}{H^{3\beta}} \sum_{k=0}^{i+1} g_k^{3\beta} u_{i-k+1}^j. \quad \dots(10) \end{aligned}$$

Formula (10) represent the Hybrid explicit finite difference technique with Adomian polynomial for nonlinear term.

3.2 Algorithm of Hybrid Method:

Input : $\alpha, \beta, \varepsilon, \mu$ and η , number of space fragmentation n and for time m .

Step 1: Find the values of space step size $H = (L - a)/n$, and time step size $\tau = \frac{t}{m}$.

Step 2: Evaluate $\vartheta = \Gamma(2 - \alpha) \tau^\alpha$, $r_1 = \frac{\vartheta}{H^\beta}$, $r_2 = \frac{\vartheta}{H^{2\beta}}$ and $r_3 = \frac{\vartheta}{H^{3\beta}}$.

Step 3: Compute the initial condition for $i = 0, 1, 2, \dots, n$.
 $u(x, 0) = u_0(x), \quad a < x < L$.

Step 4: For all $j = 1, 2, 3, \dots, m - 1$.

Step 5: Set $i = 1, 3, \dots, n$.

Set $\Lambda_1 = 0; \Lambda_2 = 0; \Lambda_3 = 0; \Lambda_4 = 0; \Lambda_5 = 0; \Lambda_6 = 0$.

Step 6: Compute for all $k = 0, 1, 2, \dots, i + 1$.

$$\begin{aligned} \Lambda_1 &= \Lambda_1 + (-1)^k \times \frac{\beta(\beta-1)\dots(\beta-k+1)}{k!} u_{i-k+1}^j \cdot \\ \Lambda_2 &= \Lambda_2 + (-1)^k \times \frac{(2\beta)((2\beta)-1)\dots((2\beta)-k+1)}{k!} u_{i-k+1}^j \cdot \\ \Lambda_3 &= \Lambda_3 + (-1)^k \times \frac{(3\beta)((3\beta)-1)\dots((3\beta)-k+1)}{k!} u_{i-k+1}^j \cdot \\ \Lambda_4 &= \Lambda_4 + u_{i-k+1}^j \cdot \\ \Lambda_5 &= \Lambda_5 + (\Lambda_4 \Lambda_1) \cdot \end{aligned}$$

Step 6: Compute for $k = 1, 2, \dots, j + 1$.

$$\Lambda_6 = \Lambda_6 + ((k + 1)^{1-\alpha} - k^{1-\alpha}) [u(x_i, \tau_{j+1-k}) - u(x_i, \tau_{j-k})] \cdot$$

Step 7: Use the formula in equation (10) to evaluate the numerical solution of equation (1):

$$u(x_i, \tau_{j+1}) = u(x_i, \tau_j) - \Lambda_6 - \varepsilon \vartheta \Lambda_5 - \mu \vartheta \Lambda_2 - \eta \vartheta \Lambda_3 \cdot$$

Step 8: Print the numerical solution $u(x, t)$.

4. Numerical Application.

The exact solution of the FKDV-B equation (1) proposed by Cevikel[6]:

$$U(x, t) = \frac{-\frac{12bv^2}{25\mu} e^{-\left(\frac{vx\beta}{5\mu\Gamma(1+\beta)} + \frac{6v^3t^\alpha}{125\mu^2\Gamma(1+\alpha)}\right)}}{\frac{b^2}{4c} e^{\left(\frac{vx\beta}{5\mu\Gamma(1+\beta)} + \frac{6v^3t^\alpha}{125\mu^2\Gamma(1+\alpha)}\right)} + b+c e^{-\left(\frac{vx\beta}{5\mu\Gamma(1+\beta)} + \frac{6v^3t^\alpha}{125\mu^2\Gamma(1+\alpha)}\right)}}. \quad (11)$$

where b and c are arbitrary parameters, $\Gamma(\cdot)$ is the gamma function. In Tables 1 and 2, compare the numerical results of the EFD method and the HEFD method with the exact solution by using the absolute error (ABSE) and mean square

