Al-Rafidain Journal of Computer Sciences and Mathematics (RJCM)

www.csmj.mosuljournals.com

Construction of a complete $(\mathrm{K}, \mathscr{b}) - \boldsymbol{span}$ in $\boldsymbol{PG}(3, 13)$

Hamid Mohamed khalaf[1],*Nada Yassen Kasm Yahya[2]

Department of Mathematics, College of Basic Education, University of Telafer, Tall'Afar, Mosul, Iraq[1],
Department of Mathematics, College of Education for Pure Science, University of Mosul, Mosul, Iraq[2]
**Corresponding author. Email: hamid_math_85@yahoo.com [1]*
drnadaqasim3@uomosul.edu.iq[2]

Article information

Article history:
Received : 7/2/2022
Accepted : 19/5/2022
Available online :

Keywords:
projective 3-space, spreads.

Correspondence:
Author :Hamid Mohamed Khalaf
Email: hamid_math_85@yahoo.com

Abstract

The main goal of this work is to find a spread of PG(3,13). By construct a complete $(\mathrm{K}, \mathscr{b})$ −span which represents applications of algebraic geometry in 3-dimensional projective space PG(3,q). We prove that the maximum $(\mathrm{K}, \mathscr{b})$-span in PG(3,13) is $(170, \mathscr{b})$-span, which is a spread.

1. INTRODUCTION

A projective 3-space $PG(3, q)$ over Galois field (denoted by, $GF(q)$, where $q = p^h$ for some prime number p and some integer h), is a 3-dimensional projective space. [1], [2]. The Projective 3 – space satisfies the following axioms:

1- Any two distinct planes intersect in a unique line.
2- Any two distinct points are contained in a unique line.
3- Any two distinct coplanar lines intersect at a unique point.
4- Any line not on a given plane intersects the plane at a unique point.
5- Any three distinct non-collinear points, with the lines and points that lie not on them are contained in a unique plane. [3]

Two lines intersect in at most one point. This happens precisely if the two lines lie in a plane. Otherwise, they are called skew.

A projective 3-space PG(3,q) satisfies the following:

1- Any two planes intersect in exactly q + 1 points, any line is on exactly q + 1 planes and any two points on exactly q + 1 planes."
2- Every plane contains exactly $q^2 + q + 1$ points (lines) and every point is on exactly $q^2 + q + 1$ planes.
3- There exist $q^3 + q^2 + q + 1$ of points and there exists $q^3 + q^2 + q + 1$ of planes.
4- Every line contains exactly q + 1 points and every point is on exactly q + 1 lines. [4]

By using computer programs A, we found the points, lines and planes which are in Tables (1 and 2). The purpose of this study is to investigate a geometric construction $(\mathrm{K}, \mathscr{b})$-span in 3-dimensional projective space and find the maximum complete $(170, \mathscr{b})$-span in $PG(3{,}13)$, equal to all the points of the space.

2. Preliminaries

2.1 Definition Plane Π

In $PG(3, q)$ a plane Π is the set of all points $p(y_1, y_2, y_3, y_4)$ satisfying a linear equation $a_1y_1 + a_2y_2 + a_3y_3 + a_4y_4 = 0$. This plane is denoted by $\Pi[a_1, a_2, a_3, a_4]$, where a_1, a_2, a_3, a_4 are elements in $GF(q)$ with the exception of the quadrable consisting of four zero elements [3], [4].

2.2 Theorem

The points of $PG(3, q)$ have a unique forms, which are (1,0,0,0), $(x_1,1,0,0)$, $(x_1, x_2,1,0)$ and $(x_1, x_2, x_3,1)$ for all x_1, x_2, x_3 in $GF(q)$, which are (1,0,0,0) is one point, $(x_1,1,0,0)$, q points, $(x_1, x_2,1,0)$, q^2 points, and $(x_1, x_2, x_3,1)$, q^3 points, for all x_1, x_2, x_3 in $GF(q)$ [3].

2.3 Theorem

The planes of $PG(3,q)$ have unique forms, which are [1,0,0,0], [x_1,1,0,0], [x_1, x_2,1,0], [x_1, x_2, x_3,1] for all x_1, x_2, x_3 in $GF(q)$. which are [1,0,0,0] is one plane, [x_1,1,0,0], q planes, [x_1, x_2,1,0], q^2 planes and [x_1, x_2, x_3,1], q^3 planes, for all x_1, x_2, x_3 in $GF(q)$ [3].

2.4 Theorem

There exists $(q^2 + 1)(q^2 + q + 1)$ of lines, in PG (3,q) [3], [5].

2.5 The projective space in $PG(3,13)$

The projective space $PG(3,13)$ contains (2380) points, and (2380) planes, such that each point is on (183) planes and every plane contains (183) points, any line contains (14) points, which is the intersection of (14) planes. The points, planes and lines of $PG(3,13)$ given in Tables (1 and 2).

3. Spread and $(K, \mathscr{b})$-span

3.1 Definition

A $(K, \mathscr{b})$-span, $\mathscr{b} \geq 1$, is a set of K spaces $\Pi_{\mathscr{b}}$ (K lines) no two of which intersect [6], [7].

3.2 Definition

A maximum $(K, \mathscr{b})$-span is a set of K spaces $\Pi_{\mathscr{b}}$ which are every points of $PG(3,q)$ lies in exactly one line of $\Pi_{\mathscr{b}}$ and every two lines of $\Pi_{\mathscr{b}}$ are disjoint.

A K-span is a $(K, 0)$-span, that is, a set of K points.

A $(K, \mathscr{b})$-span is complete if it is not contained in a $(K + 1, \mathscr{b})$-span [7],[8].

3.3 Theorem

In $PG(3,q), q > 2$, there exists a complete K-span with $K = q^2 - q + 1$, or $K = q^2 - q + 2$ [7].

3.4 Theorem

In $PG(3,q), q$ odd and $q > 3$, there exists a complete $(q^2 - q + 2)$-span [7].

3.5 Definition

In $PG(3,q)$, a spread S is a set of $(q^2 + 1)$ lines, which are pairwise disjoint and thus, partition the set of points.

A partial spread ß is a set of mutually skew lines and if |ß|=s, then ß is also called a s-span. Hence, a $(q^2 + 1)$-span is a spread of $PG(3,q)$.

A partial spread is called maximal when it is not contained properly in a larger partial spread. [7], [9], [10].

3.6 Corollary

A K-span with $K > q^2 - \sqrt{q}$, can be completed uniquely to a spread [3].

3.7 Theorem

In $PG(3,q)$, a partial spread containing more than $q^2 + 1 - \sqrt{2q}$ lines, in contained in a spread of $PG(3,q)$ [11], [12].

4. Algorithm

- Choose the lines who containing all points of $PG(3,q)$.
- Choose the first line of any plane that we take contains $q + 1$ of the ordered and series points which is $1,2,3,\ldots,q + 1$, of $PG(3,q)$.
- Searching to next line starting with point $q + 2$, but does not intersect the first line.
- searching to next line starting with point $q + 3$, but does not intersect the first and second lines.
- searching to next line starting with point $q + 4$, but does not intersect the first, second and third lines.
- We continue in this way until we get the line that begins with point $q^2 + q + 1$. It must also be maintained that there is no intersection between all the previous selected lines. In this case, we get that K is equal to $q^2 + 1$.

Table (1): Points and Plans of $PG(3,13)$

I	p_i	π_i								
1	(1,0,0,0)	2	15	28	41	54	67	80	93	106
			119	132	145	158	171	184	197	210
			223	236	249	262	275	288	301	314
			327	340	353	366	379	392	405	418
			431	444	457	470	483	496	509	522
			535	548	561	574	587	600	613	626
			639	652	665	678	691	704	717	730
			743	756	769	782	795	808	821	834
			847	860	873	886	899	912	925	938
			951	964	977	990	1003	1016	1029	1042
			1055	1068	1081	1094	1107	1120	1133	1146
			1159	1172	1185	1198	1211	1224	1237	1250
			1263	1276	1289	1302	1315	1328	1341	1354
			1367	1380	1393	1406	1419	1432	1445	1458
			1471	1484	1497	1510	1523	1536	1549	1562
			1575	1588	1601	1614	1627	1640	1653	1666
			1679	1692	1705	1718	1731	1744	1757	1770
			1783	1796	1809	1822	1835	1848	1861	1874
			1887	1900	1913	1926	1939	1952	1965	1978
			1991	2004	2017	2030	2043	2056	2069	2082
			2095	2108	2121	2134	2147	2160	2173	2186
			2199	2212	2225	2238	2251	2264	2277	2290
			2303	2316	2329	2342	2355	2368		
2	(0,1,0,0)	1	15	16	17	18	19	20	21	22
			23	24	25	26	27	184	185	186
			187	188	189	190	191	192	193	194
			195	196	353	354	355	356	357	358
			359	360	361	362	363	364	365	522
			523	524	525	526	527	528	529	530
			531	532	533	534	691	692	693	694
			695	696	697	698	699	700	701	702
			703	860	861	862	863	864	865	866
			867	868	869	870	871	872	1029	1030
			1031	1032	1033	1034	1035	1036	1037	1038
			1039	1040	1041	1198	1199	1200	1201	1202
			1203	1204	1205	1206	1207	1208	1209	1210
			1367	1368	1369	1370	1371	1372	1373	1374
			1375	1376	1377	1378	1379	1536	1537	1538
			1539	1540	1541	1542	1543	1544	1545	1546
			1547	1548	1705	1706	1707	1708	1709	1710
			1711	1712	1713	1714	1715	1716	1717	1874
			1875	1876	1877	1878	1879	1880	1881	1882
			1883	1884	1885	1886	2043	2044	2045	2046
			2047	2048	2049	2050	2051	2052	2053	2054
			2055	2212	2213	2214	2215	2216	2217	2218
			2219	2220	2221	2222	2223	2224		
⋮										
2380	(12,12,12,1)	14	27	39	51	63	75	87	99	111
			123	135	147	159	171	185	197	222
			234	246	258	270	282	294	306	318
			330	342	353	378	390	402	414	426
			438	450	462	474	486	498	510	534
			546	558	570	582	594	606	618	630
			642	654	666	678	702	714	726	738
			730	762	774	786	798	810	822	834
			839	870	882	894	906	918	930	942
			954	966	978	990	1015	1027	1038	1050
			1062	1074	1086	1098	1110	1122	1134	1146
			1171	1183	1195	1206	1218	1230	1242	1254
			1266	1278	1290	1302	1327	1339	1351	1363
			1374	1386	1398	1410	1422	1434	1446	1458
			1483	1495	1507	1519	1531	1542	1554	1566
			1578	1590	1602	1614	1639	1651	1663	1675
			1687	1699	1710	1722	1734	1746	1758	1770
			1795	1807	1819	1831	1843	1855	1867	1878
			1890	1902	1914	1926	1951	1963	1975	1987
			1999	2011	2023	2035	2046	2058	2070	2082
			2107	2119	2131	2143	2155	2167	2179	2191
			2203	2214	2226	2238	2263	2275	2287	2299
			2311	2323	2335	2347	2359	2371		

Table (2): Plane and lines of PG(3,13)

