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Solving Integral Equations via a hybrid method between the Modified Adomian Decomposition Method and Bee Algorithm

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Abstract

Abstract— In this paper, we used the modified Adomian decomposition(MADM) method to solve integral equations of the Volterra and Fredholm type as well, and then the Adomian method was combined with the bee algorithm(ABC) and obtained values for the parameter λ that improved the results obtained by solving some examples and were more accurate than the default method, These results are illustrated by calculating the maximum absolute errors (MAE) and mean squared errors (MSE).

Keywords:
Voltera and Fredholm integral equations, Adomian decomposition method, Bee Colony Algorithm.

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I. INTRODUCTION (HEADING 1)

Many applications of integral equations may be found in physics, engineering, and mathematics. Yet, analytical solutions to integral equations either it has no solution or it is difficult to find a solution to it. Many numerical techniques have been tested and developed to solve integral equations precisely because of this [1]. We will focus on two fundamental categories of integral equations, including the Volterra integral equation, which has the following equation:

$$u(x) = f(x) + \lambda \int_a^x K(x,t)u(t)dt, \qquad \ldots (1)$$

and Fredholm integral equation whose equation is as follows [2]:

$$u(x) = f(x) + \lambda \int_a^b k(x,t)u(t)dt, \qquad \ldots (2)$$

where the to-be-determined unknown function $u(x)$ appears both within and outside the integral sign. Real-valued functions are provided for the function $f(x)$ and the kernel $k(x,t)$, and a parameter is λ. Integral equations, whether linear or non-linear, are used to express all natural, physical, engineering, and other phenomena. Several of these integral equations in the method of solution are not directly solvable, thus it was from To address issues and find a solution, it is required to employ approximations, as it is in the technique (MADM) [3]. The Adomian decomposition technique for solving differential and integral equations, both linear and non-linear, has received a lot of attention recently Because it gives an approximate solution in a sequential manner that is close to the exact solution.[4]

One of the most recent swarm-based algorithms is the artificial bee colony (ABC) algorithm. The ABC algorithm mimics a honey bee swarm's clever foraging behavior. In this study, a significant of numerical test parameters and outcomes produced by the modified ADM is improved using the ABC method[5].

2. Some important concepts

We will learn some crucial definitions of the study subject.

2.1.The MAE (Maximum Absolute Error) [6]: The following formula is used to calculate the MAE.

$$\|z_{exact}(y) - \varphi_m(y)\|_\infty = \max_{a \le y \le b}\{|z_{exact}(y) - \varphi_m(y)|\}$$

2.2. MSE (Mean Square Error) [6]

The mean square error is defined as follows:

$$\text{MSE} = \frac{\sum_{i=1}^{n}(Ex(y_i) - \varphi(y_i))^2}{n}$$

Where y_i is a vector, $i = 1,2,3,\cdots,$, which is the sum of the square of the exact solution minus the square of the approximate solution, and then divided by the number of iterations n

3.The Modified Decomposition Method (MADM) [7]

The ADM is an iterative technique for solving Volterra integral equations that offers the answer in an endless series of elements (1) Determine if the integral equation is of the Volterra or Fredholm type and whether the inhomogeneous term $f(x)$ is a polynomial. Nevertheless, assessing the components is necessary if the function $f(x)$ consists of binomials, two or more polynomials, trigonometric functions, hyperbolic functions, etc. It's noteworthy to note that the decomposition approach, also known as the rate method, relies heavily on splitting the function $f(x)$ into two term and cannot be used when the function $f(x)$ has just one part. Keep in mind that the recurrence relation may be used under the ADM,

$$u_0(x) = f(x)$$

$$u_{k+1}(x) = \lambda \int_0^x k(x,t) u_k(t)dt, k \geq 0 \quad \ldots(3)$$

where the solution $u(x)$ is expressed by an infinite sum of components defined

$$u(x) = \sum_{n=0}^{\infty} u_n(x).$$

The function $f(x)$ will be divided into two functions $f_1(x)$ and $f_2(x)$, we can set $f(x) = f_1(x) + f_2(x)$.

