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Solving special systems of integral equations by using Sumudu transform with a semi-analytical method

Waleed Al-Hayani

Department of Mathematics, College of Computer Science and Mathematics, University of Mosul, Iraq
Corresponding author. Email: waleedalhayani@uomosul.edu.iq / waleedalhayani@yahoo.es
https://orcid.org/0000-0001-9918-857

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Abstract

In this paper, specific forms of non-linear systems of integral equations have been solved using the Sumudu transform and the Adomian decomposition method. To illustrate the method, three examples of various forms in this class of functional equations have been developed. The method has produced an approximate-exact solution for several systems. The method's accuracy, efficiency, and simplicity are demonstrated by the results of applying it to various systems of these kinds of integral equations.

Keywords:
Double Sumudu transform; Adomian decomposition method (ADM); Adomian polynomials; Special systems of integral equations.

Correspondence:
Author: Waleed Al-Hayani
Email: waleedalhayani@uomosul.edu.iq

I. INTRODUCTION

Watugala [1] proposed the Sumudu transform in the early 1990s, and it was used to solve ordinary differential equations in control engineering problems. The broad and fundamental properties of the Sumudu transform are given by Asiru [2], Belgacem et. al. [3,4], and Al-Taee and Al-Hayani [5]. Watugala [6] has applied the Sumudu transform to two-variable functions and solved partial differential equations. Tchuenche and Mbare [7] used the double Sumudu transform to solve a population dynamics evolution equation. Kilicman and Gadain [8] have used the double Laplace transform and double Sumudu transform to solve non-homogeneous wave equations. Ahmed et. al. [9] established the convergence of the double Sumudu transform.

G. Adomian [10,11] proposed a new and fruitful approach for solving ordinary, partial, and integral equations termed the Adomian decomposition method (ADM) in the early 1980s. The ADM is a semi-analytical method, this strategy has been found to result in a rapid convergence of the solution series. The non-linear term is decomposed into Adomian's polynomials, which are a set of special polynomials.

The main goal of this work is to solve special non-linear systems of integral equations with functions of two variables by using the Sumudu transform with the semi-analytical Adomian Decomposition Method (ST-ADM).

II. SUMUDU TRANSFORM PRELIMINARIES

Definition 1. [1,12]: The Sumudu transform is defined over the set of functions

$$A = \begin{Bmatrix} f(t) : \exists\, M, \tau_1, \tau_2 > 0 \text{ such that} \\ |f(t)| < Me^{\frac{|t|}{\tau_j}},\ \text{ if } t \in (-1)^j \times [0, \infty) \end{Bmatrix} \qquad \dots (1)$$

by the following formula

$$F(r) = \mathbb{S}\{f(t)\}(r) = \frac{1}{r}\int_0^\infty f(t)e^{-\frac{t}{r}}dt,\ t > 0, r \in (-\tau_1, \tau_2) \quad (2)$$

provided the integral exists for some r, where $F(r)$ is the Sumudu transform of the function $f(t)$. We refer to $f(t)$ as the inverse Sumudu transform of $F(r)$ is given by

$$f(t) = \mathbb{S}^{-1}\{F(r)\} = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} \frac{1}{r}F\left(\frac{1}{r}\right)e^{tr}dr$$
$$= \sum \text{residues of } \left[\frac{1}{r}F\left(\frac{1}{r}\right)e^{tr}\right], r > 0 \qquad \dots (3)$$

The basic properties of the Sumudu transform can be found in [3,4].

Definition 2. [7,12]: If $f(x,y)$ is a convergent infinite series, then its double Sumudu transform is as follows:

$$F(p,q) = \mathbb{S}[f(x,y);(p,q)] = \frac{1}{pq}\int_0^\infty \int_0^\infty f(x,y)e^{-\left(\frac{x}{p}+\frac{y}{q}\right)}dxdy, \quad (4)$$

The double inverse Sumudu transform of $F(p,q)$ can be written as

$$f(x,y) = \mathbb{S}^{-1}\{F(p,q)\} = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{\frac{x}{p}}dp \frac{1}{2\pi i}\int_{\delta-i\infty}^{\delta+i\infty} e^{\frac{y}{q}} f(p,q)dq, \quad (5)$$

where $f(x,y)$ is an analytic function for all p and q.

In the following Table 1, we give the double Sumudu transforms for some functions of two variables x,y.