	15	2	171	93	67	…	54	67	93	171	171
	184	184	184	184	184	…	314	301	275	197	197
	353	197	509	431	405	…	353	353	353	353	353
	522	210	665	678	626	…	561	574	600	678	678
	691	223	821	756	847	…	769	795	847	834	834
	860	236	977	1003	899	…	977	1016	925	990	990
	1029	249	1133	1081	1120	…	1185	1068	1172	1146	1146
	1198	262	1289	1328	1341	…	1224	1289	1250	1302	1302
1	1367	275	1445	1406	1393	…	1432	1510	1497	1458	1458
	1536	288	1601	1653	1614	…	1640	1562	1575	1614	1614
	1705	301	1757	1731	1835	…	1848	1783	1822	1770	1770
	1874	314	1913	1978	1887	…	1887	2004	1900	1926	1926
	2043	327	2069	2056	2108	…	2095	2056	2147	2082	2082
	2212	340	2225	2303	2329	…	2303	2277	2225	2238	2238
.	.	.	.	.	.	….	.	.	.	.	
.	.	.	.	.	.	….	.	.	.	.	
.	.	.	.	.	.	….	.	.	.	.	
.	.	.	.	.	.	….	.	.	.	.	
.	.	.	.	.	.	….	.	.	.	.	
	171	27	14	159	87	…	111	159	147	14	171
	197	185	353	222	294	…	318	294	222	1374	197
	353	353	378	353	353	…	414	438	510	1386	353
	678	534	390	666	594	…	666	582	642	1398	678
	834	702	402	810	822	…	762	726	774	1410	834
	990	870	414	954	894	…	1027	870	906	1422	990
	1146	1038	426	1098	1122	…	1110	1183	1038	1434	1146
	1302	1206	438	1242	1363	…	1206	1327	1339	1446	1302
2380	1458	1374	450	1386	1422	…	1458	1458	1458	1458	1458
	1614	1542	462	1699	1663	…	1554	1602	1590	1483	1614
	1770	1710	474	1843	1722	…	1819	1746	1722	1495	1770
	1926	1878	486	1987	1963	…	1902	1890	2023	1507	1926
	2082	2046	498	2131	2191	…	2167	2203	2155	1519	2082
	2238	2214	2301	2282	2307	…	2301	2213	2332	2326	2276

4.1 Construction of spread in $PG(3,13)$

4.1.1 Theorem

The maximum (K, $\mathscr{b}$)-span in $\boldsymbol{PG(3,13)}$ is $(\mathbf{170}, \mathscr{b})$-span.

Proof: In Table (2), any two non-intersecting lines can be taken in $PG(3,13)$, say $\mathscr{b}_1 = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$ and $\mathscr{b}_2 = \{15, 184, 353, 522, 691, 860, 1029, 1198, 1367, 1536, 1705, 1874, 2043, 2212\}$, then $K_1 = \{\mathscr{b}_1, \mathscr{b}_2\}$ is a $(2, \mathscr{b})-$span. Can add another line $\mathscr{b}_3 = \{16, 197, 367, 537, 707, 877, 1047, 1217, 1387, 1557, 1727, 1897, 2067, 2237\}$, then $K_2 = \{\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3\}$ is a $(3, \mathscr{b}) -$ span, since $\mathscr{b}_3$ cannot intersect $\mathscr{b}_1$ or $\mathscr{b}_2$. Add another lines:

The line $\mathscr{b}_4 = \{17, 210, 381, 552, 723, 894, 1065, 1236, 1394, 1565, 1736, 1907, 2078, 2249\}$, this line can not intersect with any line of K_2, then

$K_3 = K_2 \cup \mathscr{b}_4 = \{\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \mathscr{b}_4\}$ is a $(4, \mathscr{b}) -$ span.

The line $\mathscr{b}_5 = \{18, 223, 395, 567, 739, 911, 1070, 1242, 1414, 1586, 1745, 1917, 2089, 2261\}$, this line can not intersect with any line of K_3, then $K_4 = K_3 \cup \mathscr{b}_5 = \{\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \mathscr{b}_4, \mathscr{b}_5\}$ is a $(5, \mathscr{b}) -$ span.

The line $\mathscr{b}_6 = \{19, 236, 409, 582, 755, 915, 1088, 1261, 1421, 1594, 1767, 1927, 2100, 2273\}$, this line can not intersect with any line of K_4, then

$K_5 = K_4 \cup \mathscr{b}_6 = \{\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \mathscr{b}_4, \mathscr{b}_5, \mathscr{b}_6\}$ is a $(6, \mathscr{b}) -$ span. The line $\mathscr{b}_7 = \{20, 249, 423, 597, 758, 932, 1106, 1267, 1441, 1602, 1776, 1950, 2111, 2285\}$, this line can not intersect with any line of K_5, then

$K_6 = K_5 \cup \mathscr{b}_7 = \{\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \mathscr{b}_4, \mathscr{b}_5, \mathscr{b}_6, \mathscr{b}_7\}$ is a $(7, \mathscr{b}) -$ span.

The line $\mathscr{b}_8 = \{21, 262, 437, 612, 774, 949, 1111, 1286, 1448, 1623, 1785, 1960, 2122, 2297\}$, this line can not intersect with any line of K_6, then

$K_7 = K_6 \cup \mathscr{b}_8 = \{\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \mathscr{b}_4, \mathscr{b}_5, \mathscr{b}_6, \mathscr{b}_7, \mathscr{b}_8\}$ is a $(8, \mathscr{b}) -$ span.

The line $\mathscr{b}_9 = \{22, 275, 451, 614, 790, 953, 1129, 1292, 1468, 1631, 1807, 1970, 2146, 2309\}$, this line can not intersect with any line of K_7, then

$K_8 = K_7 \cup \mathscr{b}_9 = \{\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \mathscr{b}_4, \mathscr{b}_5, \mathscr{b}_6, \mathscr{b}_7, \mathscr{b}_8, \mathscr{b}_9\}$ is a $(9, \mathscr{b}) -$ span.

The line $\mathscr{b}_{10} = \{23, 288, 465, 629, 806, 970, 1134, 1311, 1475, 1652, 1816, 1980, 2157, 2321\}$, this line can not intersect with any line of K_8, then

$K_9 = K_8 \cup \mathscr{b}_{10} =$
$\{\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \mathscr{b}_4, \mathscr{b}_5, \mathscr{b}_6, \mathscr{b}_7, \mathscr{b}_8, \mathscr{b}_9, \mathscr{b}_{10}\}$ is a $(10, \mathscr{b}) -$ span.

The line $\mathscr{b}_{11} = \{24, 301, 479, 644, 809, 987, 1152, 1317, 1495, 1660, 1825, 2003, 2168, 2333\}$, this line can not intersect with any line of K_9, then

$K_{10} = K_9 \cup \mathscr{b}_{11} =$
$\{\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \mathscr{b}_4, \mathscr{b}_5, \mathscr{b}_6, \mathscr{b}_7, \mathscr{b}_8, \mathscr{b}_9, \mathscr{b}_{10}, \mathscr{b}_{11}\}$ is a $(11, \mathscr{b}) -$ span.

The line $\mathscr{b}_{12} = \{25, 314, 493, 659, 825, 991, 1170, 1336, 1502, 1668, 1847, 2013, 2179, 2345\}$, this line can not intersect with any line of K_{10}, then $K_{11} = K_{10} \cup \mathscr{b}_{12} = \{\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \mathscr{b}_4, \mathscr{b}_5, \mathscr{b}_6, \mathscr{b}_7, \mathscr{b}_8, \mathscr{b}_9, \mathscr{b}_{10}, \mathscr{b}_{11}, \mathscr{b}_{12}\}$ is a $(12, \mathscr{b}) -$ span.

The line $\mathscr{b}_{13} = \{26, 327, 507, 674, 841, 1008, 1175, 1342, 1522, 1689, 1856, 2023, 2190, 2357\}$, this line can not intersect with any line of K_{11}, then $K_{12} = K_{11} \cup \mathscr{b}_{13} = \{\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \mathscr{b}_4, \mathscr{b}_5, \mathscr{b}_6, \mathscr{b}_7, \mathscr{b}_8, \mathscr{b}_9, \mathscr{b}_{10}, \mathscr{b}_{11}, \mathscr{b}_{12}, \mathscr{b}_{13}\}$ is a $(13, \mathscr{b}) -$ span.

The line $\mathscr{b}_{14} = \{27, 340, 521, 689, 857, 1025, 1193, 1361, 1529, 1697, 1865, 2033, 2201, 2369\}$, this line can not intersect with any line of K_{12}, then $K_{13} = K_{12} \cup \mathscr{b}_{14} = \{\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \mathscr{b}_4, \mathscr{b}_5, \mathscr{b}_6, \mathscr{b}_7, \mathscr{b}_8, \mathscr{b}_9, \mathscr{b}_{10}, \mathscr{b}_{11}, \mathscr{b}_{12}, \mathscr{b}_{13}, \mathscr{b}_{14}\}$ is a $(14, \mathscr{b}) -$ span.

The line $\mathscr{b}_{15} = \{28, 191, 373, 555, 737, 919, 1101, 1283, 1465, 1647, 1829, 2011, 2193, 2375\}$, this line can not intersect with any line of K_{13}, then $K_{14} = K_{13} \cup \mathscr{b}_{15} = \{\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \mathscr{b}_4, \mathscr{b}_5, \mathscr{b}_6, \mathscr{b}_7, \mathscr{b}_8, \mathscr{b}_9, \mathscr{b}_{10}, \mathscr{b}_{11}, \mathscr{b}_{12}, \mathscr{b}_{13}, \mathscr{b}_{14}, \mathscr{b}_{15}\}$ is a $(15, \mathscr{b}) -$ span.

The line $\mathscr{b}_{16} = \{29, 204, 387, 570, 753, 936, 1119, 1289, 1472, 1655, 1838, 2021, 2204, 2218\}$, this line can not intersect with any line of K_{14}, then

$K_{15} = K_{14} \cup \mathscr{b}_{16} = \{\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \dots, \mathscr{b}_{16}\}$ is a $(16, \mathscr{b}) -$ span.

The line $\mathscr{b}_{17} =$ {30, 217, 401, 585, 756, 940, 1124, 1308, 1492, 1676, 1860, 2031, 2046, 2230}, this line can not intersect with any line of K_{15}, then $\mathrm{K}_{16} = \mathrm{K}_{15} \cup \mathscr{b}_{17} = \{\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \ldots, \mathscr{b}_{17}\}$ is a $(17, \mathscr{b}) -$ span.

The line $\mathscr{b}_{18} =$ {31, 230, 415, 587, 772, 957, 1142, 1327, 1499, 1684, 1869, 1885, 2057, 2242}, this line can not intersect with any line of K_{16}, then

$\mathrm{K}_{17} = \mathrm{K}_{16} \cup \mathscr{b}_{18} = \{\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \ldots, \mathscr{b}_{18}\}$ is a $(18, \mathscr{b}) -$ span.

The line $\mathscr{b}_{19} =${32, 243, 429, 602, 788, 974, 1147, 1333, 1519, 1692, 1709, 1895, 2081, 2254}, this line can not intersect with any line of K_{17}, then

$\mathrm{K}_{18} = \mathrm{K}_{17} \cup \mathscr{b}_{19} = \{\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \ldots, \mathscr{b}_{19}\}$ is a $(19, \mathscr{b}) -$ span.

The line $\mathscr{b}_{20} =$ {33, 256, 443, 617, 804, 978, 1165, 1352, 1526, 1544, 1718, 1905, 2092, 2266}, this line can not intersect with any line of K_{18}, then

$\mathrm{K}_{19} = \mathrm{K}_{18} \cup \mathscr{b}_{20} = \{\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \ldots, \mathscr{b}_{20}\}$ is a $(20, \mathscr{b}) -$ span.