We are now introducing a modification in the default formula, Relation of recurrence We specify to decrease the size of the computations The component $u_0(x)$ is one part of $f(x)$, i.e., we may add another portion of $f(x)$ to component $u_1(x)$ by selecting either $f_1(x)$ or $f_2(x)$.The modified recurrence relation is therefore introduced using the modified decomposition method:

$$u_0(x) = f_1(x),$$

$$u_1(x) = f_2(x) + \lambda \int_0^x k(x,t)\, u_0(t)dt \quad \ldots(4)$$

$$u_{k+1}(x) = \lambda \int_0^x k(x,t) u_k(t)dt, \; k \geq 1.$$

we observe that only the first two components, $u_0(x)$ and $u_1(x)$, are contained in the updated recurrence relation (4) In relationships of recurrence, other elements uj stay constant. While the configuration of $u_0(x)$ and $u_1(x)$. differs only very little, this variation significantly speeds up convergence of the solution and decreases the amount of arithmetic effort. Moreover, limiting the number of words in $f_1(x)$ has an impact on additional components in addition to component $u_1(x)$. $u_1(x)$. Several research has supported the conclusion. Regarding the improved approach in this case, two crucial points can be made. First, the correct selection of the functions $f_1(x)$ and $f_2(x)$ The solution $u(x)$ can be obtained using very few iterations, sometimes By taking only two ingredients. The success of this modification depends, On the correct selection of $f_1(x)$ and $f_2(x)$. Secondly, If $f(x)$ consists of only one term The decomposition method can be used in this case, It is worth noting that the modified decomposition method will it is used in Volterra and Fredholm integral equations and linear and nonlinear equation.

4. Bee colony Algorithm (ABC)

Tasks are carried out by some specialist people in an actual bee colony. These specialist bees use effective task division and self-control to strive to optimize the amount of nectar stored in the cell. The ABC algorithm, or artificial bee colony, a recently developed optimization technique for modeling food behavior that replicates a bee colony's foraging behavior, was first proposed by Karaboga in the year 2005. The ABC algorithm simulates a honey bee colony with three different types :

1. Worker bees
2. Onlooker bees
3. Scout bees

Worker bees make up half of the colony, while spectator bees make up the other half. The bees are in charge of making use of the nectar sources that were found before and after work and providing the waiting bees (spectator bees) in the hive with essential information about the locations of the discovered food sources. Bees that are observers wait in the hive and choose a food source to exploit depending on the knowledge that the worker bees have imparted. The scout randomly scans the area in quest of another food source. This developing intelligence in bee food may be summed up as follows:

1. The bees start randomly scouting the area in the initial stage of foraging in search of a food source.

2. After discovering a food source, the bee transforms into a foraging worker and starts to use the new food supply. The worker bee then brings the nectar back to the hive and empties it. It can immediately go back to the detected source site after releasing the nectar, or it can dance in the dance area to reveal its to the source location. If its resource is exhausted, it transforms into a scout and begins seeking at random for a new source.

3. The location of the source and its importance are chosen by the spectator bees based on the dances that take place in the hive and finding sources rich in food.[8]

In the ABC algorithm, the scouts manage the exploration process while the worker bees and spectator bees carry out the exploitation process in the search space. A comprehensive code for the(ABC algorithm) is given below: [5]

Step 1): Make the population of solutions start out $xi, i = 1, \ldots, SN$
Step 2): Assess the population
Step 3): cycle = 1
Step 4): repeat
Step 5): Using (step 7) to create fresh solutions $U(i)$ for the employed bees, then assess them
Step 6): Employ the avaricious selection procedure to the hired bees.
Step7): Determine the pi-values of probability for the answers provided by (step6)
Step 8): Produce the new solutions U_i for the onlookers from the solutions xi selected depending on p_i and evaluate them
Step 9): Choose the most avaricious candidates for the spectators.
Step 10): Find the scout's abandoned solution, if one exists, and replace it with a fresh, randomly generated solution x_i (step 8)
Step 11): Memorize the best solution reached so far
Step 12): cycle = cycle + 1
Step 13): until cycle = MCN(Maximum Cycle Number)

5. Pade approximations (PA):[9]

Due to their extensive use in chemistry, physics, engineering, and other practical sciences, The topic of approximations is one of the important topics, especially for difficult functions in the numerical analysis [10,11]. A specific and traditional sort of approximation for a rational fraction is the Pade approximation. In order to compute the numerator and denominator coefficients using the coefficients of the Taylor series expansion of a function (x), a function is expanded as a ratio of two power series[10]. The Pade approximation is the most accurate rational function of a specific order to approximate a function [10]. Although George Freobenius presented the concept and looked into the characteristics of rational approximations of power series, Henri Pade is credited with developing the approach somewhere in about 1890. When functions include poles, the Pade approximation is typically preferable since rational functions are better able to express them [10]. When the Taylor series does not converge, the Pade approximation may still be applicable since it frequently provides a more accurate estimate of the function [10]. The Pade approximation is widely utilized in computer calculations due to these factors.