TABLE 1. [7,12] Double Sumudu transform of some functions

Function $f(x,y)$	Double Sumudu Transform $F(p,q)$
$f(x,y)=k,$	$F(p,q)=k, \quad k\in\mathbb{R}$
$f(x,y)=x^m y^n,$	$F(p,q)=m!\,n!\,p^m q^n, \quad m,n\in\mathbb{R}$
$f(x,y)=e^{ax+by},$	$F(p,q)=\frac{1}{(1-ap)(1-bq)},$
$f(x,y)=\sin(ax+by),$	$F(p,q)=\frac{ap+bq}{[1+(ap)^2][1+(bq)^2]},$
$f(x,y)=\cos(ax+by),$	$F(p,q)=\frac{1-(ap)(bq)}{[1+(ap)^2][1+(bq)^2]},$
$f(x,y)=\sinh(ax+by),$	$F(p,q)=\frac{ap+bq}{[1-(ap)^2][1-(bq)^2]},$
$f(x,y)=\cosh(ax+by),$	$F(p,q)=\frac{1-(ap)(bq)}{[1-(ap)^2][1-(bq)^2]}.$

III. APPLICATION OF ST-ADM

The following problems have been selected from the current literature and are studied in [13,14].

Problem 1

Firstly, solve the following non-linear inhomogeneous system of integral equations by using ST-ADM [13,14]:

$$\begin{cases} u(x,t) = x + e^{x-t} - \int_0^x uv dx, \\ v(x,t) = -x + e^{-x+t} + \int_0^x u^2v^2 dx. \end{cases} \quad \dots (6)$$

On both sides of the system (6), applying the Sumudu transform yields

$$\begin{cases} \mathbb{S}\{u(x,t)\} = \mathbb{S}\{x + e^{x-t}\} - \mathbb{S}\left\{\int_0^x uv dx\right\}, \\ \mathbb{S}\{v(x,t)\} = \mathbb{S}\{-x + e^{-x+t}\} + \mathbb{S}\left\{\int_0^x u^2v^2 dx\right\}, \end{cases}$$

so that,

$$\begin{cases} U(p,r) = p + \frac{1}{(1-p)(1+r)} - \mathbb{S}\left\{\int_0^x uv dx\right\}, \\ V(p,r) = -p + \frac{1}{(1+p)(1+r)} + \mathbb{S}\left\{\int_0^x u^2v^2 dx\right\}, \end{cases} \quad \dots (7)$$

where $\mathbb{S}(u(x,t)) = U(p,r)$ and $\mathbb{S}(v(x,t)) = V(p,r)$.

The standard of ADM [10,11] consists of approximating solution of the system (6) as an infinite series

$$U(p,r) = \sum_{n=0}^{\infty} U_n(p,r), \quad V(p,r) = \sum_{n=0}^{\infty} V_n(p,r) \quad \dots (8)$$

and decomposing the non-linear terms as

$$uv = \sum_{n=0}^{\infty} A_n, \quad u^2v^2 = \sum_{n=0}^{\infty} B_n \quad \dots (9)$$

where A_n and B_n are Adomian's polynomials of $u_0, u_1, \dots, u_n$ and $v_0, v_1, \dots, v_n$ respectively [10,11] given by

$$A_n = \sum_{i=0}^{n} u_i v_{n-i}, \quad n \ge i, \quad n \ge 0$$

$$B_n = \left(\sum_{i=0}^{n} u_i u_{n-i}\right)\left(\sum_{i=0}^{n} v_i v_{n-i}\right), \quad n \ge i, \quad n \ge 0$$

The proofs of the series convergence $\sum_{n=0}^{\infty} U_n(p,r)$, $\sum_{n=0}^{\infty} V_n(p,r)$, $\sum_{n=0}^{\infty} A_n$ and $\sum_{n=0}^{\infty} B_n$ are given in [10,15-18]. Substituting (8) and (9) in the system (7) and using the recursive relation, we get

$$\begin{cases} U_0(p,r) = p + \frac{1}{(1-p)(1+r)}, \\ V_0(p,r) = -p + \frac{1}{(1+p)(1-r)}, \\ U_{n+1}(p,r) = -\mathbb{S}\left\{\int_0^x A_n dx\right\}, \\ V_{n+1}(p,r) = \mathbb{S}\left\{\int_0^x B_n dx\right\}, \quad n \ge 0 \end{cases} \quad \dots (10)$$

