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A Sufficient Descent Property for a Different Parameter to Enhance Three-Term Method

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Abstract

In this paper, we derive a new parameter μ_{k-1} for the three-term CG (N3T) algorithm for solving unconstrained optimization problems. As demonstrated by its calculations and proof, the parameter μ_{k-1} worth is determined by τ , and the study mentions four different types of τ. The search directions of this algorithm are always sufficiently descent when using strong Wolfe line search (SWC). Under reasonable assumptions, the proposed algorithm achieves global convergence. The numerical comparison demonstrates that our proposed method works well for solving unconstrained optimization problems.

Keywords:
Unconstrained optimization, Three term, Conjugate gradient, strong Wolfe condition, sufficient descent method, global convergent.

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1. INTRODUCTION

Consider the following unconstrained nonlinear optimization problem:

$$Min\ r(x), x \in \mathcal{R}^n \tag{1}$$

with $r: \mathcal{R}^n \to \mathcal{R}$ being soft and $g(x) = \nabla r(x)$. One method for obtaining the smallest amount (1) [1] is the nonlinear CG method, which does not demand any matrices. This is how iterative CG methods look.

$$x_k = x_{k-1} + \omega_{k-1} u_{k-1}, \qquad k = 0,1,2,3, \ldots, \tag{2}$$

such that ω_k denotes a positive step size and u_k denotes the new search direction, which is typically calculated as follows:

$$u_k = \begin{cases} -g_k, & k = 0 \\ -g_k + \beta_k u_{k-1}, & k \geq 1 \end{cases}, \tag{3}$$

The scalar parameter $\beta_k \in \mathcal{R}$ is typically selected so that (2) - (3) can be diminished to the linear CG method [2].

Similarly, if r(x) If the exact line search yields a purely convex quadratic function, then (ELS). [3 - 9] define six pioneering forms of β_k.

$$\beta_k^{HS} = \frac{g_k^T y_{k-1}}{y_{k-1}^T u_{k-1}}; \quad \beta_k^{FR} = \frac{g_k^T g_k}{g_{k-1}^T g_{k-1}}; \quad \beta_k^{PRP} = \frac{g_k^T y_{k-1}}{g_{k-1}^T g_{k-1}};$$

$$\beta_k^{CD} = \frac{g_k^T g_k}{y_{k-1}^T u_{k-1}}; \quad \beta_k^{LS} = \frac{g_k^T y_{k-1}}{-g_{k-1}^T u_{k-1}}; \quad \beta_k^{DY} = \frac{g_k^T g_k}{y_{k-1}^T u_{k-1}}.$$

However, for large-scale problems, an exact line search is usually not possible, so any value of ω_k that meets certain properties known as SWC is accepted.

$$r(x_k + \omega_k u_k) \leq r(x_{k-1}) + \delta_1 \omega_{k-1} u_{k-1} \tag{4}$$

$$|g(x_k + \omega_k u_k)^T u_k| \leq \delta_2 |g_{k-1}^T u_{k-1}| \tag{5}$$

Where $0 < \delta_1 < \delta_2 < 1$,and d_k that is a path to the minimum must be descent [10].

2. MOTIVATION AND FORMULA

Many scholars have recently proposed plenty of three-term CG (TTCG) methods for unconstrained optimization problems as modifications of the classical CG algorithms Using the Armijo line search, Zhang et al. [11] suggested a three-term MPRP procedure and demonstrated its global convergence

$$u_k = \begin{cases} -g_k\,, & k=0 \\ -g_k + \frac{g_k^T y_{k-1}}{g_{k-1}^T g_{k-1}} u_{k-1} - \frac{g_k^T u_{k-1}}{g_{k-1}^T g_{k-1}} y_{k-1}, & k \geq 1 \end{cases} \quad (6)$$

such that $y_{k-1} = g_k - g_{k-1}$,the TTHS method was created by Zhang et al. [12] in a similar content. This is written as:

$$u_k = \begin{cases} -g_k\,, & k=0 \\ -g_k + \frac{g_k^T y_{k-1}}{y_{k-1}^T u_{k-1}} u_{k-1} - \frac{g_k^T u_{k-1}}{y_{k-1}^T u_{k-1}} u_{k-1}, & k \geq 1 \end{cases}, \quad (7)$$