To give a clear overview of the modified method, we now introduce the following Examples, in which we apply the MADM method to linear and nonlinear integral equations (Volterra and Fredholm) We compare the obtained results with the bee algorithm(ABC) and Pade approximations (PA) associated with the examples using Maple 13.

Example 1: Consider the linear Volterra integral equation of the second kind

$u(x) = x + \int_0^x (t - x)u(t),\ 0 < x < 1$, with the initial $a = 0$, $b = 1$

and the exact solution is $y_{Exact}(x) = \sin(x)$.

We gain the following iterations by means of making use of the MADM:

$u_0(x) = 0,$

$u_1(x) = x,$

$u_2(x) = -\frac{\lambda\, x^3}{6},$

$u_3(x) = \frac{\lambda^2 x^5}{120}.$

Now, by adding the previous terms we get the next series

$$\phi_3(x) = \sum_{i=0}^{2} u_i(x)$$

$$= -\frac{1}{6}\, x^3\, \lambda + x,$$

when compensating for a value λ=1, we get these results:

$u_0(x) = 0,$

$u_1(x) = x,$

$u_2(x) = -\frac{x^3}{6}x,$

$\phi_3(x) = -\frac{1}{6}x^3 + x,$

and when using the ABC algorithm with the modified method, we get these results when we substitute the value of $\lambda = 0.9585$

$u_0(x) = 0,$

$u_1(x) = x$

$u_2(x) = -0.159750x^3 + x,$

$\phi_3(x) = -0.159750\, x^3 + x,$

and when using the Pade approximations with the ABC algorithm with the modified method, we get these results when we substitute the value of $\lambda = 0.9585$

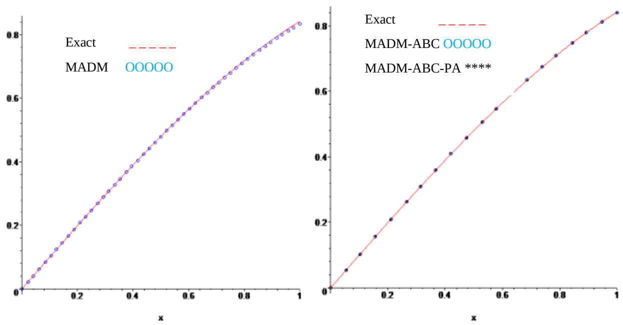
$PA(x) = 0.999999x - 0.159750x^3.$

Table 1: Numerical results of the MAE for example

x	MADM	MADM-ABC	MADM-ABC-PA
0.0	0.0	0.0	0.0
0.1	8.331349E-08	6.833353E-06	6.833353E-06
0.2	2.664128E-06	5.266920E-05	5.266920E-05
0.3	2.020666E-05	1.665433E-05	1.665433E-04
0.4	8.500897E-05	3.576576E-04	3.576576E-04
0.5	2.588719E-04	6.057113E-04	6.057113E-04
0.6	6.424733E-04	8.515266E-04	8.515266E-04
0.7	1.384353E-03	9.880627E-04	9.880627E-04
0.8	2.689424E-03	8.519091E-04	8.519091E-04
0.9	4.826909E-03	2.153403E-04	2.153403E-04
1.0	8.137651E-03	1.220984E-03	1.220984E-03

Table 2: The results (MSE

MADM	MADM-ABC	MADM-ABC-PA
9.014299E-06	4.081502E-07	4.081502E-07



a **b**

Fig 1. Exact solution compared with the proposed method and PA

a. Exact with (MADM)when λ=1.
b. Exact with (MADM-ABC, MADM-PA) when λ=0.9585.

Example 2: Let us consider the following Volterra integral equation of the second kind

$$u(x) = \frac{1}{4}x\,cos(2x) + \sin(x) - \frac{1}{4}\,x + \int_0^x x\,cos(t)u(t)dt, x \in [0,1]$$

and the exact solution is $u_{Exact}(x) = 2\sin(x)$.