On both sides of the system (10), applying the inverse Sumudu transform yields

$$\begin{cases} u_0(x,t) = x + e^{x-t}, \\ v_0(x,t) = -x + e^{-x+t}, \\ u_{n+1}(x,t) = -\mathbb{S}^{-1}\left\{\mathbb{S}\left\{\int_0^x A_n dx\right\}\right\}, \\ v_{n+1}(x,t) = \mathbb{S}^{-1}\left\{\mathbb{S}\left\{\int_0^x B_n dx\right\}\right\}, \quad n \ge 0 \end{cases} \quad \dots (11)$$

where $\mathbb{S}^{-1}(U_{n+1}(p,r)) = u_{n+1}(x,t)$ and $\mathbb{S}^{-1}(V_{n+1}(p,r)) = v_{n+1}(x,t)$. Once the A_n and B_n are known, all components of u and v can be determined. The n −term approximant to the solutions u and v is thus defined by $\phi_n[u] = \sum_{i=0}^{n-1} u_i$, and $\psi_n[v] = \sum_{i=0}^{n-1} v_i$, respectively. Then from the system (11) the iterations are

$$A_0 = u_0(x,t)v_0(x,t)$$
$$= -x^2 + 1 - xe^{x-t} + xe^{-x+t},$$
$$B_0 = u_0^2(x,t)v_0^2(x,t)$$
$$= x^4 - 4x^2 + 1 + (2x^3 - 2x)e^{x-t}$$
$$+(-2x^3 + 2x)e^{-x+t} + x^2e^{2x-2t} + x^2e^{-2x+2t},$$
$$u_1(x,t) = -\mathbb{S}^{-1}\left\{\mathbb{S}\left\{\int_0^x A_0 dx\right\}\right\}$$

$$= \frac{1}{3}x^3 - x + (x-1)e^{x-t} + (x+1)e^{-x+t} - e^t + e^{-t},$$

$$v_1(x,t) = \mathbb{S}^{-1}\left\{\mathbb{S}\left\{\int_0^x B_0 dx\right\}\right\}$$
$$= \frac{1}{5}x^5 - \frac{4}{3}x^3 + x + 2(x^3 - 3x^2 + 5x - 5)e^{x-t} + 2(x^3 + 3x^2 + 5x + 5)e^{-x+t} + \frac{1}{2}\left(x^2 - x + \frac{1}{2}\right)e^{2x-2t} - \frac{1}{2}\left(x^2 + x + \frac{1}{2}\right)e^{-2x+2t} + \left(-10 + \frac{1}{4}e^t\right)e^t + \left(10 - \frac{1}{4}e^{-t}\right)e^{-t},$$

and so on, obtaining the remaining iterations in this manner. As a result, the approximate solutions in the series form are as follows:

$$\phi_6[u] = \sum_{i=0}^{5} u_i(x,t), \quad \psi_6[v] = \sum_{i=0}^{5} v_i(x,t).$$

Between the components, the noise terms disappear. Using multivariate Taylor series expansion for exponential terms e^{ix-it} and e^{-ix+it} $(i = 1,2,\dots)$ give

$$\phi_6[u] = \left[1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5 + \frac{1}{6!}x^6 + \frac{1}{7!}x^7 + O(x^8)\right] . \left[1 - t + \frac{1}{2!}t^2 - \frac{1}{3!}t^3 + \frac{1}{4!}t^4 - \frac{1}{5!}t^5 + \frac{1}{6!}t^6 - \frac{1}{7!}t^7 + O(t^8)\right],$$

$$\psi_6[v] = \left[1 - x + \frac{1}{2!}x^2 - \frac{1}{3!}x^3 + \frac{1}{4!}x^4 - \frac{1}{5!}x^5 + \frac{1}{6!}x^6 - \frac{1}{7!}x^7 + O(x^8)\right] . \left[1 + t + \frac{1}{2!}t^2 + \frac{1}{3!}t^3 + \frac{1}{4!}t^4 + \frac{1}{5!}t^5 + \frac{1}{6!}t^6 + \frac{1}{7!}t^7 + O(t^8)\right].$$

There is a closed form for these series as $n \to \infty$ gives $u(x,t) = e^x . e^{-t} = e^{x-t}$ and $v(x,t) = e^{-x} . e^t = e^{-x+t}$, which are the exact solutions of the system (6).