The TTHS method has the steepest descent capability; when an ELS would be used, it is reduced to the classic HS method. Furthermore, to ensure the global convergence properties of the search direction specified in (7), an MTTHS algorithm on the search direction is used:

$$u_k = \begin{cases} -g_k\,, & k=0 \\ -g_k + \frac{g_k^T z_{k-1}}{z_{k-1}^T u_{k-1}} d_{k-1} - \frac{g_k^T u_{k-1}}{z_{k-1}^T u_{k-1}} y_{k-1}, & k \geq 1 \end{cases}, (8)$$

Offered that MTTHS were presented in (7) to indicate the search direction's global convergence properties, one could guess why (7) is not used to prove the search direction's global convergence properties. Rather than disregarding (7), it should be made efficient and globally convergent. As a result, (7) can be tweaked to meet the global convergence requirements. This modification is expected to outperform the MTTCG algorithm in the sense of numerical effectiveness. Zhang et al. [13] proposed a new Dai-Liao-based TTCG method motivated by this appealing descent property:

$$u_k = -g_k + \frac{{g_k}^T (y_{k-1} - \tau s_{k-1})}{s_{k-1}^T\, y_{k-1}} s_{k-1} - \frac{g_k^T\, s_{k-1}}{s_{k-1}^T\, y_{k-1}} (y_{k-1} - \tau s_{k-1}) \quad (9)$$

Such that $u_0 = -g_0$ and $\tau \geq 0$. Once again, regardless of the line search method used, the sufficient descent process occurs. i.e. for this method, $g_k^T u_k = -\|g_k\|^2$ for all k. Al-Bayati and Sharif [14] developed a specialization of the TTCG given by (9) that evaluates the search direction as

$$u_k = -g_k + \beta_k^{DL+} s_{k-1} - \frac{g_k^T\, s_{k-1}}{s_{k-1}^T\, y_{k-1}} (y_{k-1} - \tau s_{k-1}), \quad (10)$$

Where $\beta_k^{DL+} = max\left\{\frac{g_k^T\, y_{k-1}}{s_{k-1}^T\, y_{k-1}}, 0\right\} - \tau \frac{g_k^T\, s_{k-1}}{s_{k-1}^T\, y_{k-1}}$ and

$\tau = \frac{||y_{k-1}||^2}{s_{k-1}^T\, y_{k-1}}$. It is clear that (10) meets the sufficient descent condition regardless of the line search method used.

We propose the following three conjugate gradient terms:

$$u_k = -g_k + \beta_{k-1} u_{k-1} - \mu_{k-1} y_{k-1} \quad (11)$$

Many scholars have studied the values of μ_{k-1} and β_{k-1}which are critical for the algorithm's global convergence and numerical efficiency (see [15-19]).

When investigating the sufficient descent condition, the step-size ω_k is critical

$$g_k^T u_k < -\alpha_k^n \|g_k\|^2, \alpha_k^n > 0, \forall k,\ n = 1, 2, 3 \;\&\; 4 \quad (12)$$

as well as global convergence properties

$$\lim_{k\to\infty} \|g_k\|^2 = 0. \quad (13)$$

We start with a motivation explanation before delving into the specifics of our approach. Mandara and colleagues [20] posited an RMIL+ based new CG formula.

$$\beta_{k-1}^{\text{MMWA}} = \frac{g_k^T (g_k - g_{k-1} + u_{k-1})}{||u_{k-1}||^2}. \quad (14)$$

They keep the RMIL+ denominator but change the sign of the numerator from negative to positive.