The line $\mathscr{b}_{21} =$ {34, 269, 444, 632, 820, 995, 1183, 1358, 1377, 1552, 1740, 1915, 2103, 2278}, this line can not intersect with any line of K_{19}, then

$\mathrm{K}_{20} = \mathrm{K}_{19} \cup \mathscr{b}_{21} = \{\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \ldots, \mathscr{b}_{21}\}$ is a $(21, \mathscr{b}) -$ span.

The line $\mathscr{b}_{22} =$ {35, 282, 458, 647, 823, 1012, 1188, 1208, 1384, 1573, 1749, 1938, 2114, 2290}, this line can not intersect with any line of K_{20}, then

$\mathrm{K}_{21} = \mathrm{K}_{20} \cup \mathscr{b}_{22} = \{\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \ldots, \mathscr{b}_{22}\}$ is a $(22, \mathscr{b}) -$ span.

The line $\mathscr{b}_{23} =$ {36, 295, 472, 662, 839, 1016, 1037, 1214, 1404, 1581, 1758, 1948, 2125, 2315}, this line can not intersect with any line of K_{21}, then

$\mathrm{K}_{22} = \mathrm{K}_{21} \cup \mathscr{b}_{23} = \{\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \ldots, \mathscr{b}_{23}\}$ is a $(23, \mathscr{b}) -$ span.

The line $\mathscr{b}_{24} =$ {37, 308, 486, 677, 855, 864, 1042, 1233, 1411, 1589, 1780, 1958, 2136, 2327}, this line can not intersect with any line of K_{22}, then

$\mathrm{K}_{23} = \mathrm{K}_{22} \cup \mathscr{b}_{24} = \{\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \ldots, \mathscr{b}_{24}\}$ is a $(24, \mathscr{b}) -$ span.

The line $\mathscr{b}_{25} =$ {38, 321, 500, 679, 702, 881, 1060, 1239, 1431, 1610, 1789, 1968, 2147, 2339}, this line can not intersect with any line of K_{23}, then

$\mathrm{K}_{24} = \mathrm{K}_{23} \cup \mathscr{b}_{25} = \{\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \ldots, \mathscr{b}_{25}\}$ is a $(25, \mathscr{b}) -$ span.

The line $\mathscr{b}_{26} =$ {39, 334, 514, 525, 705, 898, 1078, 1258, 1438, 1618, 1798, 1978, 2171, 2351}, this line can not intersect with any line of K_{24}, then

$\mathrm{K}_{25} = \mathrm{K}_{24} \cup \mathscr{b}_{26} = \{\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \ldots, \mathscr{b}_{26}\}$ is a $(26, \mathscr{b}) -$ span.

The line $\mathscr{b}_{27} =${40, 347, 359, 540, 721, 902, 1083, 1264, 1445, 1639, 1820, 2001, 2182, 2363}, this line can not intersect with any line of K_{25}, then

$\mathrm{K}_{26} = \mathrm{K}_{25} \cup \mathscr{b}_{27} = \{\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \ldots, \mathscr{b}_{27}\}$ is a $(27, \mathscr{b}) -$ span.

The line $\mathscr{b}_{28} =$ {41, 185, 380, 575, 770, 965, 1160, 1355, 1381, 1576, 1771, 1966, 2161, 2356}, this line can not intersect with any line of K_{26}, then

$\mathrm{K}_{27} = \mathrm{K}_{26} \cup \mathscr{b}_{28} = \{\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \ldots, \mathscr{b}_{28}\}$ is a $(28, \mathscr{b}) -$ span.

The line $\mathscr{b}_{29} =$ {42, 198, 394, 590, 786, 982, 1178, 1205, 1401, 1597, 1793, 1989, 2185, 2368}, this line can not intersect with any line of K_{27}, then

$\mathrm{K}_{28} = \mathrm{K}_{27} \cup \mathscr{b}_{29} = \{\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \ldots, \mathscr{b}_{29}\}$ is a $(29, \mathscr{b}) -$ span.

The line $\mathscr{b}_{30} =${43, 211,,408, 605, 802, 999, 1196, 1211, 1408, 1605, 1802, 1999, 2196, 2224}, this line can not intersect with any line of K_{28}, then

$\mathrm{K}_{29} = \mathrm{K}_{28} \cup \mathscr{b}_{30} = \{\mathscr{b}_1, \mathscr{b}_2, \mathscr{b}_3, \ldots, \mathscr{b}_{30}\}$ is a $(30, \mathscr{b}) -$ span.

The line $\mathscr{b}_{31} =$ {44, 224, 422, 620, 818, 1003, 1032, 1230, 1428, 1626, 1811, 2009, 2207, 2236}, this line can not intersect with any line of K_{29}, then

$\mathrm{K}_{30} = \mathrm{K}_{29} \cup \mathscr{b}_{31} = \{\mathscr{b}_1, \mathscr{b}_2, \ldots, \mathscr{b}_{31}\}$ is a $(31, \mathscr{b}) -$ span.

The line $\mathscr{b}_{32} =$ {45, 237, 436, 635, 821, 1020, 1050, 1249, 1435, 1634, 1833, 2019, 2049, 2248}, this line can not intersect with any line of K_{30}, then

$\mathrm{K}_{31} = \mathrm{K}_{30} \cup \mathscr{b}_{32} = \{\mathscr{b}_1, \mathscr{b}_2, \ldots, \mathscr{b}_{32}\}$ is a $(32, \mathscr{b}) -$ span.

The line $\mathscr{b}_{33} =$ {46, 250, 450, 650, 837, 868, 1055, 1255, 1455, 1642, 1842, 2042, 2060, 2260}, this line can not intersect with any line of K_{31}, then

$\mathrm{K}_{32} = \mathrm{K}_{31} \cup \mathscr{b}_{33} = \{\mathscr{b}_1, \mathscr{b}_2, \ldots, \mathscr{b}_{33}\}$ is a $(33, \mathscr{b}) -$ span.

The line $\mathscr{b}_{34} =$ {47, 263, 464, 652, 853, 885, 1073, 1274, 1462, 1663, 1851, 1883, 2071, 2272}, this line can not intersect with any line of K_{32}, then

$\mathrm{K}_{33} = \mathrm{K}_{32} \cup \mathscr{b}_{34} = \{\mathscr{b}_1, \mathscr{b}_2, \ldots, \mathscr{b}_{34}\}$ is a $(34, \mathscr{b}) -$ span.

The line $\mathscr{b}_{35} =$ {48, 276, 478, 667, 700, 889, 1091, 1280, 1482, 1671, 1873, 1893, 2082, 2284}, this line can not intersect with any line of K_{33}, then

$\mathrm{K}_{34} = \mathrm{K}_{33} \cup \mathscr{b}_{35} = \{\mathscr{b}_1, \mathscr{b}_2, \ldots, \mathscr{b}_{35}\}$ is a $(35, \mathscr{b}) -$ span.

The line $\mathscr{b}_{36} =$ {49, 289, 492, 682, 716, 906, 1096, 1299, 1489, 1679, 1713, 1903, 2106, 2296}, this line can not intersect with any line of K_{34}, then

$\mathrm{K}_{35} = \mathrm{K}_{34} \cup \mathscr{b}_{36} = \{\mathscr{b}_1, \mathscr{b}_2, \ldots, \mathscr{b}_{36}\}$ is a $(36, \mathscr{b}) -$ span.

The line $\mathscr{b}_{37} = \{50, 302, 506, 528, 719, 923, 1114, 1305, 1509, 1700, 1722, 1913, 2117, 2308\}$, this line can not intersect with any line of K_{35}, then

$K_{36} = K_{35} \cup \mathscr{b}_{37} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{37}\}$ is a $(37, \mathscr{b}) -$ span.

The line $\mathscr{b}_{38} = \{51, 315, 520, 543, 735, 927, 1132, 1324, 1516, 1539, 1731, 1936, 2128, 2320\}$, this line can not intersect with any line of K_{36}, then

$K_{37} = K_{36} \cup \mathscr{b}_{38} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{38}\}$ is a $(38, \mathscr{b}) -$ span.

The line $\mathscr{b}_{39} = \{52, 328, 365, 558, 751, 944, 1137, 1330, 1523, 1560, 1753, 1946, 2139, 2332\}$, this line can not intersect with any line of K_{37}, then

$K_{38} = K_{37} \cup \mathscr{b}_{39} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{39}\}$ is a $(39, \mathscr{b}) -$ span.

The line $\mathscr{b}_{40} = \{53, 341, 366, 573, 767, 961, 1155, 1349, 1374, 1568, 1762, 1956, 2150, 2344\}$, this line can not intersect with any line of K_{38}, then

$K_{39} = K_{38} \cup \mathscr{b}_{40} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{40}\}$ is a $(40, \mathscr{b}) -$ span.

The line $\mathscr{b}_{41} = \{54, 192, 400, 608, 816, 1024, 1063, 1271, 1479, 1687, 1726, 1934, 2142, 2350\}$, this line can not intersect with any line of K_{39}, then

$K_{40} = K_{39} \cup \mathscr{b}_{41} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{41}\}$ is a $(41, \mathscr{b}) -$ span.

The line $\mathscr{b}_{42} = \{55, 205, 414, 623, 832, 872, 1068, 1277, 1486, 1695, 1735, 1944, 2153, 2362\}$, this line can not intersect with any line of K_{40}, then

$K_{41} = K_{40} \cup \mathscr{b}_{42} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{42}\}$ is a $(42, \mathscr{b}) -$ span.

The line $\mathscr{b}_{43} = \{56, 218, 428, 638, 835, 876, 1086, 1296, 1506, 1547, 1744, 1954, 2164, 2374\}$, this line can not intersect with any line of K_{41}, then

$K_{42} = K_{41} \cup \mathscr{b}_{43} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{43}\}$ is a $(43, \mathscr{b}) -$ span.

The line $\mathscr{b}_{44} = \{57, 231, 442, 640, 851, 893, 1104, 1302, 1513, 1555, 1766, 1977, 2175, 2217\}$, this line can not intersect with any line of K_{42}, then

$K_{43} = K_{42} \cup \mathscr{b}_{44} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{44}\}$ is a $(44, \mathscr{b}) -$ span.

The line $\mathscr{b}_{45} = \{58, 244, 456, 655, 698, 910, 1109, 1321, 1533, 1563, 1775, 1987, 2186, 2229\}$, this line can not intersect with any line of K_{43}, then

$K_{44} = K_{43} \cup \mathscr{b}_{45} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{45}\}$ is a $(45, \mathscr{b}) -$ span.

The line $\mathscr{b}_{46} = \{59, 257, 457, 670, 714, 914, 1127, 1340, 1371, 1584, 1784, 1997, 2210, 2241\}$, this line can not intersect with any line of K_{44}, then

$K_{45} = K_{44} \cup \mathscr{b}_{46} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{46}\}$ is a $(46, \mathscr{b}) -$ span.

The line $\mathscr{b}_{47} = \{60, 270, 471, 685, 717, 931, 1145, 1346, 1391, 1592, 1806, 2007, 2052, 2253\}$, this line can not intersect with any line of K_{45}, then

$K_{46} = K_{45} \cup \mathscr{b}_{47} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{47}\}$ is a $(47, \mathscr{b}) -$ span.