error (MSE)[16]:

$$ABSE = |U_i(x, t) - u_i(x, t)|.$$

$$MSE = \sqrt{\frac{\sum_{i=0}^n (U_i(x, t) - u_i(x, t))^2}{n}}$$

Taking the FKDV-B equation as in the form equation (1) with

$$\left. \begin{aligned} u(a, t) &= \frac{-\frac{12bv^2}{25\mu} e^{-\left(\frac{6v^3t^\alpha}{125\mu^2\Gamma(1+\alpha)}\right)}}{\frac{b^2}{4c} e^{\left(\frac{6v^3t^\alpha}{125\mu^2\Gamma(1+\alpha)}\right)} + b+c e^{-\left(\frac{6v^3t^\alpha}{125\mu^2\Gamma(1+\alpha)}\right)}} \cdot \\ u(L, t) &= \frac{-\frac{12bv^2}{25\mu} e^{-\left(\frac{vL\beta}{5\mu\Gamma(1+\beta)} + \frac{6v^3t^\alpha}{125\mu^2\Gamma(1+\alpha)}\right)}}{\frac{b^2}{4c} e^{\left(\frac{vL\beta}{5\mu\Gamma(1+\beta)} + \frac{6v^3t^\alpha}{125\mu^2\Gamma(1+\alpha)}\right)} + b+c e^{-\left(\frac{vL\beta}{5\mu\Gamma(1+\beta)} + \frac{6v^3t^\alpha}{125\mu^2\Gamma(1+\alpha)}\right)}} \cdot \end{aligned} \right\} (12)$$

and the initial condition:

$$u(x, 0) = \frac{-\frac{12bv^2}{25\mu} e^{-\left(\frac{vx\beta}{5\mu\Gamma(1+\beta)}\right)}}{\frac{b^2}{4c} e^{\left(\frac{vx\beta}{5\mu\Gamma(1+\beta)}\right)} + b+c e^{-\left(\frac{vx\beta}{5\mu\Gamma(1+\beta)}\right)}}, \quad a \leq x \leq L. \quad \dots(13)$$

When equation (1) is solved by using the formula in Equation (10), the results are shown in Tables 1 and 2. In all computations we fixed $b = c = 10, \varepsilon = -6, \mu = 3, \eta = 6, a = 0$ and $L = 40$, taking various values of α and β and at different times, so we focus on the effectiveness of the proposed technique by comparing it with EFD method and the exact solution. All computations evaluate using the Matlab 2021a software.

Table 1. Compares the HEFD and EFD Methods with the Analytic Solution with values of $\alpha = 0.75, \beta = 0.25$ and various times

x	Time=0.5		Time=1.0		Time=1.5	
	ABSE HEFD	ABSE EFD	ABSE HEFD	ABSE EFD	ABS HEFD	ABSE EFD
0	0	0	0	0	0	0
4	5.0918E-04	5.6682E-03	1.9350E-04	8.7712E-03	1.4742E-03	1.0524E-02
8	5.3600E-04	5.3171E-03	8.4678E-04	8.2486E-03	3.0737E-03	9.9340E-03
12	5.5107E-04	5.0656E-03	1.2677E-03	7.8759E-03	4.0799E-03	9.5136E-03
16	5.6052E-04	4.8717E-03	1.5658E-03	7.5896E-03	4.7798E-03	9.1907E-03
20	5.6703E-04	4.7152E-03	1.7891E-03	7.3587E-03	5.2973E-03	8.9304E-03
24	5.7184E-04	4.5846E-03	1.9630E-03	7.1664E-03	5.6961E-03	8.7135E-03
28	5.7561E-04	4.4730E-03	2.1022E-03	7.0020E-03	6.0127E-03	8.5279E-03
32	5.7864E-04	4.3748E-03	2.2163E-03	6.8493E-03	6.2692E-03	8.3522E-03
36	5.9256E-04	4.1903E-03	2.3209E-03	6.6437E-03	6.5544E-03	7.8352E-03
40	0	0	0	0	0	0
MSE	1.5294E-04	1.3166E-03	4.6874E-04	2.0480E-03	1.3788E-03	2.4796E-03