Create a globally convergent method in the face of an imprecise line search (ILS). In this part, we will implement a novel N3T method by modifying the β_k^{MMWA} as follows:

$$\beta_{k-1}^{N*} = \frac{g_k^T (y_{k-1} + u_{k-1})}{||u_{k-1}||^2 + |g_k^T u_{k-1}|}. \quad (15)$$

If ELS used, then (15) reduced to (14). The N3T method is defined as follows:

$$u_k = -g_k + \beta_{k-1}^{N*} s_{k-1} - \mu_{k-1} y_{k-1}. \quad (16)$$

Multiply the both sides Eq. (12) by y_{k-1}^T

$$y_{k-1}^T\, u_k = -y_{k-1}^T\, g_k + \beta_{k-1}^{N*}\, y_{k-1}^T s_{k-1} - \mu_{k-1}\, y_{k-1}^T\, y_{k-1}, \quad (17)$$

Actual algorithms, on the other hand, in most cases, ILS is preferred over ELS. The conjugation condition was recently replaced by Dai & Liao [21].$(i.e. u_k^T y_{k-1} = -g_k^T s_{k-1})$ by the condition,

$$y_{k-1}^T\, u_k = -\tau^n g_k^T s_{k-1}, \text{ where } \tau > 0, n = 1, 2, 3 \;\&\; 4 \quad (18)$$

Put (18) into (17), yielding:

$$-\tau^n\, g_k^T\, s_{k-1} = -g_k^T\, y_{k-1} + \beta_{k-1}^*\, s_{k-1}^T\, y_{k-1} - \mu_{k-1} \left|\left|y_{k-1}\right|\right|^2$$

We get after some algebraic operations with $s_{k-1} = \omega_{k-1} u_{k-1}$, we obtain

$$\mu_{k-1} = \frac{g_k^T (\tau^n \omega_{k-1} u_{k-1} - y_{k-1})}{\left|\left|y_{k-1}\right|\right|^2} + \beta_{k-1}^*\, \frac{u_{k-1}^T\, y_{k-1}}{\left|\left|y_{k-1}\right|\right|^2}, \quad n = 1, 2, 3, 4. \quad (19)$$

We will notice that equation (19) is dependent on four criteria of τ, implying that we obtained four new algorithms based on τ^n values. The four parameters of τ^n are defined as follows:

$\tau^1 = \frac{||y_{k-1}||}{||s_{k-1}||}$, proposed by Yao et al. [22].

$\tau^2 = \frac{2||y_{k-1}||^2}{||s_{k-1}||^2}$, proposed by Al-Bayati [14].

$\tau^3 = 1 + \frac{2||y_{k-1}||^2}{||s_{k-1}||^2}$, proposed by Andrei [23].

$\tau^4 = 0.5$.

The N3T Algorithm

Step 1: Choose $x_0 \in \mathcal{R}^n$, , $\varepsilon > 0$, $d_k = -g_k$, put k=0 .

Step 2: If $\|g_k\| \le \varepsilon$, then end; or else, go on to the next step.

Step 3: Estimate ω_{k-1} using SWC described in (4) & (5).

Step 4: Estimate x_k by (2), and Estimate g_k & f_k.

Step 5: Estimate the directions d_k by (16), (19) & τ^n .

Step 6: If $\|g_k\| \le \varepsilon$, stop; Otherwise, proceed to the next step.

Step 7: If k=n or $|g_k^T g_{k-1}| \ge 0.2\ (\|g_k\|^2)$ is satisfied, go to step 1; Or else, proceed to the following step.

Step 8: Put $k = k + 1$ and proceed to step (3).

3. N3T 's Global Convergence property

This part investigates the convergence of the suggested method. The sufficient descent condition is one of the requirements for an algorithm to converge. The proof of the sufficient descent condition of the proposed method with ILS is shown below.

Theorem (1): d_k is generated by formula (16), it is obvious that

$$u_k^T g_k \le -\alpha_k^n ||g_k||^2, \forall k \ \& \ n = 1,2,3,4. \tag{20}$$

Proof: When $k = 0$, $u_0^T g_0 = -\|g_0\|^2$, holds. We suppose it's true for

$$u_{k-1}^T g_{k-1} \le -\alpha_k^n ||g_{k-1}||^2, n = 1, 2 ,3 \ \& \ 4 .$$

When we multiply both sides of (16) by g_k^T, we get

$$g_k^T u_k = -||g_k||^2 + \beta_{k-1}^* g_k^T u_{k-1} - \mu_{k-1} g_k^T y_{k-1}$$

