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Generalized *h*-Closed Sets in Topological Space

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Abstract

This study introduce a new type of closed sets in topology called Generalized *h*-closed sets (briefly, *gh*-closed) define as follow: $E \subseteq \chi$ be *gh*-closed set if $CL_h(E) \subseteq U$ whenever $E \subseteq U$ and U is open set in (χ, τ). The relation between *gh*-closed set and other classes of closed sets (*h*-closed, *g*-closed, $g\delta$-closed, θg-closed and αg-closed) are studied. Also, the notion of *gh*-continuous mapping on topological space is introduce and some properties are proved. Finally, the separation axioms have been studied.

Keywords:
h-closed set, *gh*-closed set, *gh*-continuous mapping, separation axioms.

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I. INTRODUCTION AND PRELIMINARIES

In 1970 Levine [4] first defined and investigated the idea of a generalized closed sets (briefly, *g*-closed) sets. Dontchev and Maki, in 1999 [2,3], presented the idea of "generalized -δ ($g\delta$), θ- generalized (θg) respectively" closed sets. Abbas [1] in 2020 introduced the concept of *h*-open set (*h-os*). A subset E of (χ, τ) is called (*h-os*) if for every non empty set U in χ, $U \neq \chi$ and $U \in \tau$, such that $E \subseteq Int(E \cup U)$. The complement of (*h-os*) is called *h*-closed set (*h-cs*). Our work is divided in to three sections. In the first, *gh*-closed sets (*gh-cs*) are defined and provided numerous instances, as well as analyze the link between *gh*-closed sets and various types of closed sets. The second section is devoted to introduce new class of mappings called *gh*-continuous mapping. The relationship between *gh*- continuous and some form of continuous mapping are investigated. In section three we study some classes of separating axioms spaces by explain the relation between them namely $T_o, T_1, T_2, T_{ogh}, T_{1gh}, T_{2gh}$.We denoted the topological spaces (χ, τ) and (γ, σ) simbly by χ and γ respectively, open sets(resp. closed sets) by(os),(cs) topological spaces by TS we recall the following definitions and notations. The closure (resp. interior) of a subset E of a topological space χ is denoted by *CL(E)(resp.Int (E))*.

Definition 1.1 A subset *E* of a TS χ is said to be

1. δ- closed set ($\delta - cs$) [2], if "$E = CL_\delta(E)$" where $CL_\delta(E) = \{x \in \chi: Int\left(CL(U)\right) \cap E \neq \emptyset ,\ x \in U \in \tau\}$.The complement of δ- closed set is called δ- open set ($\delta - os$).
2. θ- closed set ($\theta - cs$) [3], if "$E = CL_\theta(E)$" where $CL_\theta(E) = \{x \in \chi: CL(U) \cap E \neq \emptyset , x \in U \in \tau\}$. The complement of θ- closed set is called θ- open set ($\theta - os$).
3. *h*-open set(*h-os*) [1], if for every non empty set U in χ, $U \neq \chi$ and $U \in \tau$, such that $E \subseteq Int(E \cup U)$. "The family of all *h*-closed (resp. δ-closed, θ-closed) sets of a TS is denoted by *hc*(χ) (resp. δc(χ), θc(χ))".

Definition 1.2 A subset *E* of a TS χ is said to be

1. "Generalized δ-closed (briefly,$g\delta$-closed)($g\delta - cm$) [2], if $CL_\delta(E) \subseteq U$ whenever $E \subseteq U$ and U is (os) in χ
2. θ-Generalized closed (briefly,θg-closed)($\theta g - cm$) [3], if $CL_\theta(E) \subseteq U$ whenever $E \subseteq U$ and U is (os) in χ
3. α-Generalized closed (briefly,αg-closed) ($\alpha g - cm$) [6], if $CL_\alpha(E) \subseteq U$ whenever $E \subseteq U$ and U is (os) in χ".

4. "Generalized semi-closed (briefly, *gs*-closed) (gs $-$ cm) [7], if CL_s $(E) \subseteq U$ whenever $E \subseteq U$ and U is (*os*) in χ.
5. Generalized closed (briefly, *g*-closed) (g $-$ cm) [4], if "$CL(E) \subseteq U$" whenever $E \subseteq U$ and U is (*os*) in χ".