The line $\mathscr{b}_{48} = \{61, 283, 485, 531, 733, 948, 1150, 1365, 1398, 1613, 1815, 2017, 2063, 2265\}$, this line can not intersect with any line of K_{46}, then

$K_{47} = K_{46} \cup \mathscr{b}_{48} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{48}\}$ is a $(48, \mathscr{b}) -$ span.

The line $\mathscr{b}_{49} = \{62, 296, 499, 546, 749, 952, 1168, 1202, 1418, 1621, 1824, 2040, 2074, 2277\}$, this line can not intersect with any line of K_{47}, then

$K_{48} = K_{47} \cup \mathscr{b}_{49} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{49}\}$ is a $(49, \mathscr{b}) -$ span.

The line $\mathscr{b}_{50} = \{63, 309, 513, 548, 765, 969, 1173, 1221, 1425, 1629, 1846, 1881, 2085, 2302\}$, this line can not intersect with any line of K_{48}, then

$K_{49} = K_{48} \cup \mathscr{b}_{50} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{50}\}$ is a $(50, \mathscr{b}) -$ span.

The line $\mathscr{b}_{51} = \{64, 322, 358, 563, 781, 986, 1191, 1227, 1432, 1650, 1855, 1891, 2096, 2314\}$, this line can not intersect with any line of K_{49}, then

$K_{50} = K_{49} \cup \mathscr{b}_{51} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{51}\}$ is a $(51, \mathscr{b}) -$ span.

The line $\mathscr{b}_{52} = \{65, 335, 372, 578, 784, 990, 1040, 1246, 1452, 1658, 1864, 1901, 2120, 2326\}$, this line can not intersect with any line of K_{50}, then

$K_{51} = K_{50} \cup \mathscr{b}_{52} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{52}\}$ is a $(52, \mathscr{b}) -$ span.

The line $\mathscr{b}_{53} = \{66, 348, 386, 593, 800, 1007, 1045, 1252, 1459, 1666, 1717, 1924, 2131, 2338\}$, this line can not intersect with any line of K_{51}, then

$K_{52} = K_{51} \cup \mathscr{b}_{53} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{53}\}$ is a $(53, \mathscr{b}) -$ span.

The line $\mathscr{b}_{54} = \{67, 186, 407, 628, 849, 901, 1122, 1343, 1395, 1616, 1837, 1889, 2110, 2331\}$, this line can not intersect with any line of K_{52}, then

$K_{53} = K_{52} \cup \mathscr{b}_{54} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{54}\}$ is a $(54, \mathscr{b}) -$ span.

The line $\mathscr{b}_{55} = \{68, 199, 421, 643, 696, 918, 1140, 1362, 1415, 1637, 1859, 1912, 2121, 2343\}$, this line can not intersect with any line of K_{53}, then

$K_{54} = K_{53} \cup \mathscr{b}_{55} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{55}\}$ is a $(55, \mathscr{b}) -$ span.

The line $\mathscr{b}_{56} = \{69, 212, 435, 658, 712, 935, 1158, 1199, 1422, 1645, 1868, 1922, 2145, 2355\}$, this line can not intersect with any line of K_{54}, then

$K_{55} = K_{54} \cup \mathscr{b}_{56} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{56}\}$ is a $(56, \mathscr{b}) -$ span.

The line $\mathscr{b}_{57} = \{70, 225, 449, 673, 728, 939, 1163, 1218, 1442, 1653, 1708, 1932, 2156, 2380\}$, this line can not intersect with any line of K_{55}, then

$K_{56} = K_{55} \cup \mathscr{b}_{57} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{57}\}$ is a $(57, \mathscr{b}) -$ span.

The line $\mathscr{b}_{58} = \{71, 238, 463, 688, 731, 956, 1181, 1224, 1449, 1674, 1730, 1942, 2167, 2223\}$, this line can not intersect with any line of K_{56}, then

$K_{57} = K_{56} \cup \mathscr{b}_{58} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{58}\}$ is a $(58, \mathscr{b}) -$ span.

The line $\mathscr{b}_{59} = \{72, 251, 477, 534, 747, 973, 1186, 1243, 1469, 1682, 1739, 1952, 2178, 2235\}$, this line can not intersect with any line of K_{57}, then

$K_{58} = K_{57} \cup \mathscr{b}_{59} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{59}\}$ is a $(59, \mathscr{b}) -$ span.

The line $\mathscr{b}_{60} = \{73, 264, 491, 536, 763, 977, 1035, 1262, 1476, 1703, 1748, 1975, 2189, 2247\}$, this line can not intersect with any line of K_{58}, then

$K_{59} = K_{58} \cup \mathscr{b}_{60} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{60}\}$ is a $(60, \mathscr{b}) -$ span.

The line $\mathcal{b}_{61} = \{74, 277, 505, 551, 779, 994, 1053, 1268, 1496, 1542, 1757, 1985, 2200, 2259\}$, this line can not intersect with any line of K_{59}, then

$K_{60} = K_{59} \cup \mathcal{b}_{61} = \{\mathcal{b}_1, \mathcal{b}_2, \dots, \mathcal{b}_{61}\}$ is a $(61, \mathcal{b}) -$ span.

The line $\mathcal{b}_{62} = \{75, 290, 519, 566, 782, 1011, 1058, 1287, 1503, 1550, 1779, 1995, 2055, 2271\}$, this line can not intersect with any line of K_{60}, then

$K_{61} = K_{60} \cup \mathcal{b}_{62} = \{\mathcal{b}_1, \mathcal{b}_2, \dots, \mathcal{b}_{62}\}$ is a $(62, \mathcal{b}) -$ span.

The line $\mathcal{b}_{63} = \{76, 303, 364, 581, 798, 1028, 1076, 1293, 1510, 1571, 1788, 2005, 2066, 2283\}$, this line can not intersect with any line of K_{61}, then

$K_{62} = K_{61} \cup \mathcal{b}_{63} = \{\mathcal{b}_1, \mathcal{b}_2, \dots, \mathcal{b}_{63}\}$ is a $(63, \mathcal{b}) -$ span.

The line $\mathcal{b}_{64} = \{77, 316, 378, 596, 814, 863, 1081, 1312, 1530, 1579, 1797, 2028, 2077, 2295\}$, this line can not intersect with any line of K_{62}, then

$K_{63} = K_{62} \cup \mathcal{b}_{64} = \{\mathcal{b}_1, \mathcal{b}_2, \dots, \mathcal{b}_{64}\}$ is a $(64, \mathcal{b}) -$ span.

The line $\mathcal{b}_{65} = \{78, 329, 379, 611, 830, 880, 1099, 1318, 1368, 1600, 1819, 2038, 2088, 2307\}$, this line can not intersect with any line of K_{63}, then

$K_{64} = K_{63} \cup \mathcal{b}_{65} = \{\mathcal{b}_1, \mathcal{b}_2, \dots, \mathcal{b}_{65}\}$ is a $(65, \mathcal{b}) -$ span.

The line $\mathcal{b}_{66} = \{79, 342, 393, 613, 846, 897, 1117, 1337, 1388, 1608, 1828, 1879, 2099, 2319\}$, this line can not intersect with any line of K_{64}, then

$K_{65} = K_{64} \cup \mathcal{b}_{66} = \{\mathcal{b}_1, \mathcal{b}_2, \dots, \mathcal{b}_{66}\}$ is a $(66, \mathcal{b}) -$ span.

The line $\mathcal{b}_{67} = \{80, 193, 427, 661, 726, 960, 1194, 1259, 1493, 1558, 1792, 2026, 2091, 2325\}$, this line can not intersect with any line of K_{65}, then

$K_{66} = K_{65} \cup \mathcal{b}_{67} = \{\mathcal{b}_1, \mathcal{b}_2, \dots, \mathcal{b}_{67}\}$ is a $(67, \mathcal{b}) -$ span.

The line $\mathcal{b}_{68} = \{81, 206, 441, 676, 742, 964, 1030, 1265, 1500, 1566, 1801, 2036, 2102, 2337\}$, this line can not intersect with any line of K_{66}, then

$K_{67} = K_{66} \cup \mathcal{b}_{68} = \{\mathcal{b}_1, \mathcal{b}_2, \dots, \mathcal{b}_{68}\}$ is a $(68, \mathcal{b}) -$ span.

The line $\mathcal{b}_{69} = \{82, 219, 455, 678, 745, 981, 1048, 1284, 1520, 1587, 1810, 1877, 2113, 2349\}$, this line can not intersect with any line of K_{67}, then

$K_{68} = K_{67} \cup \mathcal{b}_{69} = \{\mathcal{b}_1, \mathcal{b}_2, \dots, \mathcal{b}_{69}\}$ is a $(69, \mathcal{b}) -$ span.

The line $\mathcal{b}_{70} = \{83, 232, 469, 524, 761, 998, 1066, 1290, 1527, 1595, 1832, 1887, 2124, 2361\}$, this line can not intersect with any line of K_{68}, then

$K_{69} = K_{68} \cup \mathcal{b}_{70} = \{\mathcal{b}_1, \mathcal{b}_2, \dots, \mathcal{b}_{70}\}$ is a $(70, \mathcal{b}) -$ span.

The line $\mathcal{b}_{71} = \{84, 245, 470, 539, 777, 1015, 1071, 1309, 1378, 1603, 1841, 1910, 2135, 2373\}$, this line can not intersect with any line of K_{69}, then

$K_{70} = K_{69} \cup \mathcal{b}_{71} = \{\mathcal{b}_1, \mathcal{b}_2, \dots, \mathcal{b}_{71}\}$ is a $(71, \mathcal{b}) -$ span.

The line $\mathcal{b}_{72} = \{85, 258, 484, 554, 793, 1019, 1089, 1315, 1385, 1624, 1850, 1920, 2159, 2216\}$, this line can not intersect with any line of K_{70}, then

$K_{71} = K_{70} \cup \mathcal{b}_{72} = \{\mathcal{b}_1, \mathcal{b}_2, \dots, \mathcal{b}_{72}\}$ is a $(72, \mathcal{b}) -$ span.

The line $\mathcal{b}_{73} = \{86, 271, 498, 569, 796, 867, 1094, 1334, 1405, 1632, 1872, 1930, 2170, 2228\}$, this line can not intersect with any line of K_{71}, then

$K_{72} = K_{71} \cup \mathcal{b}_{73} = \{\mathcal{b}_1, \mathcal{b}_2, \dots, \mathcal{b}_{73}\}$ is a $(73, \mathcal{b}) -$ span.

The line $\mathcal{b}_{74} = \{87, 284, 512, 584, 812, 884, 1112, 1353, 1412, 1640, 1712, 1940, 2181, 2240\}$, this line can not intersect with any line of K_{72}, then

$K_{73} = K_{72} \cup \mathcal{b}_{74} = \{\mathcal{b}_1, \mathcal{b}_2, \dots, \mathcal{b}_{74}\}$ is a $(74, \mathcal{b}) -$ span.

The line $\mathcal{b}_{75} = \{88, 297, 357, 599, 828, 888, 1130, 1359, 1419, 1661, 1721, 1963, 2192, 2252\}$, this line can not intersect with any line of K_{73}, then

$K_{74} = K_{73} \cup \mathcal{b}_{75} = \{\mathcal{b}_1, \mathcal{b}_2, \dots, \mathcal{b}_{75}\}$ is a $(75, \mathcal{b}) -$ span.