Mandara and colleagues [20] proved that

$$\frac{g_k^T(g_k - g_{k-1} + u_{k-1})}{||u_{k-1}||^2} \le \frac{||g_k||^2}{||u_{k-1}||^2}.$$

So,

$$0 < \frac{g_k^T(y_{k-1} + u_{k-1})}{||u_{k-1}||^2 + |g_k^T u_{k-1}|} \le \frac{g_k^T(g_k - g_{k-1} + u_{k-1})}{||u_{k-1}||^2} \le \frac{||g_k||^2}{||u_{k-1}||^2} \tag{21}$$

We obtain by applying the second condition of the SWC (5)

$$u_{k-1}^T g_k \le -\delta_2 g_{k-1}^T u_{k-1} \le \alpha \delta_2 ||g_{k-1}||^2 \tag{22}$$

and $g_k^T u_{k-1} \le y_{k-1}^T u_{k-1} \le -(1 + \delta_2) g_{k-1}^T u_{k-1}$

$$\le \alpha\ (1 + \delta_2) ||g_{k-1}||^2 \tag{23}$$

We have, $g_k^T y_{k-1} = ||g_k||^2 - g_k^T g_{k-1}$,

since, $|g_k^T g_{k-1}| \ge 0.2\ (\|g_k\|^2)$,

so, $g_k^T g_{k-1} \le -0.2(\|g_k\|^2)$, therefore

$$g_k^T y_{k-1} \le 1.2||g_k||^2 \tag{24}$$

$$\beta_{k-1}^{N*} g_k^T u_{k-1} = \frac{g_k^T(y_{k-1} + u_{k-1})}{||d_{k-1}||^2 + |g_k^T u_{k-1}|} g_k^T u_{k-1} \ .$$

Using (21), (22), (23) and (24), we get

$$\beta_{k-1}^{N*} g_k^T u_{k-1} \le \frac{\xi_1^2}{\bar{\eta}^2} \alpha\ (1 + \delta_2) ||g_{k-1}||^2 \le \varphi_1 ||g_k||^2. \tag{25}$$

$$\mu_{k-1} g_k^T y_{k-1} = \left[\frac{g_k^T(\tau_n \omega_{k-1} d_{k-1} - y_{k-1})}{||y_{k-1}||^2} + \frac{g_k^T(y_{k-1} + d_{k-1})}{||d_{k-1}||^2 + |g_k^T d_{k-1}|} * \frac{u_{k-1}^T y_{k-1}}{||y_{k-1}||^2}\right] g_k^T y_{k-1}$$

$$\le \left[\frac{\tau_n \omega_{k-1} \alpha \delta_2 ||g_{k-1}||^2}{||y_{k-1}||^2} - \frac{1.2||g_k||^2}{||y_{k-1}||^2} + \frac{||g_k||^2}{||d_{k-1}||^2} * \frac{\alpha \delta_2 ||g_{k-1}||^2}{||y_{k-1}||^2}\right] \left(1.2||g_k||^2\right)$$

$$\le \left[\left(\tau_n \omega_{k-1} + \frac{||g_k||^2}{||d_{k-1}||^2}\right) \frac{\alpha \delta_2 ||g_{k-1}||^2}{||y_{k-1}||^2} - 1.2 \frac{||g_k||^2}{||y_{k-1}||^2}\right] \left(1.2||g_k||^2\right),$$

Let $\omega_{k-1} \le a$, then

$$\mu_{k-1} g_k^T y_{k-1} \le \left[\left(\tau_n a + \frac{\xi_1^2}{\bar{\eta}^2}\right) \frac{\alpha \delta_2 \xi_2^2}{\bar{\varepsilon}^2} - 1.2 \frac{\xi_1^2}{\bar{\varepsilon}^2}\right] \left(1.2||g_k||^2\right)$$

We have τ^n, n=1, 2, 3 & 4.