We gain the following iterations by means of making use of the MADM

$$u_0(x) = \frac{1}{4}\,x\cos(2x),$$

$$u_1(x) = \sin(x) - \frac{x}{4} - \frac{5\lambda x}{36} + \frac{1}{12}\,\lambda\,xcos(x) + \frac{1}{12}\,\lambda\,\sin(x)\,x^2 + \frac{1}{18}\,\lambda\,cos(x)^3 + \frac{1}{6}\lambda\,x^2\sin(x)\,cos(x)^2,$$

$$u_2(x) = \frac{3\lambda x}{4} + \frac{\lambda^2 x}{16} - \frac{1}{4}\lambda\sin(x)\,x^2 - \frac{1}{4}\lambda\,x\cos(x) + \frac{13}{96}\lambda^2x^2\cos(x)\sin(x) + \frac{5}{144}\,\lambda^2\,x^2\,cos(x)^3\sin(x) - \frac{5}{36}\,\lambda^2\sin(x)\,x^2 - \frac{1}{24}\lambda^2\,cos(x)^2\,x^3 - \frac{1}{24}\,\lambda^2\,cos(x)^4\,x^3 - \frac{1}{2}\,x\,\lambda\,cos(x)^2 - \frac{5}{36}\,\lambda^2\,x\cos(x) + \frac{13\lambda^2x^3}{192} - \frac{13}{192}\,x\,\lambda^2\,cos(x)^2 + \frac{5}{576}\,x\,\lambda^2\,cos(x)^4$$

Now, by adding the previous terms we get the next series

$$\phi_3(x) = -\frac{1}{24}\,x^3\lambda^2\,cos(x)^4 + \frac{5}{144}x^2\lambda^2\,cos(x)^3\sin(x) - \frac{1}{24}x^3\lambda^2\,cos(x)^2 + \frac{5}{576}\,x\,\lambda^2\,cos(x)^4 + \frac{13}{96}x^2\lambda^2\cos(x)\sin(x) + \frac{1}{6}x^2\,\lambda\,cos(x)^2\sin(x) + \frac{13x^3\lambda^2}{192} - \frac{5}{36}\,x^2\lambda^2\sin(x) + \frac{13}{192}\,x\,\lambda^2\,cos(x)^2 + \frac{1}{18}\,x\,\lambda\,cos(x)^3 - \frac{1}{6}\,x^2\,\lambda\sin(x) + \frac{5}{36}\,x\,\lambda^2\cos(x) - \frac{1}{2}\,x\,\lambda\cos(x)^2 + \frac{x\lambda^2}{16} - \frac{1}{6}\,x\,\lambda\,cos(x) + \frac{11x\lambda}{18} + \frac{1}{4}\,x\,cos(2x) - \frac{x}{4} + \sin(x),$$

when compensating for a value $\lambda = 1$ we get these results:

$$u_0(x) = \frac{1}{4}\,x\cos(2x)\,,$$

$$u_1(x) = \frac{1}{6}\,x^2\,cos(x)^2\sin(x) + \frac{1}{18}\,x\,cos(x)^3 + \frac{1}{12}\,x^2\sin(x) + \frac{1}{12}\,x\,cos(x) + \frac{7x}{18} + \sin(x)$$

$$u_2(x) = -\frac{1}{24}x^3\,cos(x)^4 + \frac{5}{144}\,x^2\,cos(x)^3\sin(x) - \frac{1}{24}\,x^3\,cos(x)^2 + \frac{5}{576}\,x\,cos(x)^4 + \frac{13}{96}x^2\cos(x)\sin(x) + \frac{13x^3}{192} - \frac{7x}{18}\,x^2\sin(x) + \frac{83}{192}\,x\,cos(x)^2 + \frac{7}{18}\,x\cos(x) + \frac{13x}{16}$$

$$\phi_3(x) = -\frac{1}{24}x^3\,cos(x)^4 + \frac{5}{144}\,x^2\,cos(x)^3\sin(x) - \frac{1}{24}x^3\,cos(x)^2 + \frac{1}{6}\,x^2\,cos(x)^2\sin(x) + \frac{5}{576}\,x\,cos(x)^4 + \frac{13}{96}x^2\cos(x)\sin(x) + \frac{1}{18}\,x\,cos(x)^3 + \frac{13x^3}{192} - \frac{11}{36}x^2\,\sin(x) - \frac{83}{192}\,x\,\,cos(x)^2 - \frac{11}{36}\,x\,\cos(x) + \frac{1}{4}\,x\cos(2x) - \frac{61\,x}{144} + \sin(x)$$

and when using ABC algorithm with the modified method we get these results when we substitute the value of λ=1.15942