The line $\mathcal{b}_{76} = \{89, 310, 371, 601, 844, 905, 1135, 1209, 1439, 1669, 1743, 1973, 2203, 2264\}$, this line can not intersect with any line of K_{74}, then

$K_{75} = K_{74} \cup \mathcal{b}_{76} = \{\mathcal{b}_1, \mathcal{b}_2, \dots, \mathcal{b}_{76}\}$ is a $(76, \mathcal{b}) -$ span.

The line $\mathcal{b}_{77} = \{90, 323, 385, 616, 847, 922, 1153, 1215, 1446, 1690, 1752, 1983, 2045, 2289\}$, this line can not intersect with any line of K_{75}, then

$K_{76} = K_{75} \cup \mathcal{b}_{77} = \{\mathcal{b}_1, \mathcal{b}_2, \dots, \mathcal{b}_{77}\}$ is a $(77, \mathcal{b}) -$ span.

The line $\mathcal{b}_{78} = \{91, 336, 399, 631, 694, 926, 1171, 1234, 1466, 1698, 1761, 1993, 2056, 2301\}$, this line can not intersect with any line of K_{76}, then

$K_{77} = K_{76} \cup \mathcal{b}_{78} = \{\mathcal{b}_1, \mathcal{b}_2, \dots, \mathcal{b}_{78}\}$ is a $(78, \mathcal{b}) -$ span.

The line $\mathcal{b}_{79} = \{92, 349, 413, 646, 710, 943, 1176, 1240, 1473, 1537, 1770, 2016, 2080, 2313\}$, this line can not intersect with any line of K_{77}, then

$K_{78} = K_{77} \cup \mathcal{b}_{79} = \{\mathcal{b}_1, \mathcal{b}_2, \dots, \mathcal{b}_{79}\}$ is a $(79, \mathcal{b}) -$ span.

The line $\mathcal{b}_{80} = \{93, 187, 434, 681, 759, 1006, 1084, 1331, 1409, 1656, 1734, 1981, 2059, 2306\}$, this line can not intersect with any line of K_{78}, then

$K_{79} = K_{78} \cup \mathcal{b}_{80} = \{\mathcal{b}_1, \mathcal{b}_2, \dots, \mathcal{b}_{80}\}$ is a $(80, \mathcal{b}) -$ span.

The line $\mathcal{b}_{81} = \{94, 200, 448, 527, 775, 1023, 1102, 1350, 1429, 1677, 1756, 1991, 2070, 2318\}$, this line can not intersect with any line of K_{79}, then

$K_{80} = K_{79} \cup \mathcal{b}_{81} = \{\mathcal{b}_1, \mathcal{b}_2, \dots, \mathcal{b}_{81}\}$ is a $(81, \mathcal{b}) -$ span.

The line $\mathcal{b}_{82} = \{95, 213, 462, 542, 791, 871, 1107, 1356, 1436, 1685, 1765, 2014, 2094, 2330\}$, this line can not intersect with any line of K_{80}, then

$K_{81} = K_{80} \cup \mathcal{b}_{82} = \{\mathcal{b}_1, \mathcal{b}_2, \dots, \mathcal{b}_{82}\}$ is a $(82, \mathcal{b}) -$ span.

The line $\mathcal{b}_{83} = \{96, 226, 476, 557, 807, 875, 1125, 1206, 1456, 1693, 1774, 2024, 2105, 2342\}$, this line can not intersect with any line of K_{81}, then

$K_{82} = K_{81} \cup \mathcal{b}_{83} = \{\mathcal{b}_1, \mathcal{b}_2, \dots, \mathcal{b}_{83}\}$ is a $(83, \mathcal{b}) -$ span.

The line $\mathcal{b}_{84} = \{97, 239, 490, 572, 810, 892, 1143, 1212, 1463, 1545, 1783, 2034, 2116, 2367\}$, this line can not intersect with any line of K_{82}, then

$K_{83} = K_{82} \cup \mathcal{b}_{84} = \{\mathcal{b}_1, \mathcal{b}_2, \dots, \mathcal{b}_{84}\}$ is a $(84, \mathcal{b}) -$ span.

The line $\mathscr{b}_{85} = \{98,\ 252,\ 504,\ 574,\ 826,\ 909,\ 1148,\ 1231,\ 1483,\ 1553,\ 1805,\ 1875,\ 2127,\ 2379\}$, this line can not intersect with any line of K_{83}, then

$K_{84} = K_{83} \cup \mathscr{b}_{85} = \{\mathscr{b}_1,\ \mathscr{b}_2, \dots,\ \mathscr{b}_{85}\}$ is a $(85, \mathscr{b}) -$ span.

The line $\mathscr{b}_{86} = \{99,\ 265,\ 518,\ 589,\ 842,\ 913,\ 1166,\ 1237,\ 1490,\ 1574,\ 1814,\ 1898,\ 2138,\ 2222\}$, this line can not intersect with any line of K_{84}, then

$K_{85} = K_{84} \cup \mathscr{b}_{86} = \{\mathscr{b}_1,\ \mathscr{b}_2, \dots,\ \mathscr{b}_{86}\}$ is a $(86, \mathscr{b}) -$ span.

The line $\mathscr{b}_{87} = \{100,\ 278,\ 363,\ 604,\ 858,\ 930,\ 1184,\ 1256,\ 1497,\ 1582,\ 1823,\ 1908,\ 2149,\ 2234\}$, this line can not intersect with any line of K_{85}, then

$K_{86} = K_{85} \cup \mathscr{b}_{87} = \{\mathscr{b}_1,\ \mathscr{b}_2, \dots,\ \mathscr{b}_{87}\}$ is a $(87, \mathscr{b}) -$ span.

The line $\mathscr{b}_{88} = \{101,\ 291,\ 377,\ 619,\ 692,\ 947,\ 1189,\ 1275,\ 1517,\ 1590,\ 1845,\ 1918,\ 2160,\ 2246\}$, this line can not intersect with any line of K_{86}, then

$K_{87} = K_{86} \cup \mathscr{b}_{88} = \{\mathscr{b}_1,\ \mathscr{b}_2, \dots,\ \mathscr{b}_{88}\}$ is a $(88, \mathscr{b}) -$ span.

The line $\mathscr{b}_{89} = \{102,\ 304,\ 391,\ 634,\ 708,\ 951,\ 1038,\ 1281,\ 1524,\ 1611,\ 1854,\ 1928,\ 2184,\ 2258\}$, this line can not intersect with any line of K_{87}, then

$K_{88} = K_{87} \cup \mathscr{b}_{89} = \{\mathscr{b}_1,\ \mathscr{b}_2, \dots,\ \mathscr{b}_{89}\}$ is a $(89, \mathscr{b}) -$ span.

The line $\mathscr{b}_{90} = \{103,\ 317,\ 392,\ 649,\ 724,\ 968,\ 1043,\ 1300,\ 1375,\ 1619,\ 1863,\ 1951,\ 2195,\ 2270\}$, this line can not intersect with any line of K_{88}, then

$K_{89} = K_{88} \cup \mathscr{b}_{90} = \{\mathscr{b}_1,\ \mathscr{b}_2, \dots,\ \mathscr{b}_{90}\}$ is a $(90, \mathscr{b}) -$ span.

The line $\mathscr{b}_{91} = \{104,\ 330,\ 406,\ 664,\ 740,\ 985,\ 1061,\ 1306,\ 1382,\ 1627,\ 1716,\ 1961,\ 2206,\ 2282\}$, this line can not intersect with any line of K_{89}, then

$K_{90} = K_{89} \cup \mathscr{b}_{91} = \{\mathscr{b}_1,\ \mathscr{b}_2, \dots,\ \mathscr{b}_{91}\}$ is a $(91, \mathscr{b}) -$ span.

The line $\mathscr{b}_{92} = \{105,\ 343,\ 420,\ 666,\ 743,\ 1002,\ 1079,\ 1325,\ 1402,\ 1648,\ 1725,\ 1971,\ 2048,\ 2294\}$, this line can not intersect with any line of K_{90}, then

$K_{91} = K_{90} \cup \mathscr{b}_{92} = \{\mathscr{b}_1,\ \mathscr{b}_2, \dots,\ \mathscr{b}_{92}\}$ is a $(92, \mathscr{b}) -$ span.

The line $\mathscr{b}_{93} = \{106,\ 194,\ 454,\ 545,\ 805,\ 896,\ 1156,\ 1247,\ 1507,\ 1598,\ 1858,\ 1949,\ 2209,\ 2300\}$, this line can not intersect with any line of K_{91}, then

$K_{92} = K_{91} \cup \mathscr{b}_{93} = \{\mathscr{b}_1,\ \mathscr{b}_2, \dots,\ \mathscr{b}_{93}\}$ is a $(93, \mathscr{b}) -$ span.

The line $\mathscr{b}_{94} = \{107,\ 207,\ 468,\ 560,\ 808,\ 900,\ 1161,\ 1253,\ 1514,\ 1606,\ 1867,\ 1959,\ 2051,\ 2312\}$, this line can not intersect with any line of K_{92}, then

$K_{93} = K_{92} \cup \mathscr{b}_{94} = \{\mathscr{b}_1,\ \mathscr{b}_2, \dots,\ \mathscr{b}_{94}\}$ is a $(94, \mathscr{b}) -$ span.

The line $\mathscr{b}_{95} = \{108,\ 220,\ 482,\ 562,\ 824,\ 917,\ 1179,\ 1272,\ 1534,\ 1614,\ 1707,\ 1969,\ 2062,\ 2324\}$, this line can not intersect with any line of K_{93}, then

$K_{94} = K_{93} \cup \mathscr{b}_{95} = \{\mathscr{b}_1,\ \mathscr{b}_2, \dots,\ \mathscr{b}_{95}\}$ is a $(95, \mathscr{b}) -$ span.

The line $\mathscr{b}_{96} = \{109,\ 233,\ 483,\ 577,\ 840,\ 934,\ 1197,\ 1278,\ 1372,\ 1635,\ 1729,\ 1979,\ 2073,\ 2336\}$, this line can not intersect with any line of K_{94}, then

$K_{95} = K_{94} \cup \mathscr{b}_{96} = \{\mathscr{b}_1,\ \mathscr{b}_2, \dots,\ \mathscr{b}_{96}\}$ is a $(96, \mathscr{b}) -$ span.

The line $\mathscr{b}_{97} = \{110,\ 246,\ 497,\ 592,\ 856,\ 938,\ 1033,\ 1297,\ 1392,\ 1643,\ 1738,\ 2002,\ 2084,\ 2348\}$, this line can not intersect with any line of K_{95}, then

$K_{96} = K_{95} \cup \mathscr{b}_{97} = \{\mathscr{b}_1,\ \mathscr{b}_2, \dots,\ \mathscr{b}_{97}\}$ is a $(97, \mathscr{b}) -$ span.

The line $\mathscr{b}_{98} = \{111,\ 259,\ 511,\ 607,\ 703,\ 955,\ 1051,\ 1303,\ 1399,\ 1664,\ 1747,\ 2012,\ 2095,\ 2360\}$, this line can not intersect with any line of K_{96}, then

$K_{97} = K_{96} \cup \mathscr{b}_{98} = \{\mathscr{b}_1,\ \mathscr{b}_2, \dots,\ \mathscr{b}_{98}\}$ is a $(98, \mathscr{b}) -$ span.