$\tau^1 = \frac{||y_{k-1}||}{||s_{k-1}||} \le \frac{\varepsilon}{\bar{\Omega}}$, then

$$\mu_{k-1} g_k^T y_{k-1} \le \left[\left(\frac{\varepsilon}{\bar{\Omega}} a + \frac{\xi_1^2}{\bar{\eta}^2}\right) \frac{\alpha \delta_2 \xi_2^2}{\bar{\varepsilon}^2} - 1.2 \frac{\xi_1^2}{\bar{\varepsilon}^2}\right] \left(1.2||g_k||^2\right)$$

$$\le \rho_1 ||g_k||^2.$$

$\tau^2 = \frac{2||y_{k-1}||^2}{||s_{k-1}||^2} \le \frac{2\varepsilon^2}{\bar{\Omega}^2}$, so

$$\mu_{k-1} g_k^T y_{k-1} \le \left[\left(\frac{2\varepsilon^2}{\bar{\Omega}^2} a + \frac{\xi_1^2}{\bar{\eta}^2}\right) \frac{\alpha \delta_2 \xi_2^2}{\bar{\varepsilon}^2} - 1.2 \frac{\xi_1^2}{\bar{\varepsilon}^2}\right] \left(1.2||g_k||^2\right)$$

$$\le \rho_2 ||g_k||^2.$$

$\tau^3 = 1 + \frac{2||y_{k-1}||^2}{||s_{k-1}||^2} \le 1 + \frac{2\varepsilon^2}{\bar{\Omega}^2}$, therefore

$$\mu_{k-1} g_k^T y_{k-1} \le \left[\left(\left(1 + \frac{2\varepsilon^2}{\bar{\Omega}^2}\right) a + \frac{\xi_1^2}{\bar{\eta}^2}\right) \frac{\alpha \delta_2 \xi_2^2}{\bar{\varepsilon}^2} - 1.2 \frac{\xi_1^2}{\bar{\varepsilon}^2}\right] \left(1.2||g_k||^2\right)$$

$$\leq \rho_3 \left\| g_k \right\|^2 .$$

Finally, when $\tau^4 = 0.5 \leq b$.

$$\mu_{k-1} g_k^T y_{k-1} \leq \left[\left(ba + \frac{\xi_1^2}{\bar{\eta}^2} \right) \frac{\alpha \delta_2 \xi_2^2}{\bar{\varepsilon}^2} - 1.2 \frac{\xi_1^2}{\bar{\varepsilon}^2} \right] \left(1.2 \left\| g_k \right\|^2 \right)$$

$$\leq \rho_4 \left\| g_k \right\|^2 .$$

So,

$$\mu_{k-1} g_k^T y_{k-1} \leq \varphi_2 \left\| g_k \right\|^2 , \varphi_2 = \rho_1, \rho_2, \rho_3 \ \& \ \rho_4 \ . \quad (26)$$

Using (25) and (26), we get

$$g_k^T u_k \leq - \left\| g_k \right\|^2 + \varphi_1 \left\| g_k \right\|^2 + \varphi_2 \left\| g_k \right\|^2$$

$$\leq - [1 - (\varphi_1 + \varphi_2)] \left\| g_k \right\|^2$$

$$g_k^T u_k \leq - \alpha_k^n \left\| g_k \right\|^2 , \forall k \ \& \ n = 1,2,3,4.$$

With $\alpha_k^n = [1 - (\varphi_1 + \varphi_2)]$.

To establish the global convergence property of an algorithm, the following basic presumptions on the objective function must be made.

Presumptions (I)

i-The level set $\Lambda = \{ x \in \mathcal{R}^n, r(x) \leq r(x_0) \}$, is bounded.

ii-The function r is continuously differentiable, and its gradient is Lipschitz continuous in a specific neighborhood $\mathbb{N}$ of Λ, with the notable exception that there is a constant $\mathcal{L} > 0$ such that:

$$\| \nabla r(x_1) - \nabla r(x_2) \| \leq \mathcal{L} \| x_1 - x_2 \| , \forall x_1, x_2 \in \mathcal{N} . \quad (27)$$

Since $\{ r(x_k) \}$ is decreasing, it is clear that the sequence $\{ x_k \}$ generated by N3T Algorithm is contained in Λ.