$u_0(x) = \frac{1}{4}\, x \cos(2x)$

$$u_1(x) = 0.193236\, x^2 \cos(x)^2 \sin(x) + 0.064412\, x \cos(x)^3 + 0.096618\, x^2 \sin(x) + 0.966183\, x \cos(x) - 0.411030\, x + \sin(x)$$

$$u_2(x) = -0.056010\, x^3 \cos(x)^4 + 0.046675\, x^2 \cos(x)^3 \sin(x) - 0.056010\, x^3 \cos(x)^2 + 0.011668\, x \cos(x)^4 + 0.182034 x^2 \cos(x) \sin(x) + 0.091017 x^3 - 0.476557\, x^2 \sin(x) - 0.488692\, x \cos(x)^2 - 0.476557\, x \cos(x) + 0.953580\, x$$

$$\phi 3(x) = -0.056010\ x^3 \cos(x)^4 + 0.046675\, x^2 \cos(x)^3 \sin(x) - 0.056010\, x^3 \cos(x)^2 + 0.193236\, x^2 \cos x^2\ \sin(x) + 0.011166\, x \cos(x)^4 + 0.182034\, x^2 \cos(x) \sin(x) + 0.091017\, x^3 - 0.379938\, x^2 \sin(x) - 0.379938\, x \cos(x) + 1/4\ x \cos(2x) + 0.542550\, x + \sin(x)$$

and when using pade approximations with the (ABC) algorithm with the modified method, we get this resul when we substitute the value of $\lambda = 1.15942$

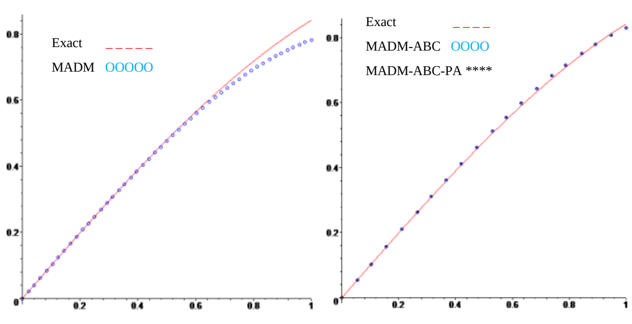
$$PA(x) = (x + 0.431023\, x^3 - 0.020134 x^5 - 0.010612\, x^7 + 0.009554\, x^9) \,/\, (1 + 0.517980\, x^2 + 0.14606\, x^4 + 0.004411\, x^8 + 0.000539\, x^{10})$$

Table 3: Numerical results of the MAE for example 2

x	MADM	MADM-ABC	MADM-ABC-PA
0.0	0.0	0.0	0.0
0.1	9.341979E-07	7.841861E-05	7.841861E-05
0.2	2.957605E-05	5.966956E-04	5.966956E-04
0.3	2.205357E-04	1.845311E-03	1.845311E-03
0.4	9.051863E-04	3.834608E-03	3.834608E-03
0.5	2.666852E-03	6.202951E-03	6.202951E-03
0.6	6.343726E-03	8.185733E-03	8.185733E-03
0.7	1.296446E-02	8.665894E-03	8.665894E-03
0.8	2.360836E-02	6.319615E-03	6.319618E-03
0.9	3.919419E-02	1.433501E-04	1.433140E-04
1.0	6.021824E-02	1.166599E-02	1.166571E-02

Table 4: The results (MSE)

MADM	MADM-ABC	MADM-ABC-PA
5.396434E-04	3.410051E-05	3.409993E-05



a b

Fig 2. Exact solution compared with the proposed method and PA

a. Exact with (MADM) when λ=1

b. Exact with (MADM-ABC, MADM-PA) when $\lambda = 1.15942$

Example 3: Let us consider the following Volterra integral equation of the second kind

$$u(x) = -x^5 + 5x^3 + \int_0^x t u(t) dt,$$

and the exact solution is $y_{Exact}(x) = 5x^3$.