The line $\mathscr{b}_{99} = \{112,\ 272,\ 356,\ 622,\ 706,\ 972,\ 1056,\ 1322,\ 1406,\ 1672,\ 1769,\ 2022,\ 2119,\ 2372\}$, this line can not intersect with any line of K_{97}, then

$K_{98} = K_{97} \cup \mathscr{b}_{99} = \{\mathscr{b}_1,\ \mathscr{b}_2, \dots,\ \mathscr{b}_{99}\}$ is a $(99, \mathscr{b}) -$ span.

The line $\mathscr{b}_{100} = \{113,\ 285,\ 370,\ 637,\ 722,\ 989,\ 1074,\ 1328,\ 1426,\ 1680,\ 1778,\ 2032,\ 2130,\ 2215\}$, this line can not intersect with any line of K_{98}, then

$K_{99} = K_{98} \cup \mathscr{b}_{100} = \{\mathscr{b}_1,\ \mathscr{b}_2, \dots, \mathscr{b}_{100}\}$ is a $(100, \mathscr{b}) -$ span.

The line $\mathscr{b}_{101} = \{114,\ 298,\ 384,\ 639,\ 738,\ 993,\ 1092,\ 1347,\ 1433,\ 1701,\ 1787,\ 1886,\ 2141,\ 2227\}$, this line can not intersect with any line of K_{99}, then

$K_{100} = K_{99} \cup \mathscr{b}_{101} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{101}\}$ is a $(101, \mathscr{b}) -$ span.

The line $\mathscr{b}_{102} = \{115,\ 311,\ 398,\ 654,\ 754,\ 1010,\ 1097,\ 1366,\ 1453,\ 1540,\ 1796,\ 1896,\ 2152,\ 2239\}$, this line can not intersect with any line of K_{100}, then $K_{101} = K_{100} \cup \mathscr{b}_{102} = \{\mathscr{b}_1,\ \mathscr{b}_2, \dots,\ \mathscr{b}_{102}\}$ is a $(102, \mathscr{b}) -$ span.

The line $\mathscr{b}_{103} = \{116,\ 324,\ 412,\ 669,\ 757,\ 1027,\ 1115,\ 1203,\ 1460,\ 1561,\ 1818,\ 1906,\ 2163,\ 2251\}$, this line can not intersect with any line of K_{101}, then $K_{102} = K_{101} \cup \mathscr{b}_{103} = \{\mathscr{b}_1,\ \mathscr{b}_2, \dots,\ \mathscr{b}_{103}\}$ is a $(103, \mathscr{b}) -$ span.

The line $\mathscr{b}_{104} = \{117,\ 337,\ 426,\ 684,\ 773,\ 862,\ 1120,\ 1222,\ 1480,\ 1569,\ 1827,\ 1916,\ 2174,\ 2276\}$, this line can not intersect with any line of K_{102}, then $K_{103} = K_{102} \cup \mathscr{b}_{104} = \{\mathscr{b}_1,\ \mathscr{b}_2, \dots,\ \mathscr{b}_{104}\}$ is a $(104, \mathscr{b}) -$ span.

The line $\mathscr{b}_{105} = \{118,\ 350,\ 440,\ 530,\ 789,\ 879,\ 1138,\ 1228,\ 1487,\ 1577,\ 1836,\ 1926,\ 2198,\ 2288\}$, this line can not intersect with any line of K_{103}, then $K_{104} = K_{103} \cup \mathscr{b}_{105} = \{\mathscr{b}_1,\ \mathscr{b}_2, \dots,\ \mathscr{b}_{105}\}$ is a $(105, \mathscr{b}) -$ span.

The line $\mathscr{b}_{106} = \{119,\ 188,\ 461,\ 565,\ 838,\ 942,\ 1046,\ 1319,\ 1423,\ 1696,\ 1800,\ 1904,\ 2177,\ 2281\}$, this line can not intersect with any line of K_{104}, then $K_{105} = K_{104} \cup \mathscr{b}_{106} = \{\mathscr{b}_1,\ \mathscr{b}_2, \dots,\ \mathscr{b}_{106}\}$ is a $(106, \mathscr{b}) -$ span.

The line $\mathscr{b}_{107} = \{120,\ 201,\ 475,\ 580,\ 854,\ 959,\ 1064,\ 1338,\ 1443,\ 1548,\ 1809,\ 1914,\ 2188,\ 2293\}$, this line can not intersect with any line of K_{105}, then $K_{106} = K_{105} \cup \mathscr{b}_{107} = \{\mathscr{b}_1,\ \mathscr{b}_2, \dots,\ \mathscr{b}_{107}\}$ is a $(107, \mathscr{b}) -$ span.

The line $\mathscr{b}_{108} = \{121,\ 214,\ 489,\ 595,\ 701,\ 976,\ 1069,\ 1344,\ 1450,\ 1556,\ 1831,\ 1937,\ 2199,\ 2305\}$, this line can not intersect with any line of K_{106}, then $K_{107} = K_{106} \cup \mathscr{b}_{108} = \{\mathscr{b}_1,\ \mathscr{b}_2, \dots,\ \mathscr{b}_{108}\}$ is a $(108, \mathscr{b}) -$ span.

The line ℓ_{109} = {122, 227, 503, 610, 704, 980, 1087, 1363, 1470, 1564, 1840, 1947, 2054, 2317}, this line can not intersect with any line of K_{107}, then $K_{108} = K_{107} \cup \ell_{109} = \{\ell_1, \ell_2, \dots, \ell_{109}\}$ is a $(109, \ell)$ – span.

The line ℓ_{110} = {123, 240, 517, 625, 720, 997, 1105, 1200, 1477, 1585, 1849, 1957, 2065, 2329}, this line can not intersect with any line of K_{108}, then $K_{109} = K_{108} \cup \ell_{110} = \{\ell_1, \ell_2, \dots, \ell_{110}\}$ is a $(110, \ell)$ – span.

The line ℓ_{111} = {124, 253, 362, 627, 736, 1014, 1110, 1219, 1484, 1593, 1871, 1967, 2076, 2354}, this line can not intersect with any line of K_{109}, then $K_{110} = K_{109} \cup \ell_{111} = \{\ell_1, \ell_2, \dots, \ell_{111}\}$ is a $(111, \ell)$ – span.

The line ℓ_{112} = {125, 266, 376, 642, 752, 1018, 1128, 1225, 1504, 1601, 1711, 1990, 2087, 2366}, this line can not intersect with any line of K_{110}, then $K_{111} = K_{110} \cup \ell_{112} = \{\ell_1, \ell_2, \dots, \ell_{112}\}$ is a $(112, \ell)$ – span.

The line ℓ_{113} = {126, 279, 390, 657, 768, 866, 1133, 1244, 1511, 1622, 1720, 2000, 2098, 2378}, this line can not intersect with any line of K_{111}, then $K_{112} = K_{111} \cup \ell_{113} = \{\ell_1, \ell_2, \dots, \ell_{113}\}$ is a $(113, \ell)$ – span.

The line ℓ_{114} = {127, 292, 404, 672, 771, 883, 1151, 1250, 1531, 1630, 1742, 2010, 2109, 2221}, this line can not intersect with any line of K_{112}, then $K_{113} = K_{112} \cup \ell_{114} = \{\ell_1, \ell_2, \dots, \ell_{114}\}$ is a $(114, \ell)$ – span.

The line ℓ_{115} = {128, 305, 405, 687, 787, 887, 1169, 1269, 1369, 1651, 1751, 2020, 2133, 2233}, this line can not intersect with any line of K_{113}, then $K_{114} = K_{113} \cup \ell_{115} = \{\ell_1, \ell_2, \dots, \ell_{115}\}$ is a $(115, \ell)$ – span.

The line ℓ_{116} = {129, 318, 419, 533, 803, 904, 1174, 1288, 1389, 1659, 1760, 2030, 2144, 2245}, this line can not intersect with any line of K_{114}, then $K_{115} = K_{114} \cup \ell_{116} = \{\ell_1, \ell_2, \dots, \ell_{116}\}$ is a $(116, \ell)$ – span.

The line ℓ_{117} = {130, 331, 433, 535, 819, 921, 1192, 1294, 1396, 1667, 1782, 1884, 2155, 2257}, this line can not intersect with any line of K_{115}, then $K_{116} = K_{115} \cup \ell_{117} = \{\ell_1, \ell_2, \dots, \ell_{117}\}$ is a $(117, \ell)$ – span.

The line ℓ_{118} = {131, 344, 447, 550, 822, 925, 1041, 1313, 1416, 1688, 1791, 1894, 2166, 2269}, this line can not intersect with any line of K_{116}, then $K_{117} = K_{116} \cup \ell_{118} = \{\ell_1, \ell_2, \dots, \ell_{118}\}$ is a $(118, \ell)$ – span.

The line ℓ_{119} = {132, 195, 481, 598, 715, 1001, 1118, 1235, 1521, 1638, 1755, 2041, 2158, 2275}, this line can not intersect with any line of K_{117}, then $K_{118} = K_{117} \cup \ell_{119} = \{\ell_1, \ell_2, \dots, \ell_{119}\}$ is a $(119, \ell)$ – span.

The line ℓ_{120} = {133, 208, 495, 600, 718, 1005, 1123, 1241, 1528, 1646, 1764, 1882, 2169, 2287}, this line can not intersect with any line of K_{118}, then $K_{119} = K_{118} \cup \ell_{120} = \{\ell_1, \ell_2, \dots, \ell_{120}\}$ is a $(120, \ell)$ – span.

The line ℓ_{121} = {134, 221, 496, 615, 734, 1022, 1141, 1260, 1379, 1654, 1773, 1892, 2180, 2299}, this line can not intersect with any line of K_{119}, then $K_{120} = K_{119} \cup \ell_{121} = \{\ell_1, \ell_2, \dots, \ell_{121}\}$ is a $(121, \ell)$ – span.

The line ℓ_{122} = {135, 234, 510, 630, 750, 870, 1146, 1266, 1386, 1675, 1795, 1902, 2191, 2311}, this line can not intersect with any line of K_{120}, then $K_{121} = K_{120} \cup \ell_{122} = \{\ell_1, \ell_2, \dots, \ell_{122}\}$ is a $(122, \ell)$ – span.

The line ℓ_{123} = {136, 247, 355, 645, 766, 874, 1164, 1285, 1393, 1683, 1804, 1925, 2202, 2323}, this line can not intersect with any line of K_{121}, then $K_{122} = K_{121} \cup \ell_{123} = \{\ell_1, \ell_2, \dots, \ell_{123}\}$ is a $(123, \ell)$ – span.

The line ℓ_{124} = {137, 260, 369, 660, 769, 891, 1182, 1291, 1413, 1704, 1813, 1935, 2044, 2335}, this line can not intersect with any line of K_{122}, then $K_{123} = K_{122} \cup \ell_{124} = \{\ell_1, \ell_2, \dots, \ell_{124}\}$ is a $(124, \ell)$ – span.

The line ℓ_{125} = {138, 273, 383, 675, 785, 908, 1187, 1310, 1420, 1543, 1822, 1945, 2068, 2347}, this line can not intersect with any line of K_{123}, then $K_{124} = K_{123} \cup \ell_{125} = \{\ell_1, \ell_2, \dots, \ell_{125}\}$ is a $(125, \ell)$ – span.

The line ℓ_{126} = {139, 286, 397, 690, 801, 912, 1036, 1316, 1440, 1551, 1844, 1955, 2079, 2359}, this line can not intersect with any line of K_{124}, then $K_{125} = K_{124} \cup \ell_{126} = \{\ell_1, \ell_2, \dots, \ell_{126}\}$ is a $(126, \ell)$ – span.