Furthermore, using the N3T Algorithm, we can deduce from Presumption (I) that there is a positive constant, resulting in $0 < \| g_k \| \leq \xi, \ \forall x \in \Lambda$ [24].

Theorem (2): Consider whether Presumption (I) is true, whether the N3T is satisfying (16), and whether the step-size ω_k satisfies (4) & (5). If

$$\sum_{k \geq 1} \frac{1}{\| u_k \|^2} = \infty \quad (28)$$

Then

$$lim_{k \to \infty} \, inf \| g_k \| = 0 \ . \quad (29)$$

Theorem (3): Suppose that Presumption (I) is correct. Let $\{ x_k \}$ be a point sequence generated by N3T. Then there's $\lim_{k \to \infty} \inf \| g_k \| = 0$.

Proof: From the direction of N3T, we have

$$\left\| u_k \right\| = \| - g_k + \beta_{k-1}^{N*} u_{k-1} - \mu_{k-1} y_{k-1} \|$$

$$\leq \left\| g_k \right\| + | \beta_{k-1}^{N*} | \, \| u_{k-1} \| + | \mu_{k-1} | \, \| y_{k-1} \| \quad (30)$$

From (21), we have

$$| \beta_{k-1}^{N*} | \, \| u_{k-1} \| \leq \frac{\left\| g_k \right\|^2}{\left\| u_{k-1} \right\|^2} \| u_{k-1} \|$$

$$\leq \frac{\xi_1}{\bar{\eta}} \left\| g_k \right\| \leq \vartheta_1 \left\| g_k \right\| . \quad (31)$$

$$| \mu_{k-1} | \, \| y_{k-1} \| \leq \frac{\tau^n | \omega_{k-1} | \| g_k \| | \| u_{k-1} \| - \| g_k \| | \| y_{k-1} \| |}{\left\| y_{k-1} \right\|^2} \| y_{k-1} \| + \frac{\left\| g_k \right\|^2}{\left\| u_{k-1} \right\|^2} * \frac{\| u_{k-1} \| \| y_{k-1} \|}{\left\| y_{k-1} \right\|^2} \| y_{k-1} \|$$

$$\leq \left[\frac{\tau^n | \omega_{k-1} | | \| u_{k-1} \| - \| y_{k-1} \| |}{\| y_{k-1} \|} + \frac{\| g_k \|}{\| u_{k-1} \|} \right] \left\| g_k \right\|$$

$$\leq \left[\frac{\tau^n a \eta - \varepsilon}{\bar{\varepsilon}} + \frac{\xi_1}{\bar{\eta}} \right] \left\| g_k \right\| \leq \vartheta_2 \left\| g_k \right\| . \quad (32)$$

For all τ^n, n=1, 2, 3 & 4, we assume that $\tau^n \leq v$, put (31) and (32) in (30), we get:

$$\left\| u_k \right\| \leq [1 + \vartheta_1 + \vartheta_2] \left\| g_k \right\|$$

$$\leq \vartheta \xi_1 = W$$

$$\| u_k \| \leq W \ \Rightarrow \| u_k \|^2 \leq W^2 \quad (33)$$

We obtain by adding the sums on both sides of (33).

$$\sum_{k \geq 1} \frac{1}{\| u_k \|^2} \geq \frac{1}{W^2} \sum_{k \geq 1} 1 = +\infty .$$

As a result of the inconsistency with the Zountendijk theorem [25],

$$\lim_{k \to \infty} inf \| g_k \| = 0. \ \blacksquare$$

4. EXPERIMENTS WITH NUMBERS

The main task of this section is to report on the performance of the N3T algorithm on a set of test problems. Fortran77 and double precision arithmetic were used to write the codes.

