

ii- Open Set in Bitopological Spaces

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ABSTRACT

In this paper, we define *ii*-open set in bitopological space as follows: Let (X, τ_1, τ_2) be a bitopological space, a subset A of X is said to be $(\tau_1\tau_2 - ii\text{-open set})$ if there exist $U, V \neq \emptyset, X$ and $U, V \in \tau_1 \cup \tau_2$ such that:

1. $A = \text{int}^1(U)$ or $A = \text{int}^2(V)$
2. $A \subseteq \text{CL}^1(A \cap U)$ or $A \subseteq \text{CL}^2(A \cap V)$

We study some characterizations and properties of this class. Also, we explain the relation between *ii*-open sets and open sets, *i*-open sets and α -open sets in bitopological space. Furthermore, we define *ii*-continuous mapping on bitopological spaces with some properties.

Keywords: α -open set, *i*-open set, *ii*-open set, bitopological space.

المجموعة المفتوحة من النمط - *ii* في الفضاءات التوبولوجية الثنائية

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الملخص

في هذا البحث، عرفنا المجموعة المفتوحة من النمط - *ii* في الفضاء التوبولوجي الثنائي على النحو التالي: ليكن (X, τ_1, τ_2) فضاء توبولوجي ثنائي، المجموعة الجزئية A من X تسمى (مجموعة مفتوحة من

النمط- *ii*) - إذا وجد $U, V \neq \emptyset, X$ و $U, V \in \tau_1 \cup \tau_2$ بحيث ان:

1. $A = \text{int}^1(U)$ or $A = \text{int}^2(V)$
2. $A \subseteq \text{CL}^1(A \cap U)$ or $A \subseteq \text{CL}^2(A \cap V)$

درسنا بعض الصفات والخواص لهذا الصنف. ايضا وضحنا العلاقة بين المجاميع المفتوحة من النمط - *ii* والمجاميع المفتوحة، المجاميع المفتوحة من النمط - *i* والمجاميع المفتوحة من النمط- α في الفضاء التوبولوجي الثنائي. اضافة الى ذلك، عرفنا التطبيق المستمر من النمط - *ii* على الفضاءات التوبولوجية الثنائية مع بعض خصائصه.

الكلمات المفتاحية: مجموعة مفتوحة من النمط - α ، مجموعة مفتوحة من النمط -*i*، مجموعة مفتوحة من النمط -*ii*، فضاء توبولوجي ثنائي.

Introduction

The concept of bitopological space was introduced by Kelly [5] in 1963. A set equipped with two topologies is called a bitopological space and is denoted by (X, τ_1, τ_2) , where $(X, \tau_1), (X, \tau_2)$ are two topological spaces. Since then many authors have contributed to the development of various bitopological properties. Mohammed and Abdullah in 2019 [1], introduced the concept of *ii*-open sets as follows: A subset A of a topological space (X, τ) is said to be *ii*-open set if there exists an open set G in the topology of X , such that: $G \neq \emptyset, X, A \subseteq CL(A \cap G)$ and $int(A)=G$. later, the same authors in [2] studied topological properties of *ii*-derived, *ii*-border, *ii*-frontier, *ii*-exterior of a set by using the concept of *ii*-open sets. It is shown in [1] that each of $\tau \subset \tau^{ii}$ (the family of all *ii*-open sets) and τ^{ii} is a topology on X . Following Mahdi in [4] and Askandar and Mohammed in [3], we introduce the concept of *ii*-open set in bitopological space as follows: Let (X, τ_1, τ_2) be a bitopological space, a subset A of X is said to be $(\tau_1\tau_2 - ii\text{-open set})$ if there exist $U, V \neq \emptyset, X$ and $U, V \in \tau_1 \cup \tau_2$ such that:

1. $A=int^1(U)$ or $A=int^2(V)$
2. $A \subseteq CL^1(A \cap U)$ or $A \subseteq CL^2(A \cap V)$

The aim of this paper is to study $\tau_1\tau_2 - ii\text{-open set}$ with their properties and characterizations in bitopological space (X, τ_1, τ_2) . Further, we study the comparison of this class with other classes namely: $\tau_1\tau_2 - \text{open}$, $\tau_1\tau_2 - i\text{-open}$ and $\tau_1\tau_2 - \alpha\text{-open}$ sets. This work consists of three sections. In the first one, we define *ii*-open sets in bitopological spaces and we give many related examples. In the second section, we discuss the relationship between *ii*-open sets with open, *i*-open and α -open sets in bitopological spaces. In the third section, we define *ii*-continuous mapping on bitopological spaces and we study the relation between this type of mapping with continuous, α -continuous [7] and *i*-continuous mapping [3].

1. *ii*-Open Sets in Bitopological Spaces

In this section, we recall the following definitions, which are useful in the sequel.

Definition 1.1 [3,6,7]: Let (X, τ_1, τ_2) be a bitopological space, a subset A of X is said to be

1. $(\tau_1\tau_2 - \text{open set})$ [6] if $A \in \tau_1 \cup \tau_2$
2. $(\tau_1\tau_2 - \alpha\text{-open set})$ [7] if $A \subseteq \tau_1 - int(\tau_2 - cl(\tau_1 - int(A)))$
3. $(\tau_1\tau_2 - i\text{-open set})$ [3] if there exists $\tau_1 - \text{open set } (G \neq \emptyset, X)$ such that $A \subseteq \tau_2 - CL(A \cap G)$. The complement of $(\tau_1\tau_2 - i\text{-open set})$ is called $(\tau_1\tau_2 - i\text{-closed set})$.