Problem 2

Solve the following non-linear inhomogeneous system of integral equations by using ST-ADM [13,14]:

$$\begin{cases} u(x,y) = 2x + 2y - 2x^2 - 4xy + \int_0^x u v_x dx, \\ v(x,y) = 2x - 2y - 2x^2 + 4xy + \int_0^x u_x v dx. \end{cases} \quad \dots (12)$$

On both sides of the system (12), applying the Sumudu transform yields

$$\begin{cases} \mathbb{S}\{u(x,y)\} = \mathbb{S}\{2x + 2y - 2x^2 - 4xy\} - \mathbb{S}\left\{\int_0^x u v_x dx\right\}, \\ \mathbb{S}\{v(x,y)\} = \mathbb{S}\{2x - 2y - 2x^2 + 4xy\} + \mathbb{S}\left\{\int_0^x u_x v dx\right\}, \end{cases}$$

so that,

$$\begin{cases} U(p,q) = 2p + 2q - 4p^2 - 4pq + \mathbb{S}\left\{\int_0^x u v_x dx\right\}, \\ V(p,q) = 2p - 2q - 4p^2 + 4pq + \mathbb{S}\left\{\int_0^x u_x v dx\right\}, \end{cases} \quad \dots (13)$$

where $\mathbb{S}(u(x,y)) = U(p,q)$ and $\mathbb{S}(v(x,y)) = V(p,q)$.

The standard of ADM [10,11] consists of approximating solution of the system (12) as an infinite series

$$U(p,q) = \sum_{n=0}^{\infty} U_n(p,q), \quad V(p,q) = \sum_{n=0}^{\infty} V_n(p,q) \quad \dots (14)$$

and decomposing the non-linear terms as

$$u v_x = \sum_{n=0}^{\infty} C_n, \quad u_x v = \sum_{n=0}^{\infty} D_n \quad \dots (15)$$

where C_n and D_n are Adomian's polynomials of $u_0, u_1, \dots, u_n$ and $v_0, v_1, \dots, v_n$ respectively [10,11] given by

$$C_n = \sum_{i=0}^{n} u_i (v_x)_{n-i}, \quad n \geq i, \quad n \geq 0$$

$$D_n = \sum_{i=0}^{n} (u_x)_i v_{n-i}, \quad n \geq i, \quad n \geq 0$$

The proofs of the series convergence $\sum_{n=0}^{\infty} U_n(p,q)$, $\sum_{n=0}^{\infty} V_n(p,q)$, $\sum_{n=0}^{\infty} C_n$ and $\sum_{n=0}^{\infty} D_n$ are given in [10,15,16,17]. Substituting (14) and (15) in the system (13) and using the recursive relation, we get

$$\begin{cases} U_0(p,q) = 2p + 2q - 4p^2 - 4pq, \\ V_0(p,q) = 2p - 2q - 4p^2 + 4pq, \\ U_{n+1}(p,q) = \mathbb{S}\left\{\int_0^x C_n dx\right\}, \\ V_{n+1}(p,q) = \mathbb{S}\left\{\int_0^x D_n dx\right\}, \quad n \geq 0 \end{cases} \quad \dots (16)$$

On both sides of the system (16), applying the inverse Sumudu transform yields

$$\begin{cases} u_0(x,y) = 2x + 2y - 2x^2 - 4xy, \\ v_0(x,y) = 2x - 2y - 2x^2 + 4xy, \\ u_{n+1}(x,y) = \mathbb{S}^{-1}\left\{\mathbb{S}\left\{\int_0^x C_n dx\right\}\right\}, \\ v_{n+1}(x,y) = \mathbb{S}^{-1}\left\{\mathbb{S}\left\{\int_0^x D_n dx\right\}\right\}, \quad n \geq 0 \end{cases} \quad \dots (17)$$

where $\mathbb{S}^{-1}(U_{n+1}(p,q)) = u_{n+1}(x,y)$ and $\mathbb{S}^{-1}(V_{n+1}(p,q)) = v_{n+1}(x,y)$. Once the C_n and D_n are known, all components of u and v can be determined. The n −term approximant to the solutions u and v is defined by $\phi_n[u] = \sum_{i=0}^{n-1} u_i$, and $\psi_n[v] = \sum_{i=0}^{n-1} v_i$, respectively. Then from the system (17) the iterations are

$$C_0 = u_0(x,y)\frac{\partial}{\partial x}v_0(x,y)$$
$$= 8x^3 + (8y - 12)x^2 - (16y^2 + 8y - 4)x + 8y^2 + 4y,$$