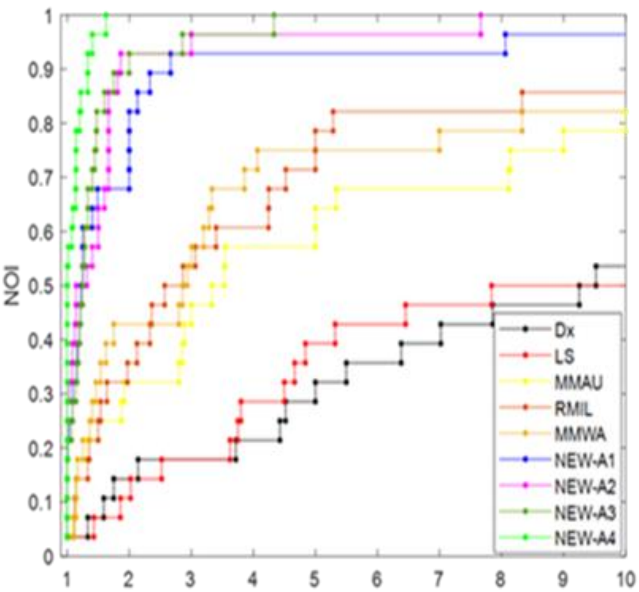
Our experiments were carried out on a set of 28 nonlinear unconstrained problems, and we compared the proposed algorithm, which contains four types of parameter μ_{k-1} that are dependent on the parameter τ^n, (NEW- A1, NEW- A2, NEW- A3, and NEW- A4), and we tried on small dimensions (n=100) and large dimensions (n=1000) of the variables with SWC with $\delta_1 = 10^{-3}$ and $\delta_2 = 0.9$, respectively. The same test functions were used to compare our algorithms' reliability to the well-known routines Dx [26], LS [27], MMAU [28], RMIL [29], and MMWA [20].

When the following stopping criteria are met $\| g_{k+1} \| \leq 1 * 10^{-6}$, all of these methods terminate. These routines are also forced to stop if the number of iterations exceeds 600.

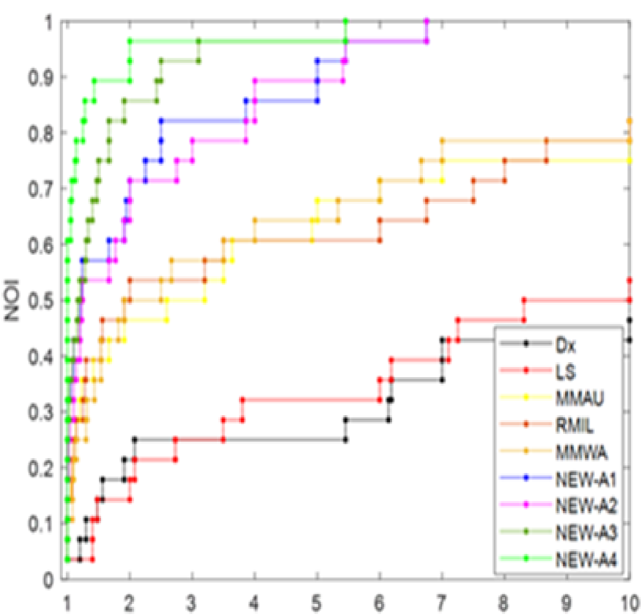
The tests were based on number of iterations (No. I) and the number of function evaluations (No. F). As shown in the figures below, the four suggested algorithms outperform the other algorithms. At the same time, when we compare the four suggested algorithms, the NEW- A4 is the best, followed by the NEW- A3, the NEW- A2, and finally the NEW- A1.

The Dolan and More' method [30] was used to plot these results for better comparison, and the results are shown in the figures below:

Fig. (1) depicts the progress of the N3T algorithm over the basic algorithms of Dx, LS, MMAU, RMIL, and MMWA in relation to the calculated (No. I) of the test functions during Dolan-More' method implementation with (n=100 & n=1000).