Table 2. Compares the HEFD and EFD Methods with the Analytic Solution with values of $\alpha = 0.5, \beta = 0.5$ and various times

x	Time=0.5		Time=1.0		Time=1.5	
	ABSE HEFD	ABSE EFD	ABSE HEFD	ABSE EFD	ABS HEFD	ABSE EFD
0	0	0	0	0	0	0
4	1.7360E-03	4.2254E-03	4.2372E-04	5.5302E-03	1.0368E-03	6.4541E-03
8	1.3924E-03	3.1289E-03	3.7567E-04	4.2850E-03	2.2014E-03	5.1445E-03
12	1.2442E-03	2.5047E-03	5.3173E-04	3.5692E-03	2.3299E-03	4.3780E-03
16	1.1650E-03	2.0881E-03	4.7981E-04	3.0815E-03	2.1377E-03	3.8464E-03
20	1.1140E-03	1.7844E-03	3.5542E-04	2.7176E-03	1.8407E-03	3.4427E-03
24	1.0755E-03	1.5503E-03	2.1063E-04	2.4299E-03	1.5205E-03	3.1181E-03
28	1.0425E-03	1.3624E-03	6.7525E-05	2.1932E-03	1.2107E-03	2.8464E-03
32	1.0114E-03	1.2062E-03	6.3917E-05	1.9895E-03	9.2579E-04	2.6094E-03
36	1.0091E-03	1.0263E-03	2.0013E-04	1.7310E-03	6.6542E-04	2.2980E-03
40	0	0	0	0	0	0
MSE	3.3258E-04	6.3134E-04	9.3287E-05	8.9180E-04	4.4892E-04	1.0903E-03

The numerical results obtained from Tables 1 and 2 reveal that the hybrid approach employed in this study surpasses the explicit finite difference method in terms of accuracy and efficiency. Specifically, the hybrid approach demonstrates superior performance when utilizing the values of $\alpha = 0.75$ and $\beta = 0.25$ for time values 0.5, 1, and 1.5. Moreover, the advantageous properties of the hybrid approach are maintained even when applying $\alpha = 0.5$ and $\beta = 0.5$, as observed in Table 2. These findings highlight the effectiveness of the hybrid approach in effectively handling the nonlinearities of the equation, resulting in precise and efficient solutions.

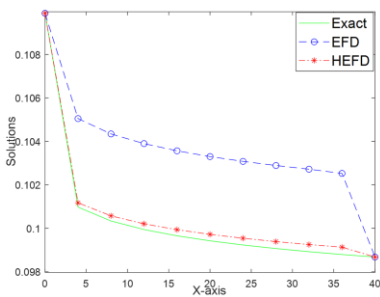


Figure1. HEFD and EFD with Analytic solution $\alpha = 0.9$ and $\beta = 0.1$ at Time=0.5

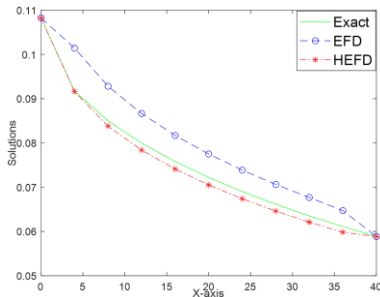


Figure2. HEFD and EFD with Analytic solution $\alpha = 0.5$ and $\beta = 0.5$ at Time 1

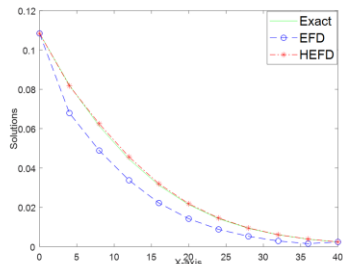


Figure 3. HEFD and EFD with Analytic solution $\alpha = 0.25$ and $\beta = 0.9$ at Time=0.8

Furthermore, the comparison of the results obtained from the hybrid technique, the explicit method, and the exact solution can be observed in Figures 1, 2, and 3 at different time instances,

considering various values of α and β ($\alpha > \beta$, $\alpha = \beta$, and $\alpha < \beta$). These figures visually represent the behavior and performance of the solutions generated by the two methods. Additionally, Figures 4, 5, and 6 illustrate the overall behavior of the solutions across the entire domain when $\beta = 0.1$ and $\alpha = 0.9$. Analyzing these figures enables a comprehensive assessment of the hybrid technique's accuracy, efficiency, and suitability in different scenarios.