Theorem 1.3

1. *Each (δ- cs) in a TS is (gδ-cs) [2].*
2. *Each (θ- cs) in a TS is (θg-cs) [3].*
3. *Each (cs) in a TS is (h-cs) [1].*
4. *Each (cs) in a TS is (g-cs) [4].*

Definition 1.4 " Let χ and γ be a TS, a mapping $f: \chi \rightarrow \gamma$ is said to be

1. Generalized δ-continuous ($g\delta - contm$) [2] suppose that the inverse image of each closed subset of γ is ($g\delta$-*cs*) in χ.
2. θ-Generalized continuous ($\theta g{-}contm$) [3] suppose that the inverse image of each closed subset of γ is (θg-*cs*) in χ.
3. α -Generalized continuous ($\alpha g{-}contm$) [6] suppose that the inverse image of each closed subset of γ is (αg-*cs*) in χ.
4. Generalized semi-continuous ($gs{-}contm$) [7] suppose that the inverse image of each closed subset of γ is (*gs-cs*) in χ.
5. *h*-continuous ($h - contm$) [1] suppose that the inverse image of each open subset of γ is (*h-os*) in χ .
6. Generalized-continuous ($g{-}contm$) [4] suppose that the inverse image of each closed subset of γ is (*g-cs*) in χ".

Definition.1.5. A TS (χ ,τ) is called

1. $T_{0h}-$ space[1] if a, b are to distinct points in χ there exists (*h-os*) U such that either $a \in U$ and $b \notin U$, or $b \in U$ and $a \notin U$.
2. $T_{1h}-$ space [1] if $a, b \in \chi$ and $a \neq b$, there exists (*h-os*) *U*, *V* containing *a*, *b* respectively, such that either $b \notin U$ and $a \notin V$.
3. $T_{2h}-$ space[1] if *a*,*b* $\in \chi$ and $a \neq b$, there exists disjoint (*h-os*) *U*, *V* containing *a*, *b* respectively.

II. Generalized (*h-cs*) in *TS*

This section introduces a new closed set class called generalized (*h-cs*) and we investigate the relationship with closed set, (*h-cs*), (*g-cs*), (*α- cs*), (θg-*cs*), ($g\delta$-*cs*), (αg- *cs*) and (*gs-cs*).

Definition 2.1."A subset E of the TS χ is said to be generalized h-closed (briefly, *gh*-closed) set, if CL_h $(E) \subseteq U$ whenever $E \subseteq U$ and U is (*os*) in χ. The complement of *gh*-closed set is called *gh*-open (*gh-os*). The set of all family *gh*-closed denoted by *ghc(χ)*".

Example 2.2 .*If* $\chi = \{2,4,6\}$ and $\tau = \{\emptyset, \chi, \{4\}, \{4,6\}\}$. Then $hc(\chi) = ghc(\chi) = \{\emptyset, \chi, \{4\}, \{2\}, \{2,4\}, \{2,6\}\}$.

Theorem 2.3. *Each (h-cs) in any TS is* (gh - *cs).*

Proof. Suppose that E be (*h-cs*) in χ such that $E \subseteq U$, where U is (*os*). Since E is (*h-cs*) by proposition (2.2) [1], $CL_h(E) = E$, and $E \subseteq U$, therefore $CL_h(E) \subseteq U$. Hence E is (*gh-cs*) in χ . ■

As shown in the following example, the converse of the preceding theorem is not true in general.

Example 2.4. *If* $\chi = \{3, 6, 9\}$ and $\tau = \{\emptyset, \chi, \{9\}\}$ then

$hc(\chi) = \{\emptyset, \chi, \{9\}, \{3,6\}\}$

$ghc(\chi) = \{\emptyset, \chi, \{3\}, \{6\}, \{9\}, \{3,6\}, \{3,9\}, \{6,9\}\}$

Let *A={6}*. *A* is (*gh-cs*) but not (*h-c*) in .

Proposition 2.5. *Let E be subset of a space χ, then* $CL_h(E) \subseteq CL_\theta(E)$

Proof. Assume that E is a subset of the space χ and let $x \in CL_h(E)$. By Theorem(2.3) [1], $(\forall\, U \in ho(\chi))(x \in U \Longrightarrow E \cap U \neq \emptyset)$. Since (all (*os*) is (*h-os*)) [1], then $(\forall U \in \tau)(x \in U \Longrightarrow E \cap U \neq \emptyset)$ Since $U \subseteq CL(U)$, then $U \cap E \subseteq CL(U) \cap E$ for all (*os*) U contain *x*, so $CL(U) \cap E \neq \emptyset$ for all (*os*) U contain *x*. Therefore $x \in CL_\theta(E)$. Hence $CL_h(E) \subseteq CL_\theta(E)$. ■