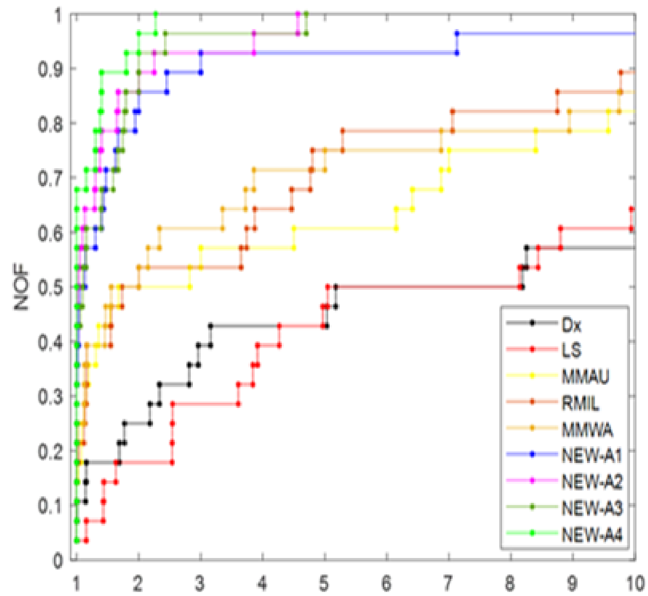
(a)



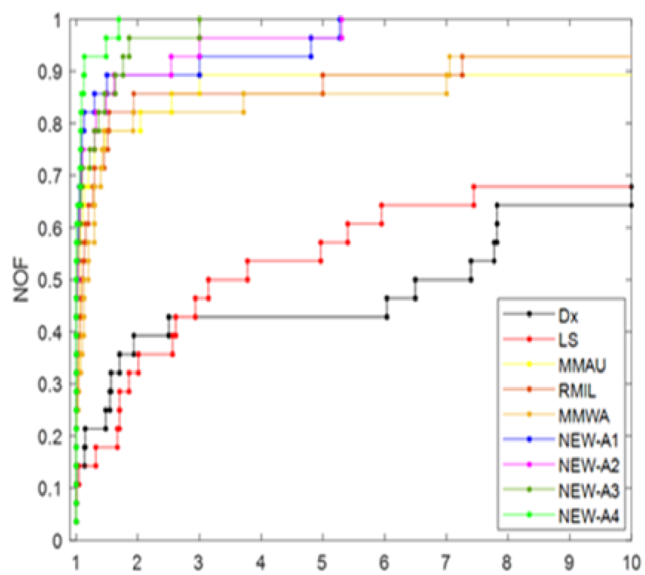
(b)

Fig 1. Performance results based on (No. I) (a) n=100 & (b) n=1000.

Fig. (2) depicts the progress of the N3T algorithm over the basic algorithms of Dx, LS, MMAU, RMIL, and MMWA in relation to the calculated (No. F) of the test functions during Dolan-More method implementation with (n=100 & n=1000).



(c)



(d)

Fig 2. Performance results based on (No. F) (c) n=100 & (d) n=1000.

Conclusion

Recent CG method research has resulted in a number of modifications to the CG method. TTCG methods are an intriguing computational innovation that results in efficient conjugate gradient algorithms. When combined with a sWLS, a N3T as a modification of the MMAW formula gives enough descent directions for the objective function in this work. The modified method's global convergence was founded by employing the same line search. Furthermore, numerical experiments show that the proposed methods are effective. outperform some traditional conjugate gradient methods that employ some test functions and inexact line search.

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خاصية الانحدار الكافِ لمعاملات مختلفة لتحسين طريقة ثلاثية الحدود

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الملخص

تم في هذا البحث اشتقاق معلمة جديدة μ_{k-1} لطريقة التدرج المترافق ثلاثية الحدود لحل مسائل الامثلية غير المقيدة. كما يتضح من الحسابات العددية والاثباتات، ان قيمة μ_{k-1} تعتمد على معلمة أخرى هي τ ، والأخيرة لها اربعة أنواع تم التطرق اليها داخل البحث. عند استخدام خطي وولف القويين فان اتجاه البحث لهذه الخوارزمية تحقق خاصية الانحدار الكاف دائما مقارنة مع خوارزميات التدرج المترافق غير الخطية. الخوارزمية المقترحة تحقق ايضاً التقارب الشمولي تحت فرضيات معينة. النتائج العددية اثبتت ان الطريقة المقترحة فعالة في حل مسائل الامثلية غير المقيدة.

الكلمات المفتاحية: الامثلية غير المقيدة، ثلاثية الحدود، التدرج المترافق، شرط وولف القوي، الانحدار الكاف، التقارب الشمولي.