We gain the following iterations by means of making use of the MADM

$u_0(x) = -x^5$

$u_1(x) = 5x^3 - \frac{1}{7}\, \lambda x^7$

$u_2(x) = -\frac{1}{63}\, \lambda^2\, x^9 + x^5$

Now, by adding the previous terms we get the next series

$$\phi_3(x) = \sum_{i=0}^{2} u_i(x)$$

$$= -\frac{1}{63}\, x^9\, \lambda^2 - \frac{1}{7}\, x^7\, \lambda + x^5\, \lambda - x^5 + 5x^3$$

when compensating for a value $\lambda = 1$ We get these results:

$u_0(x) = -x^5$

$u_1(x) = -\frac{1}{7}\, x^7 + 5x^3$

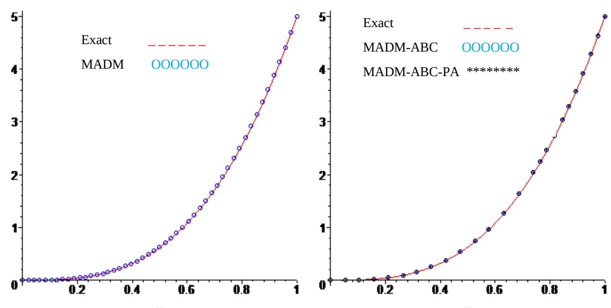
$u_2(x) = -\frac{1}{63}x^9 + x^5$
$\phi_3(x) = -\frac{1}{63}x^9 - \frac{1}{7}x^7 + 5x^3$
and when using ABC algorithm with the modified method we get these results when we substitute the value of λ=1.2
$u_0(x) = -x^5$
$u_1(x) = -0.171428x^7 + 5x^3$
$u_2(x) = -0.022857x^9 + 1.2x^5$
$\phi_3(x) = -0.022857x^9 - 0.171428x^7 + 0.2x^5 + 5x^3$
And when using Pade approximations with the ABC algorithm with the modified method, we get these results when we substitute the value of λ=1.2
$PA(x) = (5x^3 - 2.768171x^5)\ /\ (1 - 0.593634x^2 + 0.058031x^4 - 0.018102x^6)$

Table 5: Numerical results of the MAE for example 3

x	MADM	MADM-ABC	MADM-ABC-PA
0.0	0.0	0.0	0.0
0.1	1.430158E-08	1.982834E-06	1.982834E-06
0.2	1.836698E-06	6.179401E-05	6.179400E-05
0.3	3.155528E-05	4.480586E-04	4.480583E-04
0.4	2.382181E-04	1.761139E-03	1.761125E-03
0.5	1.147073E-03	4.866071E-03	4.865802E-03
0.6	4.159049E-03	1.052274E-02	1.051957E-02
0.7	1.240543E-02	1.857375E-02	1.854714E-02
0.8	3.208975E-02	2.651698E-02	2.634135E-02
0.9	7.447766E-02	2.724892E-02	2.626853E-02
1.0	1.587301E-01	5.714285E-03	8.055726E-04

Table 6: The results (MSE)

MADM	MADM-ABC	MADM-ABC-PA
2.904045E-03	1.782731E-04	1.696538E-04



a b
Fig 3. Exact solution compared with the proposed method and PA

a. Exact with (MADM) when λ=1
b. Exact with (MADM-ABC, MADM-PA) when λ=1.2

xample 4: Let us consider the following Fredholm equation of the second kind

$$u(x) = -x - \frac{\pi}{4} + 2\int_0^1 \frac{1}{4 + u(t)^2}\, dt\ , 0 \le x \le 1$$

and the exact solution is $y_{Exact}(x) = x$.
We gain the following iterations by means of making use of the MADM

$u_0(x) = -\frac{\pi}{4}$,
$u_1(x) = x + 0.433195\,\lambda$,
$u_2(x)$
$= -1.273239\,\lambda\, ln\frac{-0.607300\ 10^{30} + 0.170115\ 10^{30}\lambda}{-0.1000000\ 10^{31} + 0.170115\ 10^{30}\lambda}$.