Proposition 2.6. *Let E be subset of a space χ, then* $CL_h(E) \subseteq CL_\delta(E)$

Proof. Assume that E is a subset of the space χ and let $x \in CL_h(E)$. By Theorem(2.3) [1], $(\forall\, U \in ho(\chi))(x \in U \Longrightarrow E \cap U \neq \emptyset)$. Since (all (*os*) is (*h-os*)) [1], for all (*os*) U contain *x*, then $E \cap U \neq \emptyset$. Since $U = int(U) \subseteq int(cl(U))$, then $E \cap U \subseteq int(cl(U)) \cap E$ for all (*os*) U contain *x*. Therefore $int(Cl(U)) \cap E \neq \emptyset$, for all (*os*) U contain *x*, so $x \in CL_\delta(E)$. Hence $CL_h(E) \subseteq CL_\delta(E)$. ■

Theorem 2.7. *Each (θg-cs) in χ is* (gh - *cs).*

Proof. Suppose that E be (θg-*cs*) in χ such that $E \subseteq U$, where U is (*os*). Since E is (θg-*cs*) by proposition (2.5), then $CL_h(E) \subseteq CL_\theta(E) \subseteq U$, so we get $CL_h(E) \subseteq U$. Hence E is (*gh-cs*) in . ■

"The converse of the above the over is not true in general as shown in the following example".

Example 2.8. *If* $\chi = \{5,4,7\}$ and

$\tau = \{\emptyset, \chi, \{4\}, \{5,4\}, \{4,7\}\}$ then

$hc(\chi) = ghc(\chi) = \{\emptyset, \chi, \{7\}, \{5,4\}, \{5\}, \{4\}, \{5,7\}, \{4,7\}\}$,

$\theta o(\chi) = \{\emptyset, \chi\} = \theta c(\chi)$

Let *A={4}*. Here *A* is (*gh-cs*) in χ but not (θg-*cs*) because $CL_\theta(\{4\}) = \chi \not\subseteq \{4\}$.

Corollary 2.9. *Each (θ-cs) in χ is (gh-cs).*

Proof. By Theorem (1.3) (2) and Theorem (2.7). ■

Theorem 2.10. *All ($g\delta$-cs) in χ is* (gh - *cs).*

Proof. Suppose that E be ($g\delta$-*cs*) in χ such that $E \subseteq U$, where U is (*os*). Since E is ($g\delta$-*cs*), by proposition (2.6), then $CL_h(E) \subseteq CL_\delta(E) \subseteq U$, so we get $CL_h(E) \subseteq U$. Hence E is (*gh-cs*) in χ. ■

The converse is not true in general as shown in the following example.

Example 2.11. *If* $\chi = \{5, 2, 3\}$ and $\tau = \{\emptyset, \chi, \{3\}\}$ then

$hc(\chi) = \{\emptyset, \chi, \{3\}, \{5,2\}\}$
$ghc(\chi) = \{\emptyset, \chi, \{5\}, \{3\}, \{2\}, \{5,2\}, \{2,3\}, \{5,3\}\}$
Let *A={3}*, *A* is (*gh-cs*) in χ but not ($g\delta$*-cs*) .

Corollary 2.12. *Each (δ-cs) in χ is (gh-cs).*
Proof. By Theorem (1.3) (1) and Theorem (2.10). ■

Theorem 2.13. *Each (g-cs) in χ is (gh - cs).*
Proof. Suppose that *E* be (*g-cs*) in χ such that $E \subseteq U$, where *U* is (*os*). Since *E* is (*g-cs*) by Theorem (2.4) [1], then
$CL_h(E) \subseteq CL(E) \subseteq U$, so we get $CL_h(E) \subseteq U$. Hence *E* is (*gh-cs*) in χ. ■

"The converse is not true in general as shown in the following example".

Example 2.14. From Example (2.8)
$CL_h(\{4\})=\{4\}$, $CL(\{4\})=\chi$. Hence *{4}* is (*gh-cs*) in χ but not (*g- cs*).