The line ℓ_{127} = {140, 299, 411, 523, 817, 929, 1054, 1335, 1447, 1572, 1853, 1965, 2090, 2371}, this line can not intersect with any line of K_{125}, then $K_{126} = K_{125} \cup \ell_{127} = \{\ell_1, \ell_2, \dots, \ell_{127}\}$ is a $(127, \ell)$ – span.

The line ℓ_{128} = {141, 312, 425, 538, 833, 946, 1059, 1341, 1467, 1580, 1862, 1988, 2101, 2214}, this line can not intersect with any line of K_{126}, then $K_{127} = K_{126} \cup \ell_{128} = \{\ell_1, \ell_2, \dots, \ell_{128}\}$ is a $(128, \ell)$ – span.

The line ℓ_{129} = {142, 325, 439, 553, 836, 963, 1077, 1360, 1474, 1588, 1715, 1998, 2112, 2226}, this line can not intersect with any line of K_{127}, then $K_{128} = K_{127} \cup \ell_{129} = \{\ell_1, \ell_2, \dots, \ell_{129}\}$ is a $(129, \ell)$ – span.

The line ℓ_{130} = {143, 338, 453, 568, 852, 967, 1082, 1210, 1494, 1609, 1724, 2008, 2123, 2238}, this line can not intersect with any line of K_{128}, then $K_{129} = K_{128} \cup \ell_{130} = \{\ell_1, \ell_2, \dots, \ell_{130}\}$ is a $(130, \ell)$ – span.

The line ℓ_{131} = {144, 351, 467, 583, 699, 984, 1100, 1216, 1501, 1617, 1733, 2018, 2134, 2263}, this line can not intersect with any line of K_{129}, then $K_{130} = K_{129} \cup \ell_{131} = \{\ell_1, \ell_2, \dots, \ell_{131}\}$ is a $(131, \ell)$ – span.

The line ℓ_{132} = {145, 189, 488, 618, 748, 878, 1177, 1307, 1437, 1567, 1866, 1996, 2126, 2256}, this line can not intersect with any line of K_{130}, then $K_{131} = K_{130} \cup \ell_{132} = \{\ell_1, \ell_2, \dots, \ell_{132}\}$ is a $(132, \ell)$ – span.

The line $\mathscr{b}_{133} = \{146, 202, 502, 633, 764, 895, 1195, 1326, 1457, 1575, 1706, 2006, 2137, 2268\}$, this line can not intersect with any line of K_{131}, then $K_{132} = K_{131} \cup \mathscr{b}_{133} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{133}\}$ is a $(133, \mathscr{b}) -$ span.
The line $\mathscr{b}_{134} = \{147, 215, 516, 648, 780, 899, 1031, 1332, 1464, 1596, 1728, 2029, 2148, 2280\}$, this line can not intersect with any line of K_{132}, then $K_{133} = K_{132} \cup \mathscr{b}_{134} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{134}\}$ is a $(134, \mathscr{b}) -$ span.
The line $\mathscr{b}_{135} = \{148, 228, 361, 663, 783, 916, 1049, 1351, 1471, 1604, 1737, 2039, 2172, 2292\}$, this line can not intersect with any line of K_{133}, then $K_{134} = K_{133} \cup \mathscr{b}_{135} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{135}\}$ is a $(135, \mathscr{b}) -$ span.
The line $\mathscr{b}_{136} = \{149, 241, 375, 665, 799, 933, 1067, 1357, 1491, 1625, 1746, 1880, 2183, 2304\}$, this line can not intersect with any line of K_{134}, then $K_{135} = K_{134} \cup \mathscr{b}_{136} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{136}\}$ is a $(136, \mathscr{b}) -$ span.
The line $\mathscr{b}_{137} = \{150, 254, 389, 680, 815, 950, 1072, 1207, 1498, 1633, 1768, 1890, 2194, 2316\}$, this line can not intersect with any line of K_{135}, then $K_{136} = K_{135} \cup \mathscr{b}_{137} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{137}\}$ is a $(137, \mathscr{b}) -$ span.
The line $\mathscr{b}_{138} = \{151, 267, 403, 526, 831, 954, 1090, 1213, 1518, 1641, 1777, 1900, 2205, 2341\}$, this line can not intersect with any line of K_{136}, then $K_{137} = K_{136} \cup \mathscr{b}_{138} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{138}\}$ is a $(138, \mathscr{b}) -$ span.
The line $\mathscr{b}_{139} = \{152, 280, 417, 541, 834, 971, 1095, 1232, 1525, 1662, 1786, 1923, 2047, 2353\}$, this line can not intersect with any line of K_{137}, then $K_{138} = K_{137} \cup \mathscr{b}_{139} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{139}\}$ is a $(139, \mathscr{b}) -$ span.
The line $\mathscr{b}_{140} = \{153, 293, 418, 556, 850, 988, 1113, 1238, 1376, 1670, 1808, 1933, 2058, 2365\}$, this line can not intersect with any line of K_{138}, then $K_{139} = K_{138} \cup \mathscr{b}_{140} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{140}\}$ is a $(140, \mathscr{b}) -$ span.
The line $\mathscr{b}_{141} = \{154, 306, 432, 571, 697, 992, 1131, 1257, 1383, 1691, 1817, 1943, 2069, 2377\}$, this line can not intersect with any line of K_{139}, then $K_{140} = K_{139} \cup \mathscr{b}_{141} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{141}\}$ is a $(141, \mathscr{b}) -$ span.
The line $\mathscr{b}_{142} = \{155, 319, 446, 586, 713, 1009, 1136, 1263, 1403, 1699, 1826, 1953, 2093, 2220\}$, this line can not intersect with any line of K_{140}, then $K_{141} = K_{140} \cup \mathscr{b}_{142} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{142}\}$ is a $(142, \mathscr{b}) -$ span.
The line $\mathscr{b}_{143} = \{156, 332, 460, 588, 729, 1026, 1154, 1282, 1410, 1538, 1835, 1976, 2104, 2232\}$, this line can not intersect with any line of K_{141}, then $K_{142} = K_{141} \cup \mathscr{b}_{143} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{143}\}$ is a $(143, \mathscr{b}) -$ span.
The line $\mathscr{b}_{144} = \{157, 345, 474, 603, 732, 861, 1159, 1301, 1430, 1559, 1857, 1986, 2115, 2244\}$, this line can not intersect with any line of K_{142}, then $K_{143} = K_{142} \cup \mathscr{b}_{144} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{144}\}$ is a $(144, \mathscr{b}) -$ span.
The line $\mathscr{b}_{145} = \{158, 196, 508, 651, 794, 937, 1080, 1223, 1535, 1678, 1821, 1964, 2107, 2250\}$, this line can not intersect with any line of K_{143}, then $K_{144} = K_{143} \cup \mathscr{b}_{145} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{145}\}$ is a $(145, \mathscr{b}) -$ span.
The line $\mathscr{b}_{146} = \{159, 209, 509, 653, 797, 941, 1085, 1229, 1373, 1686, 1830, 1974, 2118, 2262\}$, this line can not intersect with any line of K_{144}, then $K_{145} = K_{144} \cup \mathscr{b}_{146} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{146}\}$ is a $(146, \mathscr{b}) -$ span.
The line $\mathscr{b}_{147} = \{160, 222, 354, 668, 813, 958, 1103, 1248, 1380, 1694, 1839, 1984, 2129, 2274\}$, this line can not intersect with any line of K_{145}, then $K_{146} = K_{145} \cup \mathscr{b}_{147} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{147}\}$ is a $(147, \mathscr{b}) -$ span.
The line $\mathscr{b}_{148} = \{161, 235, 368, 683, 829, 975, 1108, 1254, 1400, 1546, 1848, 1994, 2140, 2286\}$, this line can not intersect with any line of K_{146}, then $K_{147} = K_{146} \cup \mathscr{b}_{148} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{148}\}$ is a $(148, \mathscr{b}) -$ span.
The line $\mathscr{b}_{149} = \{162, 248, 382, 529, 845, 979, 1126, 1273, 1407, 1554, 1870, 2004, 2151, 2298\}$, this line can not intersect with any line of K_{147}, then $K_{148} = K_{147} \cup \mathscr{b}_{149} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{149}\}$ is a $(149, \mathscr{b}) -$ span.
The line $\mathscr{b}_{150} = \{163, 261, 396, 544, 848, 996, 1144, 1279, 1427, 1562, 1710, 2027, 2162, 2310\}$, this line can not intersect with any line of K_{148}, then $K_{149} = K_{148} \cup \mathscr{b}_{150} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{150}\}$ is a $(150, \mathscr{b}) -$ span.
The line $\mathscr{b}_{151} = \{164, 274, 410, 559, 695, 1013, 1149, 1298, 1434, 1583, 1719, 2037, 2173, 2322\}$, this line can not intersect with any line of K_{149}, then $K_{150} = K_{149} \cup \mathscr{b}_{151} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{151}\}$ is a $(151, \mathscr{b}) -$ span.
The line $\mathscr{b}_{152} = \{165, 287, 424, 561, 711, 1017, 1167, 1304, 1454, 1591, 1741, 1878, 2197, 2334\}$, this line can not intersect with any line of K_{150}, then $K_{151} = K_{150} \cup \mathscr{b}_{152} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{152}\}$ is a $(152, \mathscr{b}) -$ span.
The line $\mathscr{b}_{153} = \{166, 300, 438, 576, 727, 865, 1172, 1323, 1461, 1612, 1750, 1888, 2208, 2346\}$, this line can not intersect with any line of K_{151}, then $K_{152} = K_{151} \cup \mathscr{b}_{153} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{153}\}$ is a $(153, \mathscr{b}) -$ span.
The line $\mathscr{b}_{154} = \{167, 313, 452, 591, 730, 882, 1190, 1329, 1481, 1620, 1759, 1911, 2050, 2358\}$, this line can not intersect with any line of K_{152}, then $K_{153} = K_{152} \cup \mathscr{b}_{154} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{154}\}$ is a $(154, \mathscr{b}) -$ span.
The line $\mathscr{b}_{155} = \{168, 326, 466, 606, 746, 886, 1039, 1348, 1488, 1628, 1781, 1921, 2061, 2370\}$, this line can not intersect with any line of K_{153}, then $K_{154} = K_{153} \cup \mathscr{b}_{155} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{155}\}$ is a $(155, \mathscr{b}) -$ span.
The line $\mathscr{b}_{156} = \{169, 339, 480, 621, 762, 903, 1044, 1354, 1508, 1649, 1790, 1931, 2072, 2213\}$, this line can not intersect with any line of K_{154}, then $K_{155} = K_{154} \cup \mathscr{b}_{156} = \{\mathscr{b}_1, \mathscr{b}_2, \dots, \mathscr{b}_{156}\}$ is a $(156, \mathscr{b}) -$ span.

The line $b_{157} = \{170, 352, 494, 636, 778, 920, 1062, 1204, 1515, 1657, 1799, 1941, 2083, 2225\}$, this line can not intersect with any line of K_{155}, then $K_{156} = K_{155} \cup b_{157} = \{b_1, b_2, \dots, b_{157}\}$ is a $(157, b) -$ span.

The line $b_{158} = \{171, 190, 515, 671, 827, 983, 1139, 1295, 1451, 1607, 1763, 1919, 2075, 2231\}$, this line can not intersect with any line of K_{156}, then $K_{157} = K_{156} \cup b_{158} = \{b_1, b_2, \dots, b_{158}\}$ is a $(158, b) -$ span.

The line $b_{159} = \{172, 203, 360, 686, 843, 1000, 1157, 1314, 1458, 1615, 1772, 1929, 2086, 2243\}$, this line can not intersect with any line of K_{157}, then $K_{158} = K_{157} \cup b_{159} = \{b_1, b_2, \dots, b_{159}\}$ is a $(159, b) -$ span.

The line $b_{160} = \{173, 216, 374, 532, 859, 1004, 1162, 1320, 1478, 1636, 1794, 1939, 2097, 2255\}$, this line can not intersect with any line of K_{158}, then $K_{159} = K_{158} \cup b_{160} = \{b_1, b_2, \dots, b_{160}\}$ is a $(160, b) -$ span.

The line $b_{161} = \{174, 229, 388, 547, 693, 1021, 1180, 1339, 1485, 1644, 1803, 1962, 2108, 2267\}$, this line can not intersect with any line of K_{159}, then $K_{160} = K_{159} \cup b_{161} = \{b_1, b_2, \dots, b_{161}\}$ is a $(161, b) -$ span.

The line $b_{162} = \{175, 242, 402, 549, 709, 869, 1185, 1345, 1505, 1665, 1812, 1972, 2132, 2279\}$, this line can not intersect with any line of K_{160}, then $K_{161} = K_{160} \cup b_{162} = \{b_1, b_2, \dots, b_{162}\}$ is a $(162, b) -$ span.

The line $b_{163} = \{176, 255, 416, 564, 725, 873, 1034, 1364, 1512, 1673, 1834, 1982, 2143, 2291\}$, this line can not intersect with any line of K_{161}, then $K_{162} = K_{161} \cup b_{163} = \{b_1, b_2, \dots, b_{163}\}$ is a $(163, b) -$ span.

The line $b_{164} = \{177, 268, 430, 579, 741, 890, 1052, 1201, 1532, 1681, 1843, 1992, 2154, 2303\}$, this line can not intersect with any line of K_{162}, then $K_{163} = K_{162} \cup b_{164} = \{b_1, b_2, \dots, b_{164}\}$ is a $(164, b) -$ span.

The line $b_{165} = \{178, 281, 431, 594, 744, 907, 1057, 1220, 1370, 1702, 1852, 2015, 2165, 2328\}$, this line can not intersect with any line of K_{163}, then $K_{164} = K_{163} \cup b_{165} = \{b_1, b_2, \dots, b_{165}\}$ is a $(165, b) -$ span.

The line $b_{166} = \{179, 294, 445, 609, 760, 924, 1075, 1226, 1390, 1541, 1861, 2025, 2176, 2340\}$, this line can not intersect with any line of K_{164}, then $K_{165} = K_{164} \cup b_{166} = \{b_1, b_2, \dots, b_{166}\}$ is a $(166, b) -$ span.

The line $b_{167} = \{180, 307, 459, 624, 776, 928, 1093, 1245, 1397, 1549, 1714, 2035, 2187, 2352\}$, this line can not intersect with any line of K_{165}, then $K_{166} = K_{165} \cup b_{167} = \{b_1, b_2, \dots, b_{167}\}$ is a $(167, b) -$ span.

The line $b_{168} = \{181, 320, 473, 626, 792, 945, 1098, 1251, 1417, 1570, 1723, 1876, 2211, 2364\}$, this line can not intersect with any line of K_{166}, then $K_{167} = K_{166} \cup b_{168} = \{b_1, b_2, \dots, b_{168}\}$ is a $(168, b) -$ span.

The line $b_{169} = \{182, 333, 487, 641, 795, 962, 1116, 1270, 1424, 1578, 1732, 1899, 2053, 2376\}$, this line can not intersect with any line of K_{167}, then $K_{168} = K_{167} \cup b_{169} = \{b_1, b_2, \dots, b_{169}\}$ is a $(169, b) -$ span.

Finally, add the line $b_{170} = \{183, 346, 501, 656, 811, 966, 1121, 1276, 1444, 1599, 1754, 1909, 2064, 2219\}$, to K_{168}, this line can not intersect with any line of K_{168}, then $K_{169} = K_{168} \cup b_{170} = \{b_1, b_2, \dots, b_{170}\}$ is a $(170, b) -$ span, which is the maximum $(K, b) -$ span of PG(3,13) can be obtained. Thus, K_{169} is called a spread of one hundred and seventy lines of $PG(3,13)$ which partitions $PG(3,13)$; that is every point of $PG(3,13)$ lies in exactly one line of K_{169} and every two lines of K_{169} are disjoint.

4.2 Theorem

In general, the total number of $(K, b) -$ span in $\mathrm{PG}(3, q)$, $q \geq 2$ is $q^2 + 1$ [13].

4.3 New Examples of $(K, b) -$ span in $PG(3,13)$

A new example of $(1, b) -$ span in $PG(3,13)$ which are: {(1,0,0,0), (0,1,0,0), (1,1,0,0), (2,1,0,0), (3,1,0,0), (4,1,0,0), (5,1,0,0), (6,1,0,0), (7,1,0,0), (8,1,0,0), (9,1,0,0), (10,1,0,0), (11,1,0,0), (12,1,0,0)}.

A new example of $(2, b) -$ span in $PG(3,13)$ which are: {(0,0,1,0), (0,0,0,1), (0,0,1,1), (0,0,2,1), (0,0,3,1), (0,0,4,1), (0,0,5,1), (0,0,6,1), (0,0,7,1), (0,0,8,1), (0,0,9,1), (0,0,10,1), (0,0,11,1), (0,0,12,1)}.

A new example of $(3, b) -$ span in $PG(3,13)$ which are: {(1,0,1,0), (0,1,0,1), (1,1,1,1), (2,1,2,1), (3,1,3,1), (4,1,4,1), (5,1,5,1), (6,1,6,1), (7,1,7,1), (8,1,8,1), (9,1,9,1), (10,1,10,1), (11,1,11,1), (12,1,12,1)}.

So, there are a new examples of $\{(4, b), (5, b), (6, b), \dots, (170, b)\} -$ span in $PG(3,13)$.

5. Computer program to find points and planes for $PG(3, 13)$, by using MATLAB 2019

4.1 Program (A) to find a complete (Ķ,b)-span

```
clc;
clear;
q=13;
%profile on
tic
[Point,Plane]=PointPlane(q);
I=q+1;          % length of line
K=q^2+I;        % length of plane
N=q^3+K;        % length of projective space
M=N*I;
O=q^2+1;
P=Plane;
[m] = size(P,1);
```

```
                % pre-allocate memory to the cell output
matrix (which is symmetric)
    cellMtx = cell(m,m);
    for u=1:m
      for v=u+1:m
                % determine the intersection between the
two rows
          cellMtx{u,v} = intersect(P(u,1:end),P(v,1:end));
          cellMtx{v,u} = cellMtx{u,v};
      end
    end
S=[1:I];
q1=q+2;
k=0;
for j1=2:O
   index=q+j1;
     for i=1:m-1
       for j=i:m
       G=ismember(cellMtx{i,j},index);
       sum(G);
          if sum(G)>0
             if ~ismember(S,cellMtx{i,j})
               S=[S;  cellMtx{i,j}];
             end
            %return
          end
       end
     end
end
     writematrix(S,'span.txt','Delimiter','tab');
 toc
```

6. Conclusions

From the above results, we found that a $(170, \mathscr{b}) -$ span is a maximum complete $(\mathrm{K}, \mathscr{b}) -$ span in $PG(3,13)$.

Acknowledgement

The research is supported by the College of Computer Sciences and Mathematics, University of Mosul, and College of Basic Education, University of Telafer, Mosul, Republic of Iraq. The authors declare that there are no conflicts of interest regarding this work.

References

[1] Abdulla, Ali Ahmed A., and Nada Yassen Kasm Yahya. "A Geometric Construction of Surface Complete (k, r)-cap in PG (3, 7)." Journal of Physics: Conference Series. Vol. 1879. No. 2. IOP Publishing, 2021.

[2] Al-Zangana, Emad Bakr, and Nada Yassen Kasm Yahya. "Subgroups and Orbits by Companion Matrix in Three Dimensional Projective Space." Baghdad Science Journal (2022): 0805-0805.

[3] Hirschfeld, J. W. P. Projective geometries over finite fields. Oxford Mathematical Monographs. New York: Oxford University Press, 1998.

[4] Al-Mukhtar, A. Sh. Complete Arcs and Surfaces in three Dimensional Projective Space Over Galois Field. Diss. Ph. D. Thesis, University of Technology, Iraq, 2008.

[5] Yahya, Nada Yassen Kasm. "Applications geometry of space in PG (3, P)." Journal of Interdisciplinary Mathematics 25.2 (2022): 285-297.

[6] Kareem, Fatema F., and Sawsan J. Kadhum. "A (k, ℓ) Span in Three Dimensional Projective Space PG (3, p) Over Galois field where p= 4." Journal of the college of basic education 19.80/.(٢٠١٣) .

[7] Hirschfeld, J. W. P. Finite projective spaces of three dimensions. Oxford University Press, 1985.

[8] Salih, R. A. Complete Arcs in Projective Plane Over Galois Field. Diss. M. Sc. Thesis, University of Baghdad, Iraq, 1999.

[9] Hirschfeld J. W. P." Projective geometries over finite fields". Oxford University Press, 1979.

[10] Ballico, Edoardo, and Antonio Cossidente. "Curves of the projective 3-space, tangent developables and partial spreads." Bulletin of the Belgian Mathematical Society-Simon Stevin 7.3 (2000): 387-394.

[11] Blokhuis, Aart, et al. "Covers of PG (3, q) and of finite generalized quadrangles." Bulletin of the Belgian Mathematical Society Simon Stevin 5.2 (1998): 141-158.

[12] Gács, András, and Tamás Szőnyi. "On maximal partial spreads in PG (n, q)." Designs, Codes and Cryptography 29.1 (2003): 123-129.

[13] Abdulla, Ali Ahmed A., and Nada Yassen Kasm Yahya. "Application of algebraic geometry in three dimensional projective space PG (3, 7)." Journal of Physics: Conference Series. Vol. 1591. No. 1. IOP Publishing, 2020.

البناء للامتداد $- (\mathrm{K}, \mathscr{b})$ التام في $PG(3,13)$

حامد محمد خلف — hamid_math_85@yahoo.com
كلية التربية الأساسية
جامعة تلعفر،الموصل، العراق

ندى ياسين قاسم يحيى — drnadaqasim3@uomosul.edu.iq
كلية التربية للعلوم الصرفة
جامعة الموصل،الموصل، العراق

تاريخ استلام البحث: ٢٠٢٢/٢/٧   تاريخ قبول البحث: ٢٠٢٢/٥/١٩

الخلاصة

الهدف الرئيسي من البحث هو ايجاد الناشر للفضاء الاسقاطي $PG(3,13)$ بواسطة البناء للامتداد
التام$- (\mathrm{K}, \mathscr{b})$ والذي يمثل تطبيقات الهندسة الجبرية بالفضاء الاسقاطي ثلاثي الابعاد $\mathrm{PG}(3,q)$ برهنا اكبر حجم للامتداد $- (\mathrm{K}, \mathscr{b})$ في $\mathrm{PG}(3,13)$ وهو الامتداد $- (170, \mathscr{b})$ويمثل الناشر.

الكلمات المفتاحية: الهندسة الجبرية، الفضاء الاسقاطي، البناء للامتداد التام.