Now, by adding the previous terms we get the next series

$$\phi_3(x) = \sum_{i=0}^{2} u_i(x)$$

$= -1.273239\,\lambda\, ln\frac{+0.170115\ 10^{30}\lambda - 0.607300\ 10^{30}}{+0.170115\ 10^{30}\lambda - 0.1000000\ 10^{31}}$
$+x + 0.433195\,\lambda - 0.785398$,

when compensating for a value $\lambda = 1$. The iterations are:
$u_0(x) = -\frac{\pi}{4}$,
$u_1(x) = x + 0.433195$,
$u_2(x) = -1.273239\ \ln(0.526802)$,
thus, the approximate solution is
$\phi_3(x) = x - 0.463854$,
and when using ABC algorithm with the modified method we get these results when we substitute the value of $\lambda = 0.6667$
$u_0(x) = -\frac{\pi}{4}$,
$u_1(x) = x + 0.288811$,

$u_2(x) = -0.848868\ln(0.557064),$

$\phi_3(x) = x + 0.000064,$

and when using the pade approximations with the ABC algorithm with the modified method, we get this result when we substitute the value of $\lambda = 0.6667$

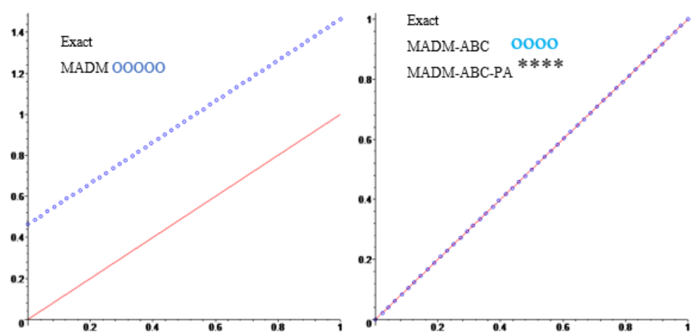
$PA(x) = 0.000064 + x.$

Table 7: Numerical results of the MAE for example

x	**MADM**	**MADM-ABC**	**MADM-ABC-PA**
0.0	4.638540E-02	6.400919E-05	6.400919E-05
0.1	4.638540E-02	6.400919E-05	6.400919E-05
0.2	4.638540E-02	6.400919E-05	6.400919E-05
0.3	4.638540E-02	6.400919E-05	6.400919E-05
0.4	4.638540E-02	6.400919E-05	6.400919E-05
0.5	4.638540E-02	6.400919E-05	6.400919E-05
0.6	4.638540E-02	6.400919E-05	6.400919E-05
0.7	4.638540E-02	6.400919E-05	6.400919E-05
0.8	4.638540E-02	6.400919E-05	6.400919E-05
0.9	4.638540E-02	6.400919E-05	6.400919E-05
1.0	4.638540E-02	6.400919E-05	6.400919E-05

Table 8: The results (MSE)

MADM	**MADM-ABC**	**MADM-ABC-PA**
2.151605E-02	4.097176E-09	4.097176E-09



a **b**

Fig 4. Exact solution compared with the proposed method and PA

a. Exact with (MADM) when λ=1

b. Exact with (MADM-ABC, MADM-PA) λ=0.6667

Conclusion

In this manuscript, the MADM was hybridized with the bee algorithm, by taking the analytical series from the MADM and then finding the best value λ using the ABC algorithm. The hybrid method proved efficient by comparing the results obtained with the solution exact, we also made another improvement to the method using the PA and finding numerical results MSE and MAE, as well as drawing using the MAPLE program.

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حل المعادلات التكاملية بطريقة هجينة بين طريقة انحلال ادوماين المعدلة وخوارزمية النحل

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الملخص

استخدمنا في هذا البحث طريقة انحلال ادوماين المعدلة (MDAM) لحل المعادلات التكاملية من نوع فولتيرا وفريدهولم ايضاً, ومن ثم دمج طريقة ادوماين المعدلة مع خوارزمية النحل (CBA) وحصلنا على قيم المعلمة (λ) التي تحسنت وكانت النتائج التي تم الحصول عليها من خلال حل بعض الأمثلة وكانت أكثر دقة من طريقة ادوماين لوحدها, وتم توضيح هذه النتائج من خلال حساب الحد الأقصى للأخطاء المطلقة (EAM) ومتوسط الأخطاء التربيعية (ESM).

الكلمات المفتاحية: معادلات فولتيرا وفريدهولم التكاملية، طريقة انحلال ادوماين، خوارزمية مستعمرة النحل