Corollary 2.15. *Each (cs) in χ is (gh-cs).*
Proof. By Theorem (1.3) (4) and Theorem (2.13). ■

Remark 2.16. *There is no relationship between (α- cs), (αg- cs) and (gs -cs) with (gh-cs) as shown in the following examples.*

Example 2.17. If $\chi = \{2, 4, 6\}$. Now,
1. If $\tau = \{\emptyset, \chi, \{4\}, \{4,6\}\}$. Then *{6}*is (*α- cs*) and (*αg- cs*) but not (*gh- cs*). Also *{6}*is semi- closed and (*sg- cs*) but not (*gh- cs*).
2. If $\tau = \{\emptyset, \chi, \{2,4\}\}$. Then *{2}*is not (*α- cs*) and not (*αg- cs)* but (*gh- cs*). Also *{4}*is not semi- closed and not (*sg- cs*) but (*gh- cs*).

Remark 2.18. *As a result of the above, we have Fig.1 below.*

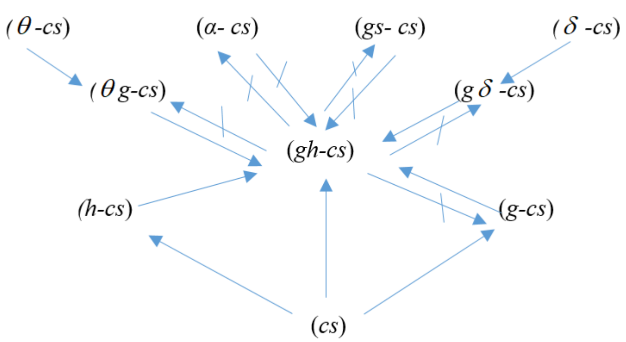

Fig. 1

Theorem 2.19. *If E is (gh-cs) and $E \subseteq B \subseteq CL_h(E)$, then B is (gh-cs).*
Proof. Assume that *U* be (*os*) in χ such that $B \subseteq U$, then $E \subseteq U$. Since *E* is (*gh-cs*). Then $CL_h(E) \subseteq U$, now $CL_h(B) \subseteq CL_h(CL_h(E)) = CL_h(E) \subseteq U$.Therefore *B* is (*gh-cs*). ■

Theorem 2.20 *Let $E \subseteq \gamma \subseteq \chi$ and suppose that E is(gh-cs) in χ,then E is(gh-cs)relative to γ.*
Proof. Because of this $E \subseteq \gamma \subseteq \chi$ and *E* is (*gh-cs*) in χ, to show that *E* is (*gh-c*) relative to γ. Let $E \subseteq U \cap \gamma$, where *U* is (*os*) in χ. Since *E* is (*gh-c*) $E \subseteq U$, implies $CL_h(E) \subseteq U$. As a consequence,$CL_h(E) \cap \gamma \subseteq U \cap \gamma$. Thus *E* is (*gh-cs*) relative to γ. ■

Theorem 2.21. *A (gh-cs) E is (h-cs) only if and only if $CL_h(E) \setminus E$ is (h-cs).*
Proof. If *E* is (*h-cs*), then $CL_h(E) \setminus E = \emptyset$. Conversely, suppose $CL_h(E) \setminus E$ is (*h-cs*) in χ . Since *E* (*gh-cs*). Then $CL_h(E) \setminus E$ there are no non empty closed sets in this collection in . Then $CL_h(E) \setminus E = \emptyset$. Hence *E* is (*h-cs*). ■

Definition 2.22. "A subset *E* of a space χ is called *gh*-open set (*gh-os*)" if $\chi \setminus E$ is (*gh-cs*). The family of all (*gh-os*) subset of a TS (χ, τ) is denoted by *gho*(χ).

All of the following results are true by using complement.

proposition 2.23. *The following statements are true:*
1. *Each (h-os) is (gh-os).*
2. *Each (os) is (gh-os).*
3. *Each (δ -os) is (gh-os) .*

Proof. By using the complement of the definition of (*gh-cs*).

III. *gh*- Continuous Mapping

The *gh*-continuous map on *TS* is introduced and studied in this section.

Definition 3.1."A mapping $f: \chi \to \gamma$ is said to be *gh*-continuous (*gh*-contm), if $f^{-1}(F)$ is (*gh-cs*) in χ for each (*cs*) *F* in γ" .

Theorem 3.2. *If $f: \chi \to \gamma$ is (contm) then it is (gh- contm)*
Proof. Assume that $f: \chi \to \gamma$ be (*contm*) and "let *F* be (*cs*) in γ. since *f* is (*contm*) then $f^{-1}(F)$ is (*cs*) in χ . By Corollary (2.15), then $f^{-1}(F)$ is (*gh-cs*) in χ. ■

The converse of the above the over is not true in general as shown in the following example".