$$D_0=\frac{\partial}{\partial x}u_0(x,y)v_0(x,y)$$
$$=8x^3-(8y+12)x^2-(16y^2-8y-4)x$$
$$+8y^2-4y,$$
$$u_1(x,y)=\mathbb{S}^{-1}\left\{\mathbb{S}\left\{\int_0^x C_0dx\right\}\right\}$$
$$=2x^4+\left(\frac{8}{3}y-4\right)x^3-(8y^2+4y-2)x^2$$
$$+(8y^2+4y)x,$$
$$v_1(x,y)=\mathbb{S}^{-1}\left\{\mathbb{S}\left\{\int_0^x D_0dx\right\}\right\}$$
$$=2x^4-\left(\frac{8}{3}y+4\right)x^3-(8y^2-4y-2)x^2$$
$$+(8y^2-4y)x,$$

and so forth, making it possible to obtain the remaining iterations in this way. Consequently, the rough answers in series form are provided by:

$$\phi_4[u]=\sum_{i=0}^{3}u_i(x,y)$$
$$=2x+2y+10x^8+\left(\frac{944}{105}y-40\right)x^7$$
$$-\left(\frac{3952}{45}y^2+\frac{472}{15}y-56\right)x^6$$
$$-\left(\frac{832}{15}y^3-\frac{3952}{15}y^2-\frac{608}{15}y+28\right)x^5$$
$$+\left(\frac{224}{3}y^4+\frac{416}{3}y^3-288y^2-20y\right)x^4$$
$$-\left(\frac{448}{3}y^4+\frac{448}{3}y^3-112y^2\right)x^3$$
$$+(128y^4+64y^3)x^2-32xy^4,$$
$$\psi_4[v]=\sum_{i=0}^{3}v_i(x,y)$$
$$=2x-2y+10x^8-\left(\frac{944}{105}y+40\right)x^7$$
$$-\left(\frac{3952}{45}y^2-\frac{472}{15}y-56\right)x^6$$
$$+\left(\frac{832}{15}y^3+\frac{3952}{15}y^2-\frac{608}{15}y-28\right)x^5$$
$$+\left(\frac{224}{3}y^4-\frac{416}{3}y^3-288y^2+20y\right)x^4$$
$$-\left(\frac{448}{3}y^4-\frac{448}{3}y^3-112y^2\right)x^3$$
$$+(128y^4-64y^3)x^2-32xy^4.$$

Between the components, the noise terms disappear when $n\to\infty$. There is a closed form for these series $u(x,y)=2x+2y$ and $v(x,y)=2x-2y$, which are the exact solutions of the system (12).

Problem 3

Finally, we solve the following non-linear inhomogeneous system of integral equations by using ST-ADM [13,14]:

$$\begin{cases}u(x,y)=4x-2x^2+2x^2y^2-v+\int_0^x uv_xdx,\\ v(x,y)=-4xy-2x^2+2x^2y^2+u+\int_0^x u_xvdx.\end{cases}\quad\ldots(18)$$

On both sides of the system (18), applying the Sumudu transform yields

$$\begin{cases}\mathbb{S}\{u(x,y)\}=\mathbb{S}\{4x-2x^2+2x^2y^2-v\}-\mathbb{S}\left\{\int_0^x uv_xdx\right\},\\ \mathbb{S}\{v(x,y)\}=\mathbb{S}\{-4xy-2x^2+2x^2y^2+u\}+\mathbb{S}\left\{\int_0^x u_xvdx\right\},\end{cases}$$

so that,

$$\begin{cases}U(p,q)=4p-4p^2+8p^2q^2\\ \quad -V(p,q)+\mathbb{S}\left\{\int_0^x uv_xdx\right\},\\ V(p,q)=-4pq-4p^2+8p^2q^2\\ \quad +U(p,q)+\mathbb{S}\left\{\int_0^x u_xvdx\right\},\end{cases}\quad\ldots(19)$$

where $\mathbb{S}(u(x,y))=U(p,q)$ and $\mathbb{S}(v(x,y))=V(p,q)$.

The standard of ADM [10,11] consists of approximating solution of the system (18) as in the problem 2. Substituting (14) and (15) in the system (19) and using the recursive relation, we get

$$\begin{cases}U_0(p,q)=4p-4p^2+8p^2q^2,\\ V_0(p,q)=-4pq-4p^2+8p^2q^2,\\ U_{n+1}(p,q)=-V_n(p,q)+\mathbb{S}\left\{\int_0^x C_ndx\right\},\\ V_{n+1}(p,q)=U_n(p,q)+\mathbb{S}\left\{\int_0^x D_ndx\right\},\ \ n\geq 0\end{cases}\quad\ldots(20)$$