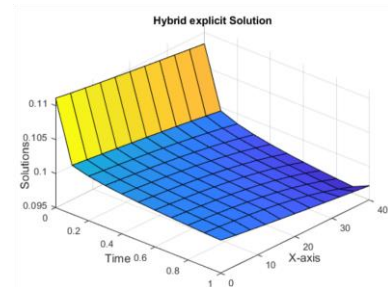


Figure 4. HEFD method with $\alpha = 0.9$ and $\beta = 0.1$

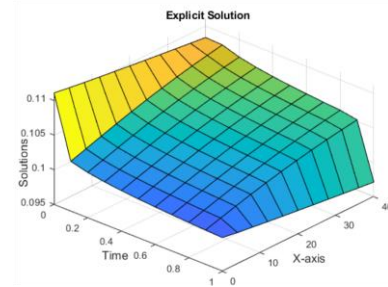


Figure 5. EFD method with $\alpha = 0.9$ and $\beta = 0.1$

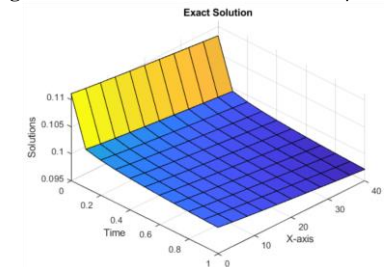


Figure 6. Exact Solution with $\alpha = 0.9$ and $\beta = 0.1$

5. Conclusion

In this research, a new hybrid technique proposes to combine the finite difference method with the Adomian polynomial to address the nonlinear term in the FKDV-B equation. This hybrid approach provides an innovative treatment for handling the nonlinearities present in the equation, enhancing the accuracy and efficiency of the solution. By leveraging the strengths of both methods, we aim to overcome the limitations of traditional numerical techniques and obtain more accurate approximations for the FKDV-B equation. The finite difference method provides a robust framework for discretizing the equation in space, while the Adomian decomposition method polynomial offers a systematic and efficient approach for solving the resulting linear sub-

problems.

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تقنية الفروقات المنتهية المهجنة لحل معادلة Korteweg-De Vries-Burger الكسرية غير الخطية

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الملخص

تقدم هذه الدراسة خوارزمية جديدة لحل معادلة Korteweg-de Vries-Burger غير الخطية (NFKDV-B) بشكل فعال باستخدام تقنية الفروقات المنتهية الصريحة المهجنة مع متعددة حدود أدوميان. التقنية المقترحة تعالج مشكلة دقة الحلول العددية لمعادلة FKDV-B بوجود المشتقات الكسرية غير الخطية.

تم الحصول على النتائج العددية من خلال حساب الخطأ المطلق ومتوسط الخطأ التريبيعي. يتم تقريب مشتقة الزمن الكسرية ومشتقات الفضاء باستخدام تعريف Caputo وصيغة Grünwald-Letnikov، على التوالي. قيمت التقنية بمقارنتها مع الحل المضبوط وطريقة الفروقات المنتهية الصريحة. تظهر النتائج تقريب ممتاز بين الحلول التقريبية والدقيقة عند قيم مختلفة للزمن. تسلط النتائج الضوء على فعالية التقنية المقترحة عند قيم مختلفة للمشتقات الكسرية و مقارنتها بالحل المضبوط وطريقة الفروقات المنتهية الصريحة.

الكلمات المفتاحية: الفروقات المنتهية المهجنة؛ معادلة NFKDV-B؛ متعددة حدود أدوميان؛ تعريف Caputo؛ صيغة Grünwald-Letnikov.