Example 3.3. *If* $\chi = \gamma = \{2, 4, 6\}$ and
$\tau = \{\emptyset, \chi, \{4\}, \{4,6\}\}$,
$\sigma = \{\emptyset, \gamma, \{6\}, \{2,6\}\}$ then
$hc(\chi) = ghc(\chi) = \{\emptyset, \chi, \{2\}, \{4\}, \{2,4\}, \{2,6\}\}$
Assume that $f: \chi \to \gamma$ be an identity map. Then *f* is (*gh- contm)* , but *f* is not (*contm*), since for the (*cs*) *{4}* in γ, $f^{-1}(\{4\}) = \{4\}$ is not (*cs*) in χ .

Theorem 3.4. *Each (g- contm) is (gh-contm).*
Proof. Assume that $f: \chi \to \gamma$ be "(*g-contm*) and let *F* be (*cs*) in . since *f* is (*g-contm*) and by Theorem (2.13), then $f^{-1}(F)$ is (*gh-cs*) in χ. ■

As shown in the following example, the converse of the preceding theorem is not true in general".

Example 3.5. *If* $\chi = \gamma = \{7, 8, 9\}$ and
$\tau = \{\emptyset, \chi, \{8\}, \{8,9\}, \{7,8\}\}$,
$\sigma = \{\emptyset, \gamma, \{7\}, \{7,8\}\}$
$gc(\chi) = \{\emptyset, \chi, \{7\}, \{9\}, \{7,9\}\}$,
$hc(\chi) = ghc(\chi) = \{\emptyset, \chi, \{7\}, \{8\}, \{9\}, \{7,8\}, \{7,9\}, \{8,9\}\}$
Assume that $f: \chi \to \gamma$ be an identity map. Then *f* is (*gh- contm)*, but *f* is not (*g- contm*) , because {8, 9} is (*cs*) in γ but $f^{-1}(\{8,9\}) = \{8,9\}$ is not (*g-cs*) in χ .

Theorem 3.6. *All (g δ- contm) is (gh-contm).*
Proof. Suppose that $f: \chi \to \gamma$ be (*g δ -contm)* and *F* be (*cs*) in γ. since *f* is (*g δ -contm)* then $f^{-1}(F)$ is (*g δ -cs*)

in χ and by Theorem (2.10), then $f^{-1}(F)$ is (*gh-cs*) in χ . ■

The converse of the above the over is not true in general as shown in the following example.

Example 3.7. *If* $\chi = \gamma = \{2, 4, 6\}$ and

$\tau = \{\emptyset, \chi, \{6\}\}$, $\sigma = \{\emptyset, \gamma, \{2,4\}\}$

$hc(\chi) = \{\emptyset, \chi, \{6\}, \{2,4\}\}$

$ghc(\chi) = \{\emptyset, \chi, \{2\}, \{4\}, \{6\}, \{2,4\}, \{2,6\}, \{4,6\}\}$

Suppose that $f: \chi \to \gamma$ be an identity map.Then f is (*gh- contm*), but f is not (*g* δ *- contm)*, because $\{6\}$is (*cs*) in γ but $f^{-1}(\{6\}) = \{6\}$ is not (*g* δ *-cs*) in χ.

Theorem 3.8. *Each (θ g- contm) is (gh-contm).*

Proof. Suppose that $f: \chi \to \gamma$ be (θ *g-contm)* and F be (*cs*) in γ. "since f is (θ *g-contm*) then $f^{-1}(F)$ is (θ *g-cs*) in χ and by Theorem (2.7), then $f^{-1}(F)$ is (*gh-cs*) in χ. ■

As shown in the following example, the converse of the preceding theorem is not true in general".

Example 3.9. If $\chi = \gamma = \{3, 4, 5\}$ and

$\tau = \{\emptyset, \chi, \{3\}, \{4\}, \{3,4\}\}$,

$\sigma = \{\emptyset, \gamma, \{4\}, \{3,4\}\}$

$hc(\chi) = \{\emptyset, \chi, \{5\}, \{3,5\}, \{4,5\}\}$

$ghc(\chi) = \{\emptyset, \chi, \{5\}, \{3,5\}, \{4,5\}, \{3\}\}$,

$\theta gc(\chi) = \{\emptyset, \chi, \{3,5\}\}$

Suppose that $f: \chi \to \gamma$ be an identity map. Then f is (*gh- contm*) , but f is not (θ *g- contm*), because $\{5\}$ is (*cs*) in γ but $f^{-1}(\{5\}) = \{5\}$ is not (θ *g-cs*) in χ .

Remark 3.10. *As a result of the above, we have Fig. 2 below.*

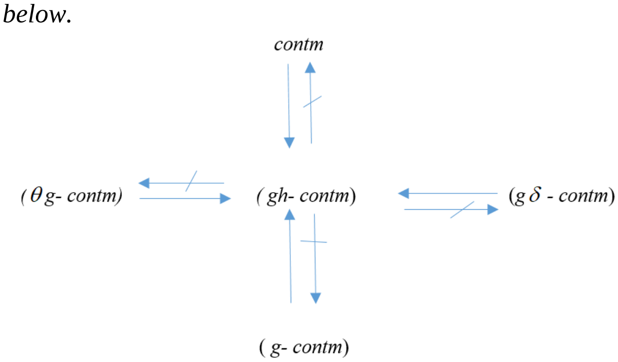

Fig. 2

Remark 3.11. *There is no relationship between (αg-contm) and (gs -contm) with (gh-contm) as shown in the following examples.*

Example 3.12. If $\chi = \gamma = \{2, 4, 6\}$ and let $f: \chi \to \gamma$ be an identity map. Now,

1. If $\tau = \{\emptyset, \chi, \{2,4\}\}$, $\sigma = \{\emptyset, \gamma, \{4,6\}\}$.

Then f is (*gh- contm*) , but f is not (α*g- contm*) and not (*gs-cnntm)*, because $\{2\}$ is (*cs*) in γ but $f^{-1}(\{2\}) = \{2\}$ is not (α*g-cs*) and not (*gs- cs*) in χ.

2. If $\tau = \{\emptyset, \chi, \{4\}, \{4,6\}\}$, $\sigma = \{\emptyset, \gamma, \{2,4\}\}$.

Then f is (α*g- contm*) and (*gs-cnntm*), but f is not (*gh-contm*) because $\{6\}$ is (cs) in γ but $f^{-1}(\{6\}) = \{6\}$ is not (*gh-cs*) in χ.

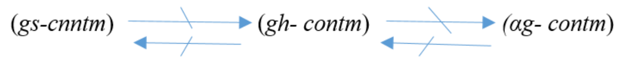

Fig. 3

Definition 3.13. A mapping $f: \chi \to \gamma$ is said to be g*h*-irresolute(*gh-irrm*), if $f^{-1}(F)$ is (*gh- cs*) in χ for each (*gh- cs*) F of γ .

Theorem 3.14. *Each (gh- irrm) is (gh-contm).*

Proof. It's obvious. ■

The converse of the above the over is not true in general as shown in the following example.

Example 3.15. *If* $\chi = \gamma = \{2, 3, 4\}$ and

$\tau = \{\emptyset, \chi, \{4\}, \{3\}, \{3,4\}\}$, $\sigma = \{\emptyset, \gamma, \{3\}, \{3,4\}\}$

$hc(\gamma) = ghc(\gamma) = \{\emptyset, \gamma, \{2\}, \{3\}, \{2,4\}, \{2,3\}\}$,

$hc(\chi) = ghc(\chi) = \{\emptyset, \chi, \{2\}, \{2,4\}, \{2,3\}\}$

Suppose that $f: \chi \to \gamma$ be an identity map. Then f is (*gh- contm*) , but f is not (*gh-irrm*) because $\{3\}$ is (*gh-cs*) in γ but $f^{-1}(\{3\}) = \{3\}$ is not (*gh-cs*) in χ .

Theorem 3.16. *A combination of two (gh- irrm) is also (gh- irrm).*

Proof. Suppose that $f: \chi \to \gamma$ and $H: \gamma \to Z$ be any two (*gh- irrm*) . let F be any (*gh-cs*) in *Z.* Since H is (*gh- irrm*), then $H^{-1}(F)$ is (*gh-cs*) in γ . Since, f is (*gh- irrm*) then $f^{-1}(H^{-1}(F)) = (H \circ f)^{-1}(F)$ *is (gh-cs)* in χ. Therefore *HoF:* $\chi \to Z$ is (*gh- irrm*). ■

Definition 3.17. "A mapping $f: \chi \to \gamma$ is said to be strongly g*h*- continuous , suppose that the inverse image of each (*gh-cs*) in γ is closed in χ" .

Theorem 3.18. *All strongly (gh- contm) it is (contm).*

Proof. Assume the following scenario: f is strongly (*gh- contm*). Let F be (cs) in γ. Since (each (*cs*) is (*gh-cs*)), then F is (*gh-cs*) in γ. Since f is strongly (*gh- contm*), $f^{-1}(F)$ is (*cs*) in . Therefore f is (*contm*). ■

As shown in the example (3.15), the converse of the preceding theorem is not true in general.

Suppose that $f: \chi \to \gamma$ be an identity map. Then f is continuous , but f is not strongly *gh-* continuous because $\{3\}$ is (*gh-cs*) in γ but $f^{-1}(\{3\}) = \{3\}$ is not (*cs*) in χ .

Theorem 3.19. *Each strongly gh- continuous map it is gh-continuous.*

Proof. Assume the following scenario: f is strongly (*gh- contm*). Let F be (*cs*) in γ. Since (all (*cs*) is (*gh-cs*)), then F is (*gh-cs*) in γ. Since f is strongly (*gh- contm*), $f^{-1}(F)$ is (*cs*) in . Since (all (*cs*) is (*gh-cs*)) , then $f^{-1}(F)$ is (*gh-cs*) in χ . ■

As shown in the example (3.15), the converse of the preceding theorem is not true in general.

Suppose that $f: \chi \to \gamma$ be an identity map. Then f is (*gh-contm*) , but f is not strongly *gh-* continuous , since for the (*gh-cs*) $\{3\}$ in γ, $f^{-1}(\{3\}) = \{3\}$ is not closed in χ.

IV. gh- Closed Sets and Separating Axioms

In this section, we introduce and study a new type of separating axioms spaces for (*gh-os*) in TS.

Definition.4.1. A TS (χ ,τ) is called

1. T_{0gh} - space if *a, b* are to distinct points in χ there

exists (*gh-os*) *U* such that either $a \in U$ and $b \notin U$, or $b \in U$ and $a \notin U$.

2. T_{1gh} - space if $a, b \in \chi$ and $a \neq b$, there exists (*gh-os*) *U, V* containing *a, b* respectively, such that either $b \notin U$ and $a \notin V$.
3. T_{2gh}-space if $a, b \in \chi$ and $a \neq b$, there exists disjoint (*gh-os*) *U, V* containing *a, b* respectively.

Theorem.4.2. *Each T_0 - space is T_{0gh} - space.*

Proof: Assume that χ be T_0 - space and *a, b* be two distinct points in χ. Since χ is T_0 - space. Then there is one an (*os*) *U* in χ such that $a \in U$ and $b \notin U$ or $b \in U$ and $a \notin U$. Since (each (*os*) is (*gh-os*)) proposition 2.23(2). Then *U* is (*gh-os*) in χ such that $a \in U$ and $b \notin U$ or $b \in U$ and $a \notin U$. Hence χ is T_{0gh} - space. ■

The converse is not true in general as shown in the following example.

Example.4.3. If $\chi = \{1, 2, 3\}$, $\tau = \{\emptyset, \chi, \{1, 2\}$. Then (χ, τ) is not T_0 -space, but $(\chi, gho(\chi))$ is T_{0gh} - space.

Theorem.4.4. *Each T_{0h} - space is T_{0gh} - space.*

Proof: Assume that χ be T_{0h} - space and *a, b* be two distinct points in χ. Since χ is T_{0h} - space. Then there is one an (*h-os*) *U* in χ such that $a \in U$ and $b \notin U$ or $b \in U$ and $a \notin U$. Since (each (*h-os*) is (*gh-os*)) proposition 2.23(1). Then *U* is (*gh-os*) in χ such that $a \in U$ and $b \notin U$ or $b \in U$ and $a \notin U$. Hence χ is T_{0gh} - space. ■

"The converse of the above the over is not true in general as shown in the following example".

Example.4.5. Let $\chi = \{3, 6, 9\}$, $\tau = \{\emptyset, \chi, \{9\}\}$. Then $(\chi, ho(\chi))$ is not T_{0h} -space, but $(\chi, gho(\chi))$ is T_{0gh} -space.

Theorem.4.6"*Each T_1- space is T_{1gh}-space*

Proof: Suppose that χ be T_1 - space and *a, b* be two distinct points in χ. Since χ is T_1- space. Then there exist two (*os*) *U, V* in χ such that $a \in U$, $b \notin U$ and $b \in V$ and $a \notin V$. Since (each (*os*) is (*gh-os*)) proposition 2.23(2).Then *U, V* are two (*gh-os*) in χ such that $a \in U$ and $b \notin U$ and $b \in V$ and $a \notin V$. Hence χ is T_{1gh} -space. ■

As shown in the following example, the converse of the preceding theorem is not true in general".

Example 4.7. *If* $\chi = \{2, 3, 5\}$, $\tau = \{\emptyset, \ , \{2\}, \{2, 3\}, \{2, 5\}\}$. Then (χ, τ) is not T_1- space, but $(\chi, gh\ o(\chi))$ is T_{1gh} - space.

Theorem.4.8. *Each T_{1h}- space is T_{1gh} −space*

Proof: Suppose that χ be T_{1h} - space and *a, b* be two distinct points in χ. Since χ is T_{1h} - space. Then there exist two (*h-os*) *U, V* in χ such that $a \in U$, $b \notin U$ and $b \in V$ and $a \notin V$. Since (each (*h-os*) is (*gh-os*)) proposition 2.23(1). Then *U, V* are two (*gh-os*) in χ such that $a \in U$ and $b \notin U$ and $b \in V$ and $a \notin V$.Hence χ is T_{1gh} – space. ■

As shown in the example (4.5), the converse of the preceding theorem is not true in general. $(\chi, ho(\chi))$ is not T_{1h} -space, but $(\chi, gho(\chi))$ is T_{1gh} - space.

Theorem.4.9. *Each T_1- space is T_{0gh} - space.*

Proof: Since each T_1- space is T_0– space and each T_0-space is T_{0gh} – space. Hence T_1- space is T_{0gh} - space. ■

As shown in the example (4.5), the converse of the preceding theorem is not true in general. (χ, τ) is not T_1-space, but $(\chi, gh\ o(\chi))$ is T_{0gh} - space.

Theorem.4.10. *Each T_{1gh} - space is T_{0gh} - space.*

Proof: It's obvious. ■

Theorem.4.11. *Each T_2 - space is T_{2gh}– space.*

Proof: Suppose that χ be T_2 - space and *a, b* be two distinct points in χ. Since χ is T_2 - space. Then there exists disjoint (*os*) *U, V* containing *a, b* respectively. From proposition 2.23(2) each (*os*) is(*gh-os*). Then *U, V* are disjoint (*gh-os*) containing *a, b* respectively. Hence χ is T_{2gh} – space. ■

As shown in the example (4.5), the converse of the preceding theorem is not true in general. (χ, τ) is not T_2-space, but $(\chi, gh\ o(\chi))$ is T_{2gh} - space.

Theorem.4.12. *"Each T_{2h} - space is T_{2gh} – space.*

Proof: Suppose that χ be T_{2h}- space and *a, b* be two distinct points in χ. Since χ is T_{2h} - space. Then there exists disjoint (*h- os*) *U, V* containing *a, b* respectively. From proposition 2.23(1) each (*h- os*) is(*gh-os*). Then *U, V* are disjoint (*gh-os*) containing *a, b* respectively. Hence χ is T_{2gh} – space. ■

As shown in the example (4.5), the converse of the preceding theorem is not true in general. $(\chi, ho(x))$ is not T_{2h}- space, but $(\chi, gh\ o(\chi))$ is T_{2gh}- space".

Theorem.4.13. *Each T_{2gh} - space is T_{1gh} – space.*

Proof: It's obvious. ■

Conclusion

The generalized h-closed set is not topological space and every closed, h-closed and g-closed sets is generalized h-closed.

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مجاميع مغلقة من النمط - h معممة في فضاء تبولوجي

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الخلاصة:

هذه الدراسة تقدم نوعا جديدا من مجموعات مغلقة في تبولوجيا تدعى مجموعات مغلقة من النمط - h معممة وباختصار *(gh-closed)* تعرف على النحو التالي: $E \subseteq \chi$ تكون مجموعة مغلقة من النمط – h معممة اذا كان $CL_h\,(E) \subseteq U$ عندما $E \subseteq U$ و U مجموعة مفتوحة في (χ, τ) . العلاقة بين مجموعة مغلقة من النمط – h معممة ومجاميع مغلقة اخرى (مغلقة من النمط - h ، مغلقة معممة، مغلقة من النمط - δ معممة ، مغلقة من النمط- θ معممة ومغلقة من النمط – α معممة) تم دراستها. ايضا، التطبيق المستمر من النمط –h المعمم على فضاء تبولوجي تم تقديمه وبعض خواص تم برهانها. اخيرا، بديهيات الفصل تم دراستها.

الكلمات المفتاحية: مجموعة مغلقة من النمط – h، مجموعة مغلقة من النمط – h معممة، تطبيق مستمر من النمط - h معمم وبديهيات الفصل.