Definition 1.2: Let (X, τ_1, τ_2) be a bitopological space, a subset A of X is said to be $\tau_1\tau_2 - ii\text{-open set}$ if there exists $U, V \neq \emptyset, X$ and $U, V \in \tau_1 \cup \tau_2$ such that:

1. $A=int^1(U)$ or $A=int^2(V)$
2. $A \subseteq CL^1(A \cap U)$ or $A \subseteq CL^2(A \cap V)$

The complement of $\tau_1\tau_2 - ii\text{-open set}$ is called $\tau_1\tau_2 - ii\text{-closed set}$.

Example 1.1: Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$

$$\tau_1 \cup \tau_2 = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}\}$$

$$\tau_1 - \text{closed sets are: } \{\emptyset, \{b, c\}, \{a, c\}, \{c\}, X\}$$

$$\tau_2 - \text{closed sets are: } \{\emptyset, \{b, c\}, \{a, b\}, \{b\}, X\}$$

1. $A = \{a\}$ $U = \{a\}$ $V = \{a, b\}$
 $\{a\} = \text{int}^1\{a\} = \{a\}$ $\{a\} \subseteq CL^1(\{a\} \cap \{a\}) = \{a, c\}$
 2. $A = \{b\}$ $U = \{b\}$ $V = \{a, b\}$
 $\{b\} = \text{int}^1\{b\} = \{b\}$ $\{b\} \subseteq CL^1(\{b\} \cap \{b\}) = \{b, c\}$
 3. $A = \{c\}$ $U = \{c\}$ $V = \{a, c\}$
 $\{c\} = \text{int}^1\{c\} = \emptyset$ $\{c\} = \text{int}^2\{c\} = \{c\}$
 $\{c\} \subseteq CL^1(\{c\} \cap \{c\}) = \{c\}$
 4. $A = \{a, b\}$ $U = \{a, b\}$ $V = \{b\}$
 $\{a, b\} = \text{int}^1\{a, b\} = \{a, b\}$ $\{a, b\} \subseteq CL^1(\{a, b\} \cap \{a, b\}) = X$
 5. $A = \{a, c\}$ $U = \{a, c\}$ $V = \{c\}$
 $\{a, c\} = \text{int}^1\{a, c\} \neq \{a\}$ $\{a, c\} = \text{int}^2\{a, c\} = \{a, c\}$
 $\{a, c\} \subseteq CL^1(\{a, c\} \cap \{a, c\}) = \{a, c\}$
 6. $A = \{b, c\}$ $U = \{c\}$ $V = \{b\}$
 $\{b, c\} = \text{int}^1\{b, c\} \neq \{b\}$ $\{b, c\} \subseteq CL^1(\{b, c\} \cap \{b\}) = \{b, c\}$
 $\{b, c\} = \text{int}^2\{b, c\} \neq \{c\}$
- $\tau_1 \tau_2 - ii - \text{open sets are: } \{ \emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\} \}$
 $\tau_1 \tau_2 - ii - \text{closed sets are: } \{ \emptyset, X, \{b, c\}, \{a, c\}, \{a, b\}, \{c\}, \{b\} \}$

Example 1.2: Let $X = \{a, b, c, d\}$,

$$\tau_1 = \{ \emptyset, X, \{a\}, \{b, c, d\}, \tau_2 = \{ \emptyset, X, \{a\}, \{c\}, \{a, c\} \}$$

$$\tau_1 \cup \tau_2 = \{ \emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, c, d\} \}$$

$$\tau_1 - \text{closed sets are: } \{ \emptyset, X, \{b, c, d\}, \{a\} \}$$

$\tau_2 - \text{closed sets are: } \{ \emptyset, X, \{b, c, d\}, \{a, b, d\}, \{b, d\} \}$. By using the definition (1.2) we get the $\tau_1 \tau_2 - ii - \text{open sets are: } \{ \emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, c, d\} \}$

2. Some Properties of ii-Open Sets in Bitopological Spaces.

Definition 2.1: Let (X, τ_1, τ_2) be a bitopological space and let A be a subset of X . Recall that:

1. The intersection of all *ii*-closed sets containing A is called *ii*-closure of A , denoted by $\tau_1 \tau_2 - \text{clii}(A)$.
2. The union of all *ii*-open sets contained in A is called *ii*-interior of A , denoted by $\tau_1 \tau_2 - \text{intii}(A)$.

Theorem 2.2: Let (X, τ_1, τ_2) be a bitopological space and $A \subseteq X$ then, the following are true:

1. $\tau_1 \tau_2 - \text{intii}(A)$ is $\tau_1 \tau_2 - ii - \text{open set}$.
2. $\tau_1 \tau_2 - \text{clii}(A)$ is $\tau_1 \tau_2 - ii - \text{closed set}$.
3. If A is $\tau_1 \tau_2 - ii - \text{closed}$ then, $A = \tau_1 \tau_2 - \text{clii}(A)$.
4. If A is $\tau_1 \tau_2 - ii - \text{open}$ then, $A = \tau_1 \tau_2 - \text{intii}(A)$.
5. If $A \subset B$ then, $\tau_1 \tau_2 - \text{clii}(A) \subset \tau_1 \tau_2 - \text{clii}(B)$.
6. $X - \tau_1 \tau_2 - \text{clii}(A) = \tau_1 \tau_2 - \text{intii}(X - A)$.
7. $X - \tau_1 \tau_2 - \text{intii}(A) = \tau_1 \tau_2 - \text{clii}(X - A)$.
8. $x \in \tau_1 \tau_2 - \text{clii}(A)$ iff for all *ii*-open set G containing A , $A \cap G \neq \emptyset$.

Proof: (1), (2), (3), (4) it follows from the definition (1.2).

5. Let $A \subset B$. Since $\tau_1 \tau_2 - clii(A)$ is the smallest ii -closed set containