



Construction of a complete (K, ℓ) – span in $PG(3, 13)$

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Abstract

The main goal of this work is to find a spread of $PG(3,13)$. By construct a complete (K, ℓ) –span which represents applications of algebraic geometry in 3-dimensional projective space $PG(3,q)$. We prove that the maximum (K, ℓ) -span in $PG(3,13)$ is $(170, \ell)$ -span, which is a spread.

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1. INTRODUCTION

A projective 3-space $PG(3, q)$ over Galois field (denoted by, $GF(q)$, where $q = p^h$ for some prime number p and some integer h), is a 3-dimensional projective space. [1], [2]. The Projective 3 – space satisfies the following axioms:

- 1- Any two distinct planes intersect in a unique line.
- 2- Any two distinct points are contained in a unique line.
- 3- Any two distinct coplanar lines intersect at a unique point.
- 4- Any line not on a given plane intersects the plane at a unique point.
- 5- Any three distinct non-collinear points, with the lines and points that lie not on them are contained in a unique plane. [3]

Two lines intersect in at most one point. This happens precisely if the two lines lie in a plane. Otherwise, they are called skew.

A projective 3-space $PG(3,q)$ satisfies the following:

- 1- Any two planes intersect in exactly $q + 1$ points, any line is on exactly $q + 1$ planes and any two points on exactly $q + 1$ planes."
- 2- Every plane contains exactly $q^2 + q + 1$ points (lines) and every point is on exactly $q^2 + q + 1$ planes.

- 3- There exist $q^3 + q^2 + q + 1$ of points and there exists $q^3 + q^2 + q + 1$ of planes.
- 4- Every line contains exactly $q + 1$ points and every point is on exactly $q + 1$ lines. [4]

By using computer programs A, we found the points, lines and planes which are in Tables (1 and 2). The purpose of this study is to investigate a geometric construction (K, ℓ) -span in 3-dimensional projective space and find the maximum complete $(170, \ell)$ -span in $PG(3,13)$, equal to all the points of the space.

2. Preliminaries

2.1 Definition Plane Π

In $PG(3, q)$ a plane Π is the set of all points $p(y_1, y_2, y_3, y_4)$ satisfying a linear equation $a_1y_1 + a_2y_2 + a_3y_3 + a_4y_4 = 0$. This plane is denoted by $\Pi[a_1, a_2, a_3, a_4]$, where a_1, a_2, a_3, a_4 are elements in $GF(q)$ with the exception of the quadrable consisting of four zero elements [3], [4].

2.2 Theorem

The points of $PG(3, q)$ have a unique forms, which are $(1,0,0,0)$, $(x_1,1,0,0)$, $(x_1, x_2,1,0)$ and $(x_1, x_2, x_3,1)$ for all x_1, x_2, x_3 in $GF(q)$, which are $(1,0,0,0)$ is one point, $(x_1,1,0,0)$, q points, $(x_1, x_2,1,0)$, q^2 points, and $(x_1, x_2, x_3,1)$, q^3 points, for all x_1, x_2, x_3 in $GF(q)$ [3].

2.3 Theorem

The planes of $PG(3, q)$ have unique forms, which are $[1,0,0,0]$, $[x_1,1,0,0]$, $[x_1, x_2,1,0]$, $[x_1, x_2, x_3,1]$ for all x_1, x_2, x_3 in $GF(q)$. which are $[1,0,0,0]$ is one plane, $[x_1,1,0,0]$, q planes, $[x_1, x_2,1,0]$, q^2 planes and $[x_1, x_2, x_3,1]$, q^3 planes, for all x_1, x_2, x_3 in $GF(q)$ [3].

2.4 Theorem

There exists $(q^2 + 1)(q^2 + q + 1)$ of lines, in $PG(3, q)$ [3], [5].

2.5 The projective space in $PG(3,13)$

The projective space $PG(3,13)$ contains (2380) points, and (2380) planes, such that each point is on (183) planes and every plane contains (183) points, any line contains (14) points, which is the intersection of (14) planes. The points, planes and lines of $PG(3,13)$ given in Tables (1 and 2).

3. Spread and (K, \mathcal{L}) -span

3.1 Definition

A (K, \mathcal{L}) -span, $\mathcal{L} \geq 1$, is a set of K spaces $\Pi_{\mathcal{L}}$ (K lines) no two of which intersect [6], [7].

3.2 Definition

A maximum (K, \mathcal{L}) -span is a set of K spaces $\Pi_{\mathcal{L}}$ which are every points of $PG(3, q)$ lies in exactly one line of $\Pi_{\mathcal{L}}$ and every two lines of $\Pi_{\mathcal{L}}$ are disjoint.

A K -span is a $(K, 0)$ -span, that is, a set of K points.

A (K, \mathcal{L}) -span is complete if it is not contained in a $(K + 1, \mathcal{L})$ -span [7],[8].

3.3 Theorem

In $PG(3, q)$, $q > 2$, there exists a complete K -span with $K = q^2 - q + 1$, or $K = q^2 - q + 2$ [7].

3.4 Theorem

In $PG(3, q)$, q odd and $q > 3$, there exists a complete $(q^2 - q + 2)$ -span [7].

3.5 Definition

In $PG(3, q)$, a spread S is a set of $(q^2 + 1)$ lines, which are pairwise disjoint and thus, partition the set of points.

A partial spread β is a set of mutually skew lines and if $|\beta|=s$, then β is also called a s -span. Hence, a $(q^2 + 1)$ -span is a spread of $PG(3, q)$.

A partial spread is called maximal when it is not contained properly in a larger partial spread. [7], [9], [10].

3.6 Corollary

A K -span with $K > q^2 - \sqrt{q}$, can be completed uniquely to a spread [3].

3.7 Theorem

In $PG(3, q)$, a partial spread containing more than $q^2 + 1 - \sqrt{2q}$ lines, in contained in a spread of $PG(3, q)$ [11], [12].

4. Algorithm

- Choose the lines who containing all points of $PG(3, q)$.
- Choose the first line of any plane that we take contains $q + 1$ of the ordered and series points which is $1, 2, 3, \dots, q + 1$, of $PG(3, q)$.
- Searching to next line starting with point $q + 2$, but does not intersect the first line.
- searching to next line starting with point $q + 3$, but does not intersect the first and second lines.

- searching to next line starting with point $q + 4$, but does not intersect the first, second and third lines.
- We continue in this way until we get the line that begins with point $q^2 + q + 1$. It must also be maintained that there is no intersection between all the previous selected lines. In this case, we get that K is equal to $q^2 + 1$.

Table (1): Points and Plans of $PG(3, 13)$

I	P _i	1	15	25	41	54	67	80	93	108	
1	(1,0,0,0)	119	132	145	158	171	184	197	210		
		223	236	249	262	275	288	301	314		
		327	340	353	366	379	392	405	418		
		431	444	457	470	483	496	509	522		
		535	548	561	574	587	600	613	626		
		639	652	665	678	691	704	717	730		
		743	756	769	782	795	808	821	834		
		847	860	873	886	899	912	925	938		
		951	964	977	990	1003	1016	1029	1042		
		1055	1068	1081	1094	1107	1120	1133	1146		
		1159	1172	1185	1198	1211	1224	1237	1250		
		1263	1276	1289	1302	1315	1328	1341	1354		
		1367	1380	1393	1406	1419	1432	1445	1458		
		1471	1484	1497	1510	1523	1536	1549	1562		
		1575	1588	1601	1614	1627	1640	1653	1666		
		1679	1692	1705	1718	1731	1744	1757	1770		
		1783	1796	1809	1822	1835	1848	1861	1874		
1887	1900	1913	1926	1939	1952	1965	1978				
1991	2004	2017	2030	2043	2056	2069	2082				
2095	2108	2121	2134	2147	2160	2173	2186				
2199	2212	2225	2238	2251	2264	2277	2290				
2303	2316	2329	2342	2355	2368						
2	(0,1,0,0)	15	16	17	18	19	20	21	22		
		23	24	25	26	27	28	29	30		
		37	38	39	40	41	42	43	44		
		53	54	55	56	57	58	59	60		
		75	76	77	78	79	80	81	82		
		97	98	99	100	101	102	103	104		
		129	130	131	132	133	134	135	136		
		151	152	153	154	155	156	157	158		
		173	174	175	176	177	178	179	180		
		195	196	197	198	199	200	201	202		
		217	218	219	220	221	222	223	224		
		246	247	248	249	250	251	252	253		
		268	269	270	271	272	273	274	275		
		290	291	292	293	294	295	296	297		
		312	313	314	315	316	317	318	319		
		334	335	336	337	338	339	340	341		
		356	357	358	359	360	361	362	363		
378	379	380	381	382	383	384	385				
400	401	402	403	404	405	406	407				
422	423	424	425	426	427	428	429				
444	445	446	447	448	449	450	451				
466	467	468	469	470	471	472	473				
488	489	490	491	492	493	494	495				
510	511	512	513	514	515	516	517				
532	533	534	535	536	537	538	539				
554	555	556	557	558	559	560	561				
576	577	578	579	580	581	582	583				
598	599	600	601	602	603	604	605				
620	621	622	623	624	625	626	627				
642	643	644	645	646	647	648	649				
664	665	666	667	668	669	670	671				
686	687	688	689	690	691	692	693				
708	709	710	711	712	713	714	715				
730	731	732	733	734	735	736	737				
752	753	754	755	756	757	758	759				
774	775	776	777	778	779	780	781				
796	797	798	799	800	801	802	803				
818	819	820	821	822	823	824	825				
840	841	842	843	844	845	846	847				
862	863	864	865	866	867	868	869				
884	885	886	887	888	889	890	891				
906	907	908	909	910	911	912	913				
928	929	930	931	932	933	934	935				
950	951	952	953	954	955	956	957				
972	973	974	975	976	977	978	979				
994	995	996	997	998	999	1000	1001				
1016	1017	1018	1019	1020	1021	1022	1023				
1038	1039	1040	1041	1042	1043	1044	1045				
1060	1061	1062	1063	1064	1065	1066	1067				
1082	1083	1084	1085	1086	1087	1088	1089				
1104	1105	1106	1107	1108	1109	1110	1111				
1126	1127	1128	1129	1130	1131	1132	1133				
1148	1149	1150	1151	1152	1153	1154	1155				
1170	1171	1172	1173	1174	1175	1176	1177				
1192	1193	1194	1195	1196	1197	1198	1199				
1214	1215	1216	1217	1218	1219	1220	1221				
1236	1237	1238	1239	1240	1241	1242	1243				
1258	1259	1260	1261	1262	1263	1264	1265				
1280	1281	1282	1283	1284	1285	1286	1287				
1302	1303	1304	1305	1306	1307	1308	1309				
1324	1325	1326	1327	1328	1329	1330	1331				
1346	1347	1348	1349	1350	1351	1352	1353				
1368	1369	1370	1371	1372	1373	1374	1375				
1390	1391	1392	1393	1394	1395	1396	1397				
1412	1413	1414	1415	1416	1417	1418	1419				
1434	1435	1436	1437	1438	1439	1440	1441				
1456	1457	1458	1459	1460	1461	1462	1463				
1478	1479	1480	1481	1482	1483	1484	1485				
1500	1501	1502	1503	1504	1505	1506	1507				
1522	1523	1524	1525	1526	1527	1528	1529				
1544	1545	1546	1547	1548	1549	1550	1551				
1566	1567	1568	1569	1570	1571	1572	1573				
1588	1589	1590	1591	1592	1593	1594	1595				
1610	1611	1612	1613	1614	1615	1616	1617				
1632	1633	1634	1635	1636	1637	1638	1639				
1654	1655	1656	1657	1658	1659	1660	1661				
1676	1677	1678	1679	1680	1681	1682	1683				
1698	1699	1700	1701	1702	1703	1704	1705				
1720	1721	1722	1723	1724	1725	1726	1727				
1742	1743	1744	1745	1746	1747	1748	1749				
1764	1765	1766	1767	1768	1769	1770	1771				
1786	1787	1788	1789	1790	1791	1792	1793				
1808	1809	1810	1811	1812	1813	1814	1815				
1830	1831	1832	1833	1834	1835	1836	1837				
1852	1853	1854	1855	1856	1857	1858	1859				
1874	1875	1876	1877	1878	1879	1880	1881				
1896	1897	1898	1899	1900	1901	1902	1903				
1918	1919	1920	1921	1922	1923	1924	1925				
1940	1941	1942	1943	1944	1945	1946	1947				
1962	1963	1964	1965	1966	1967	1968	1969				
1984	1985	1986	1987	1988	1989	1990	1991				
2006	2007	2008	2009	2010	2011	2012	2013				
2028	2029	2030	2031	2032	2033	2034	2035				
2050											

Table (2): Plane and lines of PG(3,13)

1	15	2	171	93	67	...	54	67	93	171	171
	184	184	184	184	184	...	314	301	275	197	197
	353	197	509	431	405	...	353	353	353	353	353
	522	210	665	678	626	...	561	574	600	678	678
	691	223	821	756	847	...	769	795	847	834	834
	860	236	977	1003	899	...	977	1016	925	990	990
	1029	249	1133	1081	1120	...	1185	1068	1172	1146	1146
	1198	262	1289	1328	1341	...	1224	1289	1250	1302	1302
	1367	275	1445	1406	1393	...	1432	1510	1497	1458	1458
	1536	288	1601	1653	1614	...	1640	1562	1575	1614	1614
	1705	301	1757	1731	1835	...	1848	1783	1822	1770	1770
	1874	314	1913	1978	1887	...	1887	2004	1900	1926	1926
	2043	327	2069	2056	2108	...	2095	2056	2147	2082	2082
	2212	340	2225	2303	2329	...	2303	2277	2225	2238	2238
2380	171	27	14	159	87	...	111	159	147	14	171
	197	185	353	222	294	...	318	294	222	1374	197
	353	353	378	353	353	...	414	438	510	1386	353
	678	534	390	666	594	...	666	582	642	1398	678
	834	702	402	810	822	...	762	726	774	1410	834
	990	870	414	954	894	...	1027	870	906	1422	990
	1146	1038	426	1098	1122	...	1110	1183	1038	1434	1146
	1302	1206	438	1242	1363	...	1206	1327	1339	1446	1302
	1458	1374	450	1386	1422	...	1458	1458	1458	1458	1458
	1614	1542	462	1699	1663	...	1554	1602	1590	1483	1614
	1770	1710	474	1843	1722	...	1819	1746	1722	1495	1770
	1926	1878	486	1987	1963	...	1902	1890	2023	1507	1926
	2082	2046	498	2131	2191	...	2167	2203	2155	1519	2082
	2238	2214	2301	2282	2307	...	2301	2213	2332	2326	2276

4.1 Construction of spread in PG(3,13)

4.1.1 Theorem

The maximum (K, ℓ) -span in $PG(3, 13)$ is $(170, \ell)$ -span.

Proof: In Table (2), any two non-intersecting lines can be taken in $PG(3,13)$, say

$\ell_1 = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$ and $\ell_2 = \{15, 184, 353, 522, 691, 860, 1029, 1198, 1367, 1536, 1705, 1874, 2043, 2212\}$, then $K_1 = \{\ell_1, \ell_2\}$ is a $(2, \ell)$ -span. Can add another line $\ell_3 = \{16, 197, 367, 537, 707, 877, 1047, 1217, 1387, 1557, 1727, 1897, 2067, 2237\}$, then

$K_2 = \{\ell_1, \ell_2, \ell_3\}$ is a $(3, \ell)$ -span, since ℓ_3 cannot intersect ℓ_1 or ℓ_2 . Add another lines:

The line $\ell_4 = \{17, 210, 381, 552, 723, 894, 1065, 1236, 1394, 1565, 1736, 1907, 2078, 2249\}$, this line can not intersect with any line of K_2 , then

$K_3 = K_2 \cup \ell_4 = \{\ell_1, \ell_2, \ell_3, \ell_4\}$ is a $(4, \ell)$ -span.

The line $\ell_5 = \{18, 223, 395, 567, 739, 911, 1070, 1242, 1414, 1586, 1745, 1917, 2089, 2261\}$, this line can not intersect with any line of K_3 , then $K_4 = K_3 \cup \ell_5 = \{\ell_1, \ell_2, \ell_3, \ell_4, \ell_5\}$ is a $(5, \ell)$ -span.

The line $\ell_6 = \{19, 236, 409, 582, 755, 915, 1088, 1261, 1421, 1594, 1767, 1927, 2100, 2273\}$, this line can not intersect with any line of K_4 , then

$K_5 = K_4 \cup \ell_6 = \{\ell_1, \ell_2, \ell_3, \ell_4, \ell_5, \ell_6\}$ is a $(6, \ell)$ -span. The line $\ell_7 = \{20, 249, 423, 597, 758, 932, 1106, 1267, 1441, 1602, 1776, 1950, 2111, 2285\}$, this line can not intersect with any line of K_5 , then

$K_6 = K_5 \cup \ell_7 = \{\ell_1, \ell_2, \ell_3, \ell_4, \ell_5, \ell_6, \ell_7\}$ is a $(7, \ell)$ -span.

The line $\ell_8 = \{21, 262, 437, 612, 774, 949, 1111, 1286, 1448, 1623, 1785, 1960, 2122, 2297\}$, this line can not intersect with any line of K_6 , then

$K_7 = K_6 \cup \ell_8 = \{\ell_1, \ell_2, \ell_3, \ell_4, \ell_5, \ell_6, \ell_7, \ell_8\}$ is a $(8, \ell)$ -span.

The line $\ell_9 = \{22, 275, 451, 614, 790, 953, 1129, 1292, 1468, 1631, 1807, 1970, 2146, 2309\}$, this line can not intersect with any line of K_7 , then

$K_8 = K_7 \cup \ell_9 = \{\ell_1, \ell_2, \ell_3, \ell_4, \ell_5, \ell_6, \ell_7, \ell_8, \ell_9\}$ is a $(9, \ell)$ -span.

The line $\ell_{10} = \{23, 288, 465, 629, 806, 970, 1134, 1311, 1475, 1652, 1816, 1980, 2157, 2321\}$, this line can not intersect with any line of K_8 , then

$K_9 = K_8 \cup \ell_{10} =$

$\{\ell_1, \ell_2, \ell_3, \ell_4, \ell_5, \ell_6, \ell_7, \ell_8, \ell_9, \ell_{10}\}$ is a $(10, \ell)$ -span.

The line $\ell_{11} = \{24, 301, 479, 644, 809, 987, 1152, 1317, 1495, 1660, 1825, 2003, 2168, 2333\}$, this line can not intersect with any line of K_9 , then

$K_{10} = K_9 \cup \ell_{11} =$

$\{\ell_1, \ell_2, \ell_3, \ell_4, \ell_5, \ell_6, \ell_7, \ell_8, \ell_9, \ell_{10}, \ell_{11}\}$ is a $(11, \ell)$ -span.

The line $\ell_{12} = \{25, 314, 493, 659, 825, 991, 1170, 1336, 1502, 1668, 1847, 2013, 2179, 2345\}$, this line can not intersect with any line of K_{10} , then $K_{11} = K_{10} \cup \ell_{12} = \{\ell_1, \ell_2, \ell_3, \ell_4, \ell_5, \ell_6, \ell_7, \ell_8, \ell_9, \ell_{10}, \ell_{11}, \ell_{12}\}$ is a $(12, \ell)$ -span.

The line $\ell_{13} = \{26, 327, 507, 674, 841, 1008, 1175, 1342, 1522, 1689, 1856, 2023, 2190, 2357\}$, this line can not intersect with any line of K_{11} , then $K_{12} = K_{11} \cup \ell_{13} = \{\ell_1, \ell_2, \ell_3, \ell_4, \ell_5, \ell_6, \ell_7, \ell_8, \ell_9, \ell_{10}, \ell_{11}, \ell_{12}, \ell_{13}\}$ is a $(13, \ell)$ -span.

The line $\ell_{14} = \{27, 340, 521, 689, 857, 1025, 1193, 1361, 1529, 1697, 1865, 2033, 2201, 2369\}$, this line can not intersect with any line of K_{12} , then $K_{13} = K_{12} \cup \ell_{14} = \{\ell_1, \ell_2, \ell_3, \ell_4, \ell_5, \ell_6, \ell_7, \ell_8, \ell_9, \ell_{10}, \ell_{11}, \ell_{12}, \ell_{13}, \ell_{14}\}$ is a $(14, \ell)$ -span.

The line $\ell_{15} = \{28, 191, 373, 555, 737, 919, 1101, 1283, 1465, 1647, 1829, 2011, 2193, 2375\}$, this line can not intersect with any line of K_{13} , then $K_{14} = K_{13} \cup \ell_{15} = \{\ell_1, \ell_2, \ell_3, \ell_4, \ell_5, \ell_6, \ell_7, \ell_8, \ell_9, \ell_{10}, \ell_{11}, \ell_{12}, \ell_{13}, \ell_{14}, \ell_{15}\}$ is a $(15, \ell)$ -span.

The line $\ell_{16} = \{29, 204, 387, 570, 753, 936, 1119, 1289, 1472, 1655, 1838, 2021, 2204, 2218\}$, this line can not intersect with any line of K_{14} , then

$K_{15} = K_{14} \cup \ell_{16} = \{\ell_1, \ell_2, \ell_3, \dots, \ell_{16}\}$ is a $(16, \ell)$ -span.

The line $\mathcal{L}_{17} = \{30, 217, 401, 585, 756, 940, 1124, 1308, 1492, 1676, 1860, 2031, 2046, 2230\}$, this line can not intersect with any line of K_{15} , then $K_{16} = K_{15} \cup \mathcal{L}_{17} = \{\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, \dots, \mathcal{L}_{17}\}$ is a $(17, \mathcal{L})$ – span.

The line $\mathcal{L}_{18} = \{31, 230, 415, 587, 772, 957, 1142, 1327, 1499, 1684, 1869, 1885, 2057, 2242\}$, this line can not intersect with any line of K_{16} , then

$K_{17} = K_{16} \cup \mathcal{L}_{18} = \{\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, \dots, \mathcal{L}_{18}\}$ is a $(18, \mathcal{L})$ – span.

The line $\mathcal{L}_{19} = \{32, 243, 429, 602, 788, 974, 1147, 1333, 1519, 1692, 1709, 1895, 2081, 2254\}$, this line can not intersect with any line of K_{17} , then

$K_{18} = K_{17} \cup \mathcal{L}_{19} = \{\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, \dots, \mathcal{L}_{19}\}$ is a $(19, \mathcal{L})$ – span.

The line $\mathcal{L}_{20} = \{33, 256, 443, 617, 804, 978, 1165, 1352, 1526, 1544, 1718, 1905, 2092, 2266\}$, this line can not intersect with any line of K_{18} , then

$K_{19} = K_{18} \cup \mathcal{L}_{20} = \{\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, \dots, \mathcal{L}_{20}\}$ is a $(20, \mathcal{L})$ – span.

The line $\mathcal{L}_{21} = \{34, 269, 444, 632, 820, 995, 1183, 1358, 1377, 1552, 1740, 1915, 2103, 2278\}$, this line can not intersect with any line of K_{19} , then

$K_{20} = K_{19} \cup \mathcal{L}_{21} = \{\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, \dots, \mathcal{L}_{21}\}$ is a $(21, \mathcal{L})$ – span.

The line $\mathcal{L}_{22} = \{35, 282, 458, 647, 823, 1012, 1188, 1208, 1384, 1573, 1749, 1938, 2114, 2290\}$, this line can not intersect with any line of K_{20} , then

$K_{21} = K_{20} \cup \mathcal{L}_{22} = \{\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, \dots, \mathcal{L}_{22}\}$ is a $(22, \mathcal{L})$ – span.

The line $\mathcal{L}_{23} = \{36, 295, 472, 662, 839, 1016, 1037, 1214, 1404, 1581, 1758, 1948, 2125, 2315\}$, this line can not intersect with any line of K_{21} , then

$K_{22} = K_{21} \cup \mathcal{L}_{23} = \{\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, \dots, \mathcal{L}_{23}\}$ is a $(23, \mathcal{L})$ – span.

The line $\mathcal{L}_{24} = \{37, 308, 486, 677, 855, 864, 1042, 1233, 1411, 1589, 1780, 1958, 2136, 2327\}$, this line can not intersect with any line of K_{22} , then

$K_{23} = K_{22} \cup \mathcal{L}_{24} = \{\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, \dots, \mathcal{L}_{24}\}$ is a $(24, \mathcal{L})$ – span.

The line $\mathcal{L}_{25} = \{38, 321, 500, 679, 702, 881, 1060, 1239, 1431, 1610, 1789, 1968, 2147, 2339\}$, this line can not intersect with any line of K_{23} , then

$K_{24} = K_{23} \cup \mathcal{L}_{25} = \{\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, \dots, \mathcal{L}_{25}\}$ is a $(25, \mathcal{L})$ – span.

The line $\mathcal{L}_{26} = \{39, 334, 514, 525, 705, 898, 1078, 1258, 1438, 1618, 1798, 1978, 2171, 2351\}$, this line can not intersect with any line of K_{24} , then

$K_{25} = K_{24} \cup \mathcal{L}_{26} = \{\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, \dots, \mathcal{L}_{26}\}$ is a $(26, \mathcal{L})$ – span.

The line $\mathcal{L}_{27} = \{40, 347, 359, 540, 721, 902, 1083, 1264, 1445, 1639, 1820, 2001, 2182, 2363\}$, this line can not intersect with any line of K_{25} , then

$K_{26} = K_{25} \cup \mathcal{L}_{27} = \{\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, \dots, \mathcal{L}_{27}\}$ is a $(27, \mathcal{L})$ – span.

The line $\mathcal{L}_{28} = \{41, 185, 380, 575, 770, 965, 1160, 1355, 1381, 1576, 1771, 1966, 2161, 2356\}$, this line can not intersect with any line of K_{26} , then

$K_{27} = K_{26} \cup \mathcal{L}_{28} = \{\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, \dots, \mathcal{L}_{28}\}$ is a $(28, \mathcal{L})$ – span.

The line $\mathcal{L}_{29} = \{42, 198, 394, 590, 786, 982, 1178, 1205, 1401, 1597, 1793, 1989, 2185, 2368\}$, this line can not intersect with any line of K_{27} , then

$K_{28} = K_{27} \cup \mathcal{L}_{29} = \{\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, \dots, \mathcal{L}_{29}\}$ is a $(29, \mathcal{L})$ – span.

The line $\mathcal{L}_{30} = \{43, 211, 408, 605, 802, 999, 1196, 1211, 1408, 1605, 1802, 1999, 2196, 2224\}$, this line can not intersect with any line of K_{28} , then

$K_{29} = K_{28} \cup \mathcal{L}_{30} = \{\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, \dots, \mathcal{L}_{30}\}$ is a $(30, \mathcal{L})$ – span.

The line $\mathcal{L}_{31} = \{44, 224, 422, 620, 818, 1003, 1032, 1230, 1428, 1626, 1811, 2009, 2207, 2236\}$, this line can not intersect with any line of K_{29} , then

$K_{30} = K_{29} \cup \mathcal{L}_{31} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{31}\}$ is a $(31, \mathcal{L})$ – span.

The line $\mathcal{L}_{32} = \{45, 237, 436, 635, 821, 1020, 1050, 1249, 1435, 1634, 1833, 2019, 2049, 2248\}$, this line can not intersect with any line of K_{30} , then

$K_{31} = K_{30} \cup \mathcal{L}_{32} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{32}\}$ is a $(32, \mathcal{L})$ – span.

The line $\mathcal{L}_{33} = \{46, 250, 450, 650, 837, 868, 1055, 1255, 1455, 1642, 1842, 2042, 2060, 2260\}$, this line can not intersect with any line of K_{31} , then

$K_{32} = K_{31} \cup \mathcal{L}_{33} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{33}\}$ is a $(33, \mathcal{L})$ – span.

The line $\mathcal{L}_{34} = \{47, 263, 464, 652, 853, 885, 1073, 1274, 1462, 1663, 1851, 1883, 2071, 2272\}$, this line can not intersect with any line of K_{32} , then

$K_{33} = K_{32} \cup \mathcal{L}_{34} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{34}\}$ is a $(34, \mathcal{L})$ – span.

The line $\mathcal{L}_{35} = \{48, 276, 478, 667, 700, 889, 1091, 1280, 1482, 1671, 1873, 1893, 2082, 2284\}$, this line can not intersect with any line of K_{33} , then

$K_{34} = K_{33} \cup \mathcal{L}_{35} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{35}\}$ is a $(35, \mathcal{L})$ – span.

The line $\mathcal{L}_{36} = \{49, 289, 492, 682, 716, 906, 1096, 1299, 1489, 1679, 1713, 1903, 2106, 2296\}$, this line can not intersect with any line of K_{34} , then

$K_{35} = K_{34} \cup \mathcal{L}_{36} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{36}\}$ is a $(36, \mathcal{L})$ – span.

The line $\mathcal{L}_{37} = \{50, 302, 506, 528, 719, 923, 1114, 1305, 1509, 1700, 1722, 1913, 2117, 2308\}$, this line can not intersect with any line of K_{35} , then

$K_{36} = K_{35} \cup \mathcal{L}_{37} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{37}\}$ is a $(37, \mathcal{L})$ – span.

The line $\mathcal{L}_{38} = \{51, 315, 520, 543, 735, 927, 1132, 1324, 1516, 1539, 1731, 1936, 2128, 2320\}$, this line can not intersect with any line of K_{36} , then

$K_{37} = K_{36} \cup \mathcal{L}_{38} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{38}\}$ is a $(38, \mathcal{L})$ – span.

The line $\mathcal{L}_{39} = \{52, 328, 365, 558, 751, 944, 1137, 1330, 1523, 1560, 1753, 1946, 2139, 2332\}$, this line can not intersect with any line of K_{37} , then

$K_{38} = K_{37} \cup \mathcal{L}_{39} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{39}\}$ is a $(39, \mathcal{L})$ – span.

The line $\mathcal{L}_{40} = \{53, 341, 366, 573, 767, 961, 1155, 1349, 1374, 1568, 1762, 1956, 2150, 2344\}$, this line can not intersect with any line of K_{38} , then

$K_{39} = K_{38} \cup \mathcal{L}_{40} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{40}\}$ is a $(40, \mathcal{L})$ – span.

The line $\mathcal{L}_{41} = \{54, 192, 400, 608, 816, 1024, 1063, 1271, 1479, 1687, 1726, 1934, 2142, 2350\}$, this line can not intersect with any line of K_{39} , then

$K_{40} = K_{39} \cup \mathcal{L}_{41} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{41}\}$ is a $(41, \mathcal{L})$ – span.

The line $\mathcal{L}_{42} = \{55, 205, 414, 623, 832, 872, 1068, 1277, 1486, 1695, 1735, 1944, 2153, 2362\}$, this line can not intersect with any line of K_{40} , then

$K_{41} = K_{40} \cup \mathcal{L}_{42} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{42}\}$ is a $(42, \mathcal{L})$ – span.

The line $\mathcal{L}_{43} = \{56, 218, 428, 638, 835, 876, 1086, 1296, 1506, 1547, 1744, 1954, 2164, 2374\}$, this line can not intersect with any line of K_{41} , then

$K_{42} = K_{41} \cup \mathcal{L}_{43} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{43}\}$ is a $(43, \mathcal{L})$ – span.

The line $\mathcal{L}_{44} = \{57, 231, 442, 640, 851, 893, 1104, 1302, 1513, 1555, 1766, 1977, 2175, 2217\}$, this line can not intersect with any line of K_{42} , then

$K_{43} = K_{42} \cup \mathcal{L}_{44} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{44}\}$ is a $(44, \mathcal{L})$ – span.

The line $\mathcal{L}_{45} = \{58, 244, 456, 655, 698, 910, 1109, 1321, 1533, 1563, 1775, 1987, 2186, 2229\}$, this line can not intersect with any line of K_{43} , then

$K_{44} = K_{43} \cup \mathcal{L}_{45} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{45}\}$ is a $(45, \mathcal{L})$ – span.

The line $\mathcal{L}_{46} = \{59, 257, 457, 670, 714, 914, 1127, 1340, 1371, 1584, 1784, 1997, 2210, 2241\}$, this line can not intersect with any line of K_{44} , then

$K_{45} = K_{44} \cup \mathcal{L}_{46} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{46}\}$ is a $(46, \mathcal{L})$ – span.

The line $\mathcal{L}_{47} = \{60, 270, 471, 685, 717, 931, 1145, 1346, 1391, 1592, 1806, 2007, 2052, 2253\}$, this line can not intersect with any line of K_{45} , then

$K_{46} = K_{45} \cup \mathcal{L}_{47} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{47}\}$ is a $(47, \mathcal{L})$ – span.

The line $\mathcal{L}_{48} = \{61, 283, 485, 531, 733, 948, 1150, 1365, 1398, 1613, 1815, 2017, 2063, 2265\}$, this line can not intersect with any line of K_{46} , then

$K_{47} = K_{46} \cup \mathcal{L}_{48} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{48}\}$ is a $(48, \mathcal{L})$ – span.

The line $\mathcal{L}_{49} = \{62, 296, 499, 546, 749, 952, 1168, 1202, 1418, 1621, 1824, 2040, 2074, 2277\}$, this line can not intersect with any line of K_{47} , then

$K_{48} = K_{47} \cup \mathcal{L}_{49} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{49}\}$ is a $(49, \mathcal{L})$ – span.

The line $\mathcal{L}_{50} = \{63, 309, 513, 548, 765, 969, 1173, 1221, 1425, 1629, 1846, 1881, 2085, 2302\}$, this line can not intersect with any line of K_{48} , then

$K_{49} = K_{48} \cup \mathcal{L}_{50} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{50}\}$ is a $(50, \mathcal{L})$ – span.

The line $\mathcal{L}_{51} = \{64, 322, 358, 563, 781, 986, 1191, 1227, 1432, 1650, 1855, 1891, 2096, 2314\}$, this line can not intersect with any line of K_{49} , then

$K_{50} = K_{49} \cup \mathcal{L}_{51} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{51}\}$ is a $(51, \mathcal{L})$ – span.

The line $\mathcal{L}_{52} = \{65, 335, 372, 578, 784, 990, 1040, 1246, 1452, 1658, 1864, 1901, 2120, 2326\}$, this line can not intersect with any line of K_{50} , then

$K_{51} = K_{50} \cup \mathcal{L}_{52} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{52}\}$ is a $(52, \mathcal{L})$ – span.

The line $\mathcal{L}_{53} = \{66, 348, 386, 593, 800, 1007, 1045, 1252, 1459, 1666, 1717, 1924, 2131, 2338\}$, this line can not intersect with any line of K_{51} , then

$K_{52} = K_{51} \cup \mathcal{L}_{53} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{53}\}$ is a $(53, \mathcal{L})$ – span.

The line $\mathcal{L}_{54} = \{67, 186, 407, 628, 849, 901, 1122, 1343, 1395, 1616, 1837, 1889, 2110, 2331\}$, this line can not intersect with any line of K_{52} , then

$K_{53} = K_{52} \cup \mathcal{L}_{54} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{54}\}$ is a $(54, \mathcal{L})$ – span.

The line $\mathcal{L}_{55} = \{68, 199, 421, 643, 696, 918, 1140, 1362, 1415, 1637, 1859, 1912, 2121, 2343\}$, this line can not intersect with any line of K_{53} , then

$K_{54} = K_{53} \cup \mathcal{L}_{55} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{55}\}$ is a $(55, \mathcal{L})$ – span.

The line $\mathcal{L}_{56} = \{69, 212, 435, 658, 712, 935, 1158, 1199, 1422, 1645, 1868, 1922, 2145, 2355\}$, this line can not intersect with any line of K_{54} , then

$K_{55} = K_{54} \cup \mathcal{L}_{56} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{56}\}$ is a $(56, \mathcal{L})$ – span.

The line $\mathcal{L}_{57} = \{70, 225, 449, 673, 728, 939, 1163, 1218, 1442, 1653, 1708, 1932, 2156, 2380\}$, this line can not intersect with any line of K_{55} , then

$K_{56} = K_{55} \cup \mathcal{L}_{57} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{57}\}$ is a $(57, \mathcal{L})$ – span.

The line $\mathcal{L}_{58} = \{71, 238, 463, 688, 731, 956, 1181, 1224, 1449, 1674, 1730, 1942, 2167, 2223\}$, this line can not intersect with any line of K_{56} , then

$K_{57} = K_{56} \cup \mathcal{L}_{58} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{58}\}$ is a $(58, \mathcal{L})$ – span.

The line $\mathcal{L}_{59} = \{72, 251, 477, 534, 747, 973, 1186, 1243, 1469, 1682, 1739, 1952, 2178, 2235\}$, this line can not intersect with any line of K_{57} , then

$K_{58} = K_{57} \cup \mathcal{L}_{59} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{59}\}$ is a $(59, \mathcal{L})$ – span.

The line $\mathcal{L}_{60} = \{73, 264, 491, 536, 763, 977, 1035, 1262, 1476, 1703, 1748, 1975, 2189, 2247\}$, this line can not intersect with any line of K_{58} , then

$K_{59} = K_{58} \cup \mathcal{L}_{60} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{60}\}$ is a $(60, \mathcal{L})$ – span.

The line $\mathcal{L}_{61} = \{74, 277, 505, 551, 779, 994, 1053, 1268, 1496, 1542, 1757, 1985, 2200, 2259\}$, this line can not intersect with any line of K_{59} , then

$K_{60} = K_{59} \cup \mathcal{L}_{61} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{61}\}$ is a $(61, \mathcal{L})$ – span.

The line $\mathcal{L}_{62} = \{75, 290, 519, 566, 782, 1011, 1058, 1287, 1503, 1550, 1779, 1995, 2055, 2271\}$, this line can not intersect with any line of K_{60} , then

$K_{61} = K_{60} \cup \mathcal{L}_{62} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{62}\}$ is a $(62, \mathcal{L})$ – span.

The line $\mathcal{L}_{63} = \{76, 303, 364, 581, 798, 1028, 1076, 1293, 1510, 1571, 1788, 2005, 2066, 2283\}$, this line can not intersect with any line of K_{61} , then

$K_{62} = K_{61} \cup \mathcal{L}_{63} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{63}\}$ is a $(63, \mathcal{L})$ – span.

The line $\mathcal{L}_{64} = \{77, 316, 378, 596, 814, 863, 1081, 1312, 1530, 1579, 1797, 2028, 2077, 2295\}$, this line can not intersect with any line of K_{62} , then

$K_{63} = K_{62} \cup \mathcal{L}_{64} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{64}\}$ is a $(64, \mathcal{L})$ – span.

The line $\mathcal{L}_{65} = \{78, 329, 379, 611, 830, 880, 1099, 1318, 1368, 1600, 1819, 2038, 2088, 2307\}$, this line can not intersect with any line of K_{63} , then

$K_{64} = K_{63} \cup \mathcal{L}_{65} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{65}\}$ is a $(65, \mathcal{L})$ – span.

The line $\mathcal{L}_{66} = \{79, 342, 393, 613, 846, 897, 1117, 1337, 1388, 1608, 1828, 1879, 2099, 2319\}$, this line can not intersect with any line of K_{64} , then

$K_{65} = K_{64} \cup \mathcal{L}_{66} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{66}\}$ is a $(66, \mathcal{L})$ – span.

The line $\mathcal{L}_{67} = \{80, 193, 427, 661, 726, 960, 1194, 1259, 1493, 1558, 1792, 2026, 2091, 2325\}$, this line can not intersect with any line of K_{65} , then

$K_{66} = K_{65} \cup \mathcal{L}_{67} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{67}\}$ is a $(67, \mathcal{L})$ – span.

The line $\mathcal{L}_{68} = \{81, 206, 441, 676, 742, 964, 1030, 1265, 1500, 1566, 1801, 2036, 2102, 2337\}$, this line can not intersect with any line of K_{66} , then

$K_{67} = K_{66} \cup \mathcal{L}_{68} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{68}\}$ is a $(68, \mathcal{L})$ – span.

The line $\mathcal{L}_{69} = \{82, 219, 455, 678, 745, 981, 1048, 1284, 1520, 1587, 1810, 1877, 2113, 2349\}$, this line can not intersect with any line of K_{67} , then

$K_{68} = K_{67} \cup \mathcal{L}_{69} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{69}\}$ is a $(69, \mathcal{L})$ – span.

The line $\mathcal{L}_{70} = \{83, 232, 469, 524, 761, 998, 1066, 1290, 1527, 1595, 1832, 1887, 2124, 2361\}$, this line can not intersect with any line of K_{68} , then

$K_{69} = K_{68} \cup \mathcal{L}_{70} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{70}\}$ is a $(70, \mathcal{L})$ – span.

The line $\mathcal{L}_{71} = \{84, 245, 470, 539, 777, 1015, 1071, 1309, 1378, 1603, 1841, 1910, 2135, 2373\}$, this line can not intersect with any line of K_{69} , then

$K_{70} = K_{69} \cup \mathcal{L}_{71} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{71}\}$ is a $(71, \mathcal{L})$ – span.

The line $\mathcal{L}_{72} = \{85, 258, 484, 554, 793, 1019, 1089, 1315, 1385, 1624, 1850, 1920, 2159, 2216\}$, this line can not intersect with any line of K_{70} , then

$K_{71} = K_{70} \cup \mathcal{L}_{72} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{72}\}$ is a $(72, \mathcal{L})$ – span.

The line $\mathcal{L}_{73} = \{86, 271, 498, 569, 796, 867, 1094, 1334, 1405, 1632, 1872, 1930, 2170, 2228\}$, this line can not intersect with any line of K_{71} , then

$K_{72} = K_{71} \cup \mathcal{L}_{73} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{73}\}$ is a $(73, \mathcal{L})$ – span.

The line $\mathcal{L}_{74} = \{87, 284, 512, 584, 812, 884, 1112, 1353, 1412, 1640, 1712, 1940, 2181, 2240\}$, this line can not intersect with any line of K_{72} , then

$K_{73} = K_{72} \cup \mathcal{L}_{74} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{74}\}$ is a $(74, \mathcal{L})$ – span.

The line $\mathcal{L}_{75} = \{88, 297, 357, 599, 828, 888, 1130, 1359, 1419, 1661, 1721, 1963, 2192, 2252\}$, this line can not intersect with any line of K_{73} , then

$K_{74} = K_{73} \cup \mathcal{L}_{75} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{75}\}$ is a $(75, \mathcal{L})$ – span.

The line $\mathcal{L}_{76} = \{89, 310, 371, 601, 844, 905, 1135, 1209, 1439, 1669, 1743, 1973, 2203, 2264\}$, this line can not intersect with any line of K_{74} , then

$K_{75} = K_{74} \cup \mathcal{L}_{76} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{76}\}$ is a $(76, \mathcal{L})$ – span.

The line $\mathcal{L}_{77} = \{90, 323, 385, 616, 847, 922, 1153, 1215, 1446, 1690, 1752, 1983, 2045, 2289\}$, this line can not intersect with any line of K_{75} , then

$K_{76} = K_{75} \cup \mathcal{L}_{77} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{77}\}$ is a $(77, \mathcal{L})$ – span.

The line $\mathcal{L}_{78} = \{91, 336, 399, 631, 694, 926, 1171, 1234, 1466, 1698, 1761, 1993, 2056, 2301\}$, this line can not intersect with any line of K_{76} , then

$K_{77} = K_{76} \cup \mathcal{L}_{78} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{78}\}$ is a $(78, \mathcal{L})$ – span.

The line $\mathcal{L}_{79} = \{92, 349, 413, 646, 710, 943, 1176, 1240, 1473, 1537, 1770, 2016, 2080, 2313\}$, this line can not intersect with any line of K_{77} , then

$K_{78} = K_{77} \cup \mathcal{L}_{79} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{79}\}$ is a $(79, \mathcal{L})$ – span.

The line $\mathcal{L}_{80} = \{93, 187, 434, 681, 759, 1006, 1084, 1331, 1409, 1656, 1734, 1981, 2059, 2306\}$, this line can not intersect with any line of K_{78} , then

$K_{79} = K_{78} \cup \mathcal{L}_{80} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{80}\}$ is a $(80, \mathcal{L})$ – span.

The line $\mathcal{L}_{81} = \{94, 200, 448, 527, 775, 1023, 1102, 1350, 1429, 1677, 1756, 1991, 2070, 2318\}$, this line can not intersect with any line of K_{79} , then

$K_{80} = K_{79} \cup \mathcal{L}_{81} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{81}\}$ is a $(81, \mathcal{L})$ – span.

The line $\mathcal{L}_{82} = \{95, 213, 462, 542, 791, 871, 1107, 1356, 1436, 1685, 1765, 2014, 2094, 2330\}$, this line can not intersect with any line of K_{80} , then

$K_{81} = K_{80} \cup \mathcal{L}_{82} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{82}\}$ is a $(82, \mathcal{L})$ – span.

The line $\mathcal{L}_{83} = \{96, 226, 476, 557, 807, 875, 1125, 1206, 1456, 1693, 1774, 2024, 2105, 2342\}$, this line can not intersect with any line of K_{81} , then

$K_{82} = K_{81} \cup \mathcal{L}_{83} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{83}\}$ is a $(83, \mathcal{L})$ – span.

The line $\mathcal{L}_{84} = \{97, 239, 490, 572, 810, 892, 1143, 1212, 1463, 1545, 1783, 2034, 2116, 2367\}$, this line can not intersect with any line of K_{82} , then

$K_{83} = K_{82} \cup \mathcal{L}_{84} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{84}\}$ is a $(84, \mathcal{L})$ – span.

The line $\mathcal{L}_{85} = \{98, 252, 504, 574, 826, 909, 1148, 1231, 1483, 1553, 1805, 1875, 2127, 2379\}$, this line can not intersect with any line of K_{83} , then

$K_{84} = K_{83} \cup \mathcal{L}_{85} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{85}\}$ is a $(85, \mathcal{L})$ – span.

The line $\mathcal{L}_{86} = \{99, 265, 518, 589, 842, 913, 1166, 1237, 1490, 1574, 1814, 1898, 2138, 2222\}$, this line can not intersect with any line of K_{84} , then

$K_{85} = K_{84} \cup \mathcal{L}_{86} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{86}\}$ is a $(86, \mathcal{L})$ – span.

The line $\mathcal{L}_{87} = \{100, 278, 363, 604, 858, 930, 1184, 1256, 1497, 1582, 1823, 1908, 2149, 2234\}$, this line can not intersect with any line of K_{85} , then

$K_{86} = K_{85} \cup \mathcal{L}_{87} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{87}\}$ is a $(87, \mathcal{L})$ – span.

The line $\mathcal{L}_{88} = \{101, 291, 377, 619, 692, 947, 1189, 1275, 1517, 1590, 1845, 1918, 2160, 2246\}$, this line can not intersect with any line of K_{86} , then

$K_{87} = K_{86} \cup \mathcal{L}_{88} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{88}\}$ is a $(88, \mathcal{L})$ – span.

The line $\mathcal{L}_{89} = \{102, 304, 391, 634, 708, 951, 1038, 1281, 1524, 1611, 1854, 1928, 2184, 2258\}$, this line can not intersect with any line of K_{87} , then

$K_{88} = K_{87} \cup \mathcal{L}_{89} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{89}\}$ is a $(89, \mathcal{L})$ – span.

The line $\mathcal{L}_{90} = \{103, 317, 392, 649, 724, 968, 1043, 1300, 1375, 1619, 1863, 1951, 2195, 2270\}$, this line can not intersect with any line of K_{88} , then

$K_{89} = K_{88} \cup \mathcal{L}_{90} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{90}\}$ is a $(90, \mathcal{L})$ – span.

The line $\mathcal{L}_{91} = \{104, 330, 406, 664, 740, 985, 1061, 1306, 1382, 1627, 1716, 1961, 2206, 2282\}$, this line can not intersect with any line of K_{89} , then

$K_{90} = K_{89} \cup \mathcal{L}_{91} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{91}\}$ is a $(91, \mathcal{L})$ – span.

The line $\mathcal{L}_{92} = \{105, 343, 420, 666, 743, 1002, 1079, 1325, 1402, 1648, 1725, 1971, 2048, 2294\}$, this line can not intersect with any line of K_{90} , then

$K_{91} = K_{90} \cup \mathcal{L}_{92} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{92}\}$ is a $(92, \mathcal{L})$ – span.

The line $\mathcal{L}_{93} = \{106, 194, 454, 545, 805, 896, 1156, 1247, 1507, 1598, 1858, 1949, 2209, 2300\}$, this line can not intersect with any line of K_{91} , then

$K_{92} = K_{91} \cup \mathcal{L}_{93} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{93}\}$ is a $(93, \mathcal{L})$ – span.

The line $\mathcal{L}_{94} = \{107, 207, 468, 560, 808, 900, 1161, 1253, 1514, 1606, 1867, 1959, 2051, 2312\}$, this line can not intersect with any line of K_{92} , then

$K_{93} = K_{92} \cup \mathcal{L}_{94} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{94}\}$ is a $(94, \mathcal{L})$ – span.

The line $\mathcal{L}_{95} = \{108, 220, 482, 562, 824, 917, 1179, 1272, 1534, 1614, 1707, 1969, 2062, 2324\}$, this line can not intersect with any line of K_{93} , then

$K_{94} = K_{93} \cup \mathcal{L}_{95} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{95}\}$ is a $(95, \mathcal{L})$ – span.

The line $\mathcal{L}_{96} = \{109, 233, 483, 577, 840, 934, 1197, 1278, 1372, 1635, 1729, 1979, 2073, 2336\}$, this line can not intersect with any line of K_{94} , then

$K_{95} = K_{94} \cup \mathcal{L}_{96} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{96}\}$ is a $(96, \mathcal{L})$ – span.

The line $\mathcal{L}_{97} = \{110, 246, 497, 592, 856, 938, 1033, 1297, 1392, 1643, 1738, 2002, 2084, 2348\}$, this line can not intersect with any line of K_{95} , then

$K_{96} = K_{95} \cup \mathcal{L}_{97} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{97}\}$ is a $(97, \mathcal{L})$ – span.

The line $\mathcal{L}_{98} = \{111, 259, 511, 607, 703, 955, 1051, 1303, 1399, 1664, 1747, 2012, 2095, 2360\}$, this line can not intersect with any line of K_{96} , then

$K_{97} = K_{96} \cup \mathcal{L}_{98} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{98}\}$ is a $(98, \mathcal{L})$ – span.

The line $\mathcal{L}_{99} = \{112, 272, 356, 622, 706, 972, 1056, 1322, 1406, 1672, 1769, 2022, 2119, 2372\}$, this line can not intersect with any line of K_{97} , then

$K_{98} = K_{97} \cup \mathcal{L}_{99} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{99}\}$ is a $(99, \mathcal{L})$ – span.

The line $\mathcal{L}_{100} = \{113, 285, 370, 637, 722, 989, 1074, 1328, 1426, 1680, 1778, 2032, 2130, 2215\}$, this line can not intersect with any line of K_{98} , then

$K_{99} = K_{98} \cup \mathcal{L}_{100} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{100}\}$ is a $(100, \mathcal{L})$ – span.

The line $\mathcal{L}_{101} = \{114, 298, 384, 639, 738, 993, 1092, 1347, 1433, 1701, 1787, 1886, 2141, 2227\}$, this line can not intersect with any line of K_{99} , then

$K_{100} = K_{99} \cup \mathcal{L}_{101} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{101}\}$ is a $(101, \mathcal{L})$ – span.

The line $\mathcal{L}_{102} = \{115, 311, 398, 654, 754, 1010, 1097, 1366, 1453, 1540, 1796, 1896, 2152, 2239\}$, this line can not intersect with any line of K_{100} , then $K_{101} = K_{100} \cup \mathcal{L}_{102} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{102}\}$ is a $(102, \mathcal{L})$ – span.

The line $\mathcal{L}_{103} = \{116, 324, 412, 669, 757, 1027, 1115, 1203, 1460, 1561, 1818, 1906, 2163, 2251\}$, this line can not intersect with any line of K_{101} , then $K_{102} = K_{101} \cup \mathcal{L}_{103} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{103}\}$ is a $(103, \mathcal{L})$ – span.

The line $\mathcal{L}_{104} = \{117, 337, 426, 684, 773, 862, 1120, 1222, 1480, 1569, 1827, 1916, 2174, 2276\}$, this line can not intersect with any line of K_{102} , then $K_{103} = K_{102} \cup \mathcal{L}_{104} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{104}\}$ is a $(104, \mathcal{L})$ – span.

The line $\mathcal{L}_{105} = \{118, 350, 440, 530, 789, 879, 1138, 1228, 1487, 1577, 1836, 1926, 2198, 2288\}$, this line can not intersect with any line of K_{103} , then $K_{104} = K_{103} \cup \mathcal{L}_{105} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{105}\}$ is a $(105, \mathcal{L})$ – span.

The line $\mathcal{L}_{106} = \{119, 188, 461, 565, 838, 942, 1046, 1319, 1423, 1696, 1800, 1904, 2177, 2281\}$, this line can not intersect with any line of K_{104} , then $K_{105} = K_{104} \cup \mathcal{L}_{106} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{106}\}$ is a $(106, \mathcal{L})$ – span.

The line $\mathcal{L}_{107} = \{120, 201, 475, 580, 854, 959, 1064, 1338, 1443, 1548, 1809, 1914, 2188, 2293\}$, this line can not intersect with any line of K_{105} , then $K_{106} = K_{105} \cup \mathcal{L}_{107} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{107}\}$ is a $(107, \mathcal{L})$ – span.

The line $\mathcal{L}_{108} = \{121, 214, 489, 595, 701, 976, 1069, 1344, 1450, 1556, 1831, 1937, 2199, 2305\}$, this line can not intersect with any line of K_{106} , then $K_{107} = K_{106} \cup \mathcal{L}_{108} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{108}\}$ is a $(108, \mathcal{L})$ – span.

The line $\mathcal{L}_{109} = \{122, 227, 503, 610, 704, 980, 1087, 1363, 1470, 1564, 1840, 1947, 2054, 2317\}$, this line can not intersect with any line of K_{107} , then $K_{108} = K_{107} \cup \mathcal{L}_{109} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{109}\}$ is a $(109, \mathcal{L})$ – span.

The line $\mathcal{L}_{110} = \{123, 240, 517, 625, 720, 997, 1105, 1200, 1477, 1585, 1849, 1957, 2065, 2329\}$, this line can not intersect with any line of K_{108} , then $K_{109} = K_{108} \cup \mathcal{L}_{110} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{110}\}$ is a $(110, \mathcal{L})$ – span.

The line $\mathcal{L}_{111} = \{124, 253, 362, 627, 736, 1014, 1110, 1219, 1484, 1593, 1871, 1967, 2076, 2354\}$, this line can not intersect with any line of K_{109} , then $K_{110} = K_{109} \cup \mathcal{L}_{111} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{111}\}$ is a $(111, \mathcal{L})$ – span.

The line $\mathcal{L}_{112} = \{125, 266, 376, 642, 752, 1018, 1128, 1225, 1504, 1601, 1711, 1990, 2087, 2366\}$, this line can not intersect with any line of K_{110} , then $K_{111} = K_{110} \cup \mathcal{L}_{112} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{112}\}$ is a $(112, \mathcal{L})$ – span.

The line $\mathcal{L}_{113} = \{126, 279, 390, 657, 768, 866, 1133, 1244, 1511, 1622, 1720, 2000, 2098, 2378\}$, this line can not intersect with any line of K_{111} , then $K_{112} = K_{111} \cup \mathcal{L}_{113} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{113}\}$ is a $(113, \mathcal{L})$ – span.

The line $\mathcal{L}_{114} = \{127, 292, 404, 672, 771, 883, 1151, 1250, 1531, 1630, 1742, 2010, 2109, 2221\}$, this line can not intersect with any line of K_{112} , then $K_{113} = K_{112} \cup \mathcal{L}_{114} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{114}\}$ is a $(114, \mathcal{L})$ – span.

The line $\mathcal{L}_{115} = \{128, 305, 405, 687, 787, 887, 1169, 1269, 1369, 1651, 1751, 2020, 2133, 2233\}$, this line can not intersect with any line of K_{113} , then $K_{114} = K_{113} \cup \mathcal{L}_{115} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{115}\}$ is a $(115, \mathcal{L})$ – span.

The line $\mathcal{L}_{116} = \{129, 318, 419, 533, 803, 904, 1174, 1288, 1389, 1659, 1760, 2030, 2144, 2245\}$, this line can not intersect with any line of K_{114} , then $K_{115} = K_{114} \cup \mathcal{L}_{116} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{116}\}$ is a $(116, \mathcal{L})$ – span.

The line $\mathcal{L}_{117} = \{130, 331, 433, 535, 819, 921, 1192, 1294, 1396, 1667, 1782, 1884, 2155, 2257\}$, this line can not intersect with any line of K_{115} , then $K_{116} = K_{115} \cup \mathcal{L}_{117} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{117}\}$ is a $(117, \mathcal{L})$ – span.

The line $\mathcal{L}_{118} = \{131, 344, 447, 550, 822, 925, 1041, 1313, 1416, 1688, 1791, 1894, 2166, 2269\}$, this line can not intersect with any line of K_{116} , then $K_{117} = K_{116} \cup \mathcal{L}_{118} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{118}\}$ is a $(118, \mathcal{L})$ – span.

The line $\mathcal{L}_{119} = \{132, 195, 481, 598, 715, 1001, 1118, 1235, 1521, 1638, 1755, 2041, 2158, 2275\}$, this line can not intersect with any line of K_{117} , then $K_{118} = K_{117} \cup \mathcal{L}_{119} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{119}\}$ is a $(119, \mathcal{L})$ – span.

The line $\mathcal{L}_{120} = \{133, 208, 495, 600, 718, 1005, 1123, 1241, 1528, 1646, 1764, 1882, 2169, 2287\}$, this line can not intersect with any line of K_{118} , then $K_{119} = K_{118} \cup \mathcal{L}_{120} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{120}\}$ is a $(120, \mathcal{L})$ – span.

The line $\mathcal{L}_{121} = \{134, 221, 496, 615, 734, 1022, 1141, 1260, 1379, 1654, 1773, 1892, 2180, 2299\}$, this line can not intersect with any line of K_{119} , then $K_{120} = K_{119} \cup \mathcal{L}_{121} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{121}\}$ is a $(121, \mathcal{L})$ – span.

The line $\mathcal{L}_{122} = \{135, 234, 510, 630, 750, 870, 1146, 1266, 1386, 1675, 1795, 1902, 2191, 2311\}$, this line can not intersect with any line of K_{120} , then $K_{121} = K_{120} \cup \mathcal{L}_{122} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{122}\}$ is a $(122, \mathcal{L})$ – span.

The line $\mathcal{L}_{123} = \{136, 247, 355, 645, 766, 874, 1164, 1285, 1393, 1683, 1804, 1925, 2202, 2323\}$, this line can not intersect with any line of K_{121} , then $K_{122} = K_{121} \cup \mathcal{L}_{123} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{123}\}$ is a $(123, \mathcal{L})$ – span.

The line $\mathcal{L}_{124} = \{137, 260, 369, 660, 769, 891, 1182, 1291, 1413, 1704, 1813, 1935, 2044, 2335\}$, this line can not intersect with any line of K_{122} , then $K_{123} = K_{122} \cup \mathcal{L}_{124} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{124}\}$ is a $(124, \mathcal{L})$ – span.

The line $\mathcal{L}_{125} = \{138, 273, 383, 675, 785, 908, 1187, 1310, 1420, 1543, 1822, 1945, 2068, 2347\}$, this line can not intersect with any line of K_{123} , then $K_{124} = K_{123} \cup \mathcal{L}_{125} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{125}\}$ is a $(125, \mathcal{L})$ – span.

The line $\mathcal{L}_{126} = \{139, 286, 397, 690, 801, 912, 1036, 1316, 1440, 1551, 1844, 1955, 2079, 2359\}$, this line can not intersect with any line of K_{124} , then $K_{125} = K_{124} \cup \mathcal{L}_{126} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{126}\}$ is a $(126, \mathcal{L})$ – span.

The line $\mathcal{L}_{127} = \{140, 299, 411, 523, 817, 929, 1054, 1335, 1447, 1572, 1853, 1965, 2090, 2371\}$, this line can not intersect with any line of K_{125} , then $K_{126} = K_{125} \cup \mathcal{L}_{127} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{127}\}$ is a $(127, \mathcal{L})$ – span.

The line $\mathcal{L}_{128} = \{141, 312, 425, 538, 833, 946, 1059, 1341, 1467, 1580, 1862, 1988, 2101, 2214\}$, this line can not intersect with any line of K_{126} , then $K_{127} = K_{126} \cup \mathcal{L}_{128} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{128}\}$ is a $(128, \mathcal{L})$ – span.

The line $\mathcal{L}_{129} = \{142, 325, 439, 553, 836, 963, 1077, 1360, 1474, 1588, 1715, 1998, 2112, 2226\}$, this line can not intersect with any line of K_{127} , then $K_{128} = K_{127} \cup \mathcal{L}_{129} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{129}\}$ is a $(129, \mathcal{L})$ – span.

The line $\mathcal{L}_{130} = \{143, 338, 453, 568, 852, 967, 1082, 1210, 1494, 1609, 1724, 2008, 2123, 2238\}$, this line can not intersect with any line of K_{128} , then $K_{129} = K_{128} \cup \mathcal{L}_{130} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{130}\}$ is a $(130, \mathcal{L})$ – span.

The line $\mathcal{L}_{131} = \{144, 351, 467, 583, 699, 984, 1100, 1216, 1501, 1617, 1733, 2018, 2134, 2263\}$, this line can not intersect with any line of K_{129} , then $K_{130} = K_{129} \cup \mathcal{L}_{131} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{131}\}$ is a $(131, \mathcal{L})$ – span.

The line $\mathcal{L}_{132} = \{145, 189, 488, 618, 748, 878, 1177, 1307, 1437, 1567, 1866, 1996, 2126, 2256\}$, this line can not intersect with any line of K_{130} , then $K_{131} = K_{130} \cup \mathcal{L}_{132} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{132}\}$ is a $(132, \mathcal{L})$ – span.

The line $\mathcal{L}_{133} = \{146, 202, 502, 633, 764, 895, 1195, 1326, 1457, 1575, 1706, 2006, 2137, 2268\}$, this line can not intersect with any line of K_{131} , then $K_{132} = K_{131} \cup \mathcal{L}_{133} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{133}\}$ is a $(133, \mathcal{L})$ – span.

The line $\mathcal{L}_{134} = \{147, 215, 516, 648, 780, 899, 1031, 1332, 1464, 1596, 1728, 2029, 2148, 2280\}$, this line can not intersect with any line of K_{132} , then $K_{133} = K_{132} \cup \mathcal{L}_{134} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{134}\}$ is a $(134, \mathcal{L})$ – span.

The line $\mathcal{L}_{135} = \{148, 228, 361, 663, 783, 916, 1049, 1351, 1471, 1604, 1737, 2039, 2172, 2292\}$, this line can not intersect with any line of K_{133} , then $K_{134} = K_{133} \cup \mathcal{L}_{135} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{135}\}$ is a $(135, \mathcal{L})$ – span.

The line $\mathcal{L}_{136} = \{149, 241, 375, 665, 799, 933, 1067, 1357, 1491, 1625, 1746, 1880, 2183, 2304\}$, this line can not intersect with any line of K_{134} , then $K_{135} = K_{134} \cup \mathcal{L}_{136} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{136}\}$ is a $(136, \mathcal{L})$ – span.

The line $\mathcal{L}_{137} = \{150, 254, 389, 680, 815, 950, 1072, 1207, 1498, 1633, 1768, 1890, 2194, 2316\}$, this line can not intersect with any line of K_{135} , then $K_{136} = K_{135} \cup \mathcal{L}_{137} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{137}\}$ is a $(137, \mathcal{L})$ – span.

The line $\mathcal{L}_{138} = \{151, 267, 403, 526, 831, 954, 1090, 1213, 1518, 1641, 1777, 1900, 2205, 2341\}$, this line can not intersect with any line of K_{136} , then $K_{137} = K_{136} \cup \mathcal{L}_{138} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{138}\}$ is a $(138, \mathcal{L})$ – span.

The line $\mathcal{L}_{139} = \{152, 280, 417, 541, 834, 971, 1095, 1232, 1525, 1662, 1786, 1923, 2047, 2353\}$, this line can not intersect with any line of K_{137} , then $K_{138} = K_{137} \cup \mathcal{L}_{139} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{139}\}$ is a $(139, \mathcal{L})$ – span.

The line $\mathcal{L}_{140} = \{153, 293, 418, 556, 850, 988, 1113, 1238, 1376, 1670, 1808, 1933, 2058, 2365\}$, this line can not intersect with any line of K_{138} , then $K_{139} = K_{138} \cup \mathcal{L}_{140} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{140}\}$ is a $(140, \mathcal{L})$ – span.

The line $\mathcal{L}_{141} = \{154, 306, 432, 571, 697, 992, 1131, 1257, 1383, 1691, 1817, 1943, 2069, 2377\}$, this line can not intersect with any line of K_{139} , then $K_{140} = K_{139} \cup \mathcal{L}_{141} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{141}\}$ is a $(141, \mathcal{L})$ – span.

The line $\mathcal{L}_{142} = \{155, 319, 446, 586, 713, 1009, 1136, 1263, 1403, 1699, 1826, 1953, 2093, 2220\}$, this line can not intersect with any line of K_{140} , then $K_{141} = K_{140} \cup \mathcal{L}_{142} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{142}\}$ is a $(142, \mathcal{L})$ – span.

The line $\mathcal{L}_{143} = \{156, 332, 460, 588, 729, 1026, 1154, 1282, 1410, 1538, 1835, 1976, 2104, 2232\}$, this line can not intersect with any line of K_{141} , then $K_{142} = K_{141} \cup \mathcal{L}_{143} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{143}\}$ is a $(143, \mathcal{L})$ – span.

The line $\mathcal{L}_{144} = \{157, 345, 474, 603, 732, 861, 1159, 1301, 1430, 1559, 1857, 1986, 2115, 2244\}$, this line can not intersect with any line of K_{142} , then $K_{143} = K_{142} \cup \mathcal{L}_{144} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{144}\}$ is a $(144, \mathcal{L})$ – span.

The line $\mathcal{L}_{145} = \{158, 196, 508, 651, 794, 937, 1080, 1223, 1535, 1678, 1821, 1964, 2107, 2250\}$, this line can not intersect with any line of K_{143} , then $K_{144} = K_{143} \cup \mathcal{L}_{145} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{145}\}$ is a $(145, \mathcal{L})$ – span.

The line $\mathcal{L}_{146} = \{159, 209, 509, 653, 797, 941, 1085, 1229, 1373, 1686, 1830, 1974, 2118, 2262\}$, this line can not intersect with any line of K_{144} , then $K_{145} = K_{144} \cup \mathcal{L}_{146} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{146}\}$ is a $(146, \mathcal{L})$ – span.

The line $\mathcal{L}_{147} = \{160, 222, 354, 668, 813, 958, 1103, 1248, 1380, 1694, 1839, 1984, 2129, 2274\}$, this line can not intersect with any line of K_{145} , then $K_{146} = K_{145} \cup \mathcal{L}_{147} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{147}\}$ is a $(147, \mathcal{L})$ – span.

The line $\mathcal{L}_{148} = \{161, 235, 368, 683, 829, 975, 1108, 1254, 1400, 1546, 1848, 1994, 2140, 2286\}$, this line can not intersect with any line of K_{146} , then $K_{147} = K_{146} \cup \mathcal{L}_{148} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{148}\}$ is a $(148, \mathcal{L})$ – span.

The line $\mathcal{L}_{149} = \{162, 248, 382, 529, 845, 979, 1126, 1273, 1407, 1554, 1870, 2004, 2151, 2298\}$, this line can not intersect with any line of K_{147} , then $K_{148} = K_{147} \cup \mathcal{L}_{149} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{149}\}$ is a $(149, \mathcal{L})$ – span.

The line $\mathcal{L}_{150} = \{163, 261, 396, 544, 848, 996, 1144, 1279, 1427, 1562, 1710, 2027, 2162, 2310\}$, this line can not intersect with any line of K_{148} , then $K_{149} = K_{148} \cup \mathcal{L}_{150} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{150}\}$ is a $(150, \mathcal{L})$ – span.

The line $\mathcal{L}_{151} = \{164, 274, 410, 559, 695, 1013, 1149, 1298, 1434, 1583, 1719, 2037, 2173, 2322\}$, this line can not intersect with any line of K_{149} , then $K_{150} = K_{149} \cup \mathcal{L}_{151} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{151}\}$ is a $(151, \mathcal{L})$ – span.

The line $\mathcal{L}_{152} = \{165, 287, 424, 561, 711, 1017, 1167, 1304, 1454, 1591, 1741, 1878, 2197, 2334\}$, this line can not intersect with any line of K_{150} , then $K_{151} = K_{150} \cup \mathcal{L}_{152} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{152}\}$ is a $(152, \mathcal{L})$ – span.

The line $\mathcal{L}_{153} = \{166, 300, 438, 576, 727, 865, 1172, 1323, 1461, 1612, 1750, 1888, 2208, 2346\}$, this line can not intersect with any line of K_{151} , then $K_{152} = K_{151} \cup \mathcal{L}_{153} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{153}\}$ is a $(153, \mathcal{L})$ – span.

The line $\mathcal{L}_{154} = \{167, 313, 452, 591, 730, 882, 1190, 1329, 1481, 1620, 1759, 1911, 2050, 2358\}$, this line can not intersect with any line of K_{152} , then $K_{153} = K_{152} \cup \mathcal{L}_{154} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{154}\}$ is a $(154, \mathcal{L})$ – span.

The line $\mathcal{L}_{155} = \{168, 326, 466, 606, 746, 886, 1039, 1348, 1488, 1628, 1781, 1921, 2061, 2370\}$, this line can not intersect with any line of K_{153} , then $K_{154} = K_{153} \cup \mathcal{L}_{155} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{155}\}$ is a $(155, \mathcal{L})$ – span.

The line $\mathcal{L}_{156} = \{169, 339, 480, 621, 762, 903, 1044, 1354, 1508, 1649, 1790, 1931, 2072, 2213\}$, this line can not intersect with any line of K_{154} , then $K_{155} = K_{154} \cup \mathcal{L}_{156} = \{\mathcal{L}_1, \mathcal{L}_2, \dots, \mathcal{L}_{156}\}$ is a $(156, \mathcal{L})$ – span.

The line $\ell_{157} = \{170, 352, 494, 636, 778, 920, 1062, 1204, 1515, 1657, 1799, 1941, 2083, 2225\}$, this line can not intersect with any line of K_{155} , then $K_{156} = K_{155} \cup \ell_{157} = \{\ell_1, \ell_2, \dots, \ell_{157}\}$ is a $(157, \ell)$ – span.

The line $\ell_{158} = \{171, 190, 515, 671, 827, 983, 1139, 1295, 1451, 1607, 1763, 1919, 2075, 2231\}$, this line can not intersect with any line of K_{156} , then $K_{157} = K_{156} \cup \ell_{158} = \{\ell_1, \ell_2, \dots, \ell_{158}\}$ is a $(158, \ell)$ – span.

The line $\ell_{159} = \{172, 203, 360, 686, 843, 1000, 1157, 1314, 1458, 1615, 1772, 1929, 2086, 2243\}$, this line can not intersect with any line of K_{157} , then $K_{158} = K_{157} \cup \ell_{159} = \{\ell_1, \ell_2, \dots, \ell_{159}\}$ is a $(159, \ell)$ – span.

The line $\ell_{160} = \{173, 216, 374, 532, 859, 1004, 1162, 1320, 1478, 1636, 1794, 1939, 2097, 2255\}$, this line can not intersect with any line of K_{158} , then $K_{159} = K_{158} \cup \ell_{160} = \{\ell_1, \ell_2, \dots, \ell_{160}\}$ is a $(160, \ell)$ – span.

The line $\ell_{161} = \{174, 229, 388, 547, 693, 1021, 1180, 1339, 1485, 1644, 1803, 1962, 2108, 2267\}$, this line can not intersect with any line of K_{159} , then $K_{160} = K_{159} \cup \ell_{161} = \{\ell_1, \ell_2, \dots, \ell_{161}\}$ is a $(161, \ell)$ – span.

The line $\ell_{162} = \{175, 242, 402, 549, 709, 869, 1185, 1345, 1505, 1665, 1812, 1972, 2132, 2279\}$, this line can not intersect with any line of K_{160} , then $K_{161} = K_{160} \cup \ell_{162} = \{\ell_1, \ell_2, \dots, \ell_{162}\}$ is a $(162, \ell)$ – span.

The line $\ell_{163} = \{176, 255, 416, 564, 725, 873, 1034, 1364, 1512, 1673, 1834, 1982, 2143, 2291\}$, this line can not intersect with any line of K_{161} , then $K_{162} = K_{161} \cup \ell_{163} = \{\ell_1, \ell_2, \dots, \ell_{163}\}$ is a $(163, \ell)$ – span.

The line $\ell_{164} = \{177, 268, 430, 579, 741, 890, 1052, 1201, 1532, 1681, 1843, 1992, 2154, 2303\}$, this line can not intersect with any line of K_{162} , then $K_{163} = K_{162} \cup \ell_{164} = \{\ell_1, \ell_2, \dots, \ell_{164}\}$ is a $(164, \ell)$ – span.

The line $\ell_{165} = \{178, 281, 431, 594, 744, 907, 1057, 1220, 1370, 1702, 1852, 2015, 2165, 2328\}$, this line can not intersect with any line of K_{163} , then $K_{164} = K_{163} \cup \ell_{165} = \{\ell_1, \ell_2, \dots, \ell_{165}\}$ is a $(165, \ell)$ – span.

The line $\ell_{166} = \{179, 294, 445, 609, 760, 924, 1075, 1226, 1390, 1541, 1861, 2025, 2176, 2340\}$, this line can not intersect with any line of K_{164} , then $K_{165} = K_{164} \cup \ell_{166} = \{\ell_1, \ell_2, \dots, \ell_{166}\}$ is a $(166, \ell)$ – span.

The line $\ell_{167} = \{180, 307, 459, 624, 776, 928, 1093, 1245, 1397, 1549, 1714, 2035, 2187, 2352\}$, this line can not intersect with any line of K_{165} , then $K_{166} = K_{165} \cup \ell_{167} = \{\ell_1, \ell_2, \dots, \ell_{167}\}$ is a $(167, \ell)$ – span.

The line $\ell_{168} = \{181, 320, 473, 626, 792, 945, 1098, 1251, 1417, 1570, 1723, 1876, 2211, 2364\}$, this line can not intersect with any line of K_{166} , then $K_{167} = K_{166} \cup \ell_{168} = \{\ell_1, \ell_2, \dots, \ell_{168}\}$ is a $(168, \ell)$ – span.

The line $\ell_{169} = \{182, 333, 487, 641, 795, 962, 1116, 1270, 1424, 1578, 1732, 1899, 2053, 2376\}$, this line can not intersect with any line of K_{167} , then $K_{168} = K_{167} \cup \ell_{169} = \{\ell_1, \ell_2, \dots, \ell_{169}\}$ is a $(169, \ell)$ – span.

Finally, add the line $\ell_{170} = \{183, 346, 501, 656, 811, 966, 1121, 1276, 1444, 1599, 1754, 1909, 2064, 2219\}$, to K_{168} , this line can not intersect with any line of K_{168} , then $K_{169} = K_{168} \cup \ell_{170} = \{\ell_1, \ell_2, \dots, \ell_{170}\}$ is a $(170, \ell)$ – span, which is the maximum (K, ℓ) – span of $PG(3,13)$ can be obtained. Thus, K_{169} is called a spread of one hundred and seventy lines of $PG(3,13)$ which partitions $PG(3,13)$; that is every point of $PG(3,13)$ lies in exactly one line of K_{169} and every two lines of K_{169} are disjoint.

4.2 Theorem

In general, the total number of (K, ℓ) – span in $PG(3, q)$, $q \geq 2$ is $q^2 + 1$ [13].

4.3 New Examples of (K, ℓ) – span in $PG(3,13)$

A new example of $(1, \ell)$ – span in $PG(3,13)$ which are:
 $\{(1,0,0,0), (0,1,0,0), (1,1,0,0), (2,1,0,0), (3,1,0,0), (4,1,0,0), (5,1,0,0), (6,1,0,0), (7,1,0,0), (8,1,0,0), (9,1,0,0), (10,1,0,0), (11,1,0,0), (12,1,0,0)\}$.

A new example of $(2, \ell)$ – span in $PG(3,13)$ which are:
 $\{(0,0,1,0), (0,0,0,1), (0,0,1,1), (0,0,2,1), (0,0,3,1), (0,0,4,1), (0,0,5,1), (0,0,6,1), (0,0,7,1), (0,0,8,1), (0,0,9,1), (0,0,10,1), (0,0,11,1), (0,0,12,1)\}$.

A new example of $(3, \ell)$ – span in $PG(3,13)$ which are:
 $\{(1,0,1,0), (0,1,0,1), (1,1,1,1), (2,1,2,1), (3,1,3,1), (4,1,4,1), (5,1,5,1), (6,1,6,1), (7,1,7,1), (8,1,8,1), (9,1,9,1), (10,1,10,1), (11,1,11,1), (12,1,12,1)\}$.

So, there are a new examples of

$\{(4, \ell), (5, \ell), (6, \ell), \dots, (170, \ell)\}$ – span in $PG(3,13)$.

5. Computer program to find points and planes for $PG(3, 13)$, by using MATLAB 2019

4.1 Program (A) to find a complete (K,b) -span

```

clc;
clear;
q=13;
%profile on
tic
[Point,Plane]=PointPlane(q);
I=q+1;           % length of line
K=q^2+I;        % length of plane
N=q^3+K;        % length of projective space
M=N*I;
O=q^2+1;
P=Plane;
[m] = size(P,1);

```

```

% pre-allocate memory to the cell output
matrix (which is symmetric)
cellMtx = cell(m,m);
for u=1:m
    for v=u+1:m
        % determine the intersection between the
two rows
        cellMtx{u,v} = intersect(P(u,1:end),P(v,1:end));
        cellMtx{v,u} = cellMtx{u,v};
    end
end
S=[1:I];
q1=q+2;
k=0;
for j1=2:O
    index=q+j1;
    for i=1:m-1
        for j=i:m
            G=ismember(cellMtx{i,j},index);
            sum(G);
            if sum(G)>0
                if ~ismember(S,cellMtx{i,j})
                    S=[S; cellMtx{i,j}];
                end
            end
        end
    end
end
end
writematrix(S,'span.txt','Delimiter','tab');
toc

```

6. Conclusions

From the above results, we found that a $(170, \ell) - \text{span}$ is a maximum complete $(K, \ell) - \text{span}$ in $PG(3,13)$.

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البناء للامتداد $(K, \ell) - \text{التام في } PG(3, 13)$

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الخلاصة

الهدف الرئيسي من البحث هو ايجاد الناشر للفضاء الاسقاطي $PG(3, 13)$ بواسطة البناء للامتداد التام (K, ℓ) والذي يمثل تطبيقات الهندسة الجبرية بالفضاء الاسقاطي ثلاثي الابعاد $PG(3, q)$ يرهننا اكبر حجم للامتداد (K, ℓ) في $PG(3, 13)$ وهو الامتداد $(170, \ell)$ ويمثل الناشر.

الكلمات المفتاحية: الهندسة الجبرية، الفضاء الاسقاطي، البناء للامتداد التام.