On both sides of the system (20), applying the inverse Sumudu transform yields

$$\begin{cases}u_0(x,y)=4x-2x^2+2x^2y^2,\\ v_0(x,y)=-4xy-2x^2+2x^2y^2,\\ u_{n+1}(x,y)=-v_n(x,y)+\mathbb{S}^{-1}\left\{\mathbb{S}\left\{\int_0^x C_ndx\right\}\right\},\\ v_{n+1}(x,y)=u_n(x,y)+\mathbb{S}^{-1}\left\{\mathbb{S}\left\{\int_0^x D_ndx\right\}\right\},\ \ n\geq 0\end{cases}\quad\ldots(21)$$

where $\mathbb{S}^{-1}(U_{n+1}(p,q))=u_{n+1}(x,y)$ and $\mathbb{S}^{-1}(V_{n+1}(p,q))=v_{n+1}(x,y)$. Once the C_n and D_n are known, all components of u and v can be determined. The n −term approximant to the solutions u and v is thus defined by $\phi_n[u]=\sum_{i=0}^{n-1}u_i$, and $\psi_n[v]=\sum_{i=0}^{n-1}v_i$, respectively. Then from the system (21) the iterations are

$$C_0=u_0(x,y)\frac{\partial}{\partial x}v_0(x,y)$$
$$=8(y^4-2y^2+1)x^3-8(y^3-2y^2-y+2)x^2$$
$$-16xy,$$
$$D_0=\frac{\partial}{\partial x}u_0(x,y)v_0(x,y)$$
$$=8(y^4-2y^2+1)x^3-8(2y^3-y^2-2y+1)x^2$$

$$-16xy,$$

$$u_1(x,y) = -v_0(x,y) + \mathbb{S}^{-1}\left\{\mathbb{S}\left\{\int_0^x C_0 dx\right\}\right\}$$

$$= 2(y^4 - 2y^2 + 1)x^4 - \frac{8}{3}(y^3 - 2y^2 - y + 2)x^3$$

$$-2(y^2 + 4y - 1)x^2 + 4xy,$$

$$v_1(x,y) = u_0(x,y) + \mathbb{S}^{-1}\left\{\mathbb{S}\left\{\int_0^x D_0 dx\right\}\right\}$$

$$= 2(y^4 - 2y^2 + 1)x^4 - \frac{8}{3}(2y^3 - y^2 - 2y + 1)x^3$$

$$+2(y^2 - 4y - 1)x^2 + 4x,$$

and so on, obtaining the remaining iterations in this manner. As a result, the approximate solutions in series form are as follows:

$$\phi_6[u] = \sum_{i=0}^{5} u_i(x,y) = 2x + 2xy + \text{noise terms}$$

$$\psi_4[v] = \sum_{i=0}^{5} v_i(x,y) = 2x - 2xy + \text{noise terms}$$

Between the components, the noise terms disappear.when $n \to \infty$. There is a closed form for these series $u(x,y) = 2x + 2xy$ and $v(x,y) = 2x - 2xy$, which are the exact solutions of the system (18).

Conclusion

In this paper, the Sumudu transform with the Adomian decomposition method was utilized in solving non-linear special systems of integral equations involving functions of two variables. The results demonstrate the effectiveness of this technique in addressing a variety of nonlinear integral problems. The new method reduces the computational complexity of previously commonly used methods while allowing for easy calculation adjustment. The computations in this work are done with the Maple 18 Package.

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حل أنظمة خاصة من المعادلات التكاملية بأستخدام تحويل سومودو مع طريقة شبه تحليلية

وليد الحياني

waleedalhayani@uomosul.edu.iq

كلية علوم الحاسوب والرياضيات / جامعة الموصل

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ملخص

في هذا البحث، تم حل أشكال خاصة من الأنظمة غير الخطية للمعادلات التكاملية باستخدام تحويل سومودو وطريقة انحلال ادوميان. لتوضيح الطريقة، تم تطوير ثلاثة أمثلة لأشكال مختلفة في هذه الفئة من المعادلات الدالية. أنتجت الطريقة حلاً تقريبيًا-دقيقًا لعدة أنظمة. تتضح دقة الطريقة وكفاءتها وبساطتها من خلال نتائج تطبيقها على أنظمة مختلفة من هذه الأنواع من المعادلات المتكاملة.

الكلمات المفتاحية: تحويل سومودو المزدوج، طريقة أنحلال ادوميان، كثيرات حدود ادميان، أنظمة خاصة من المعادلات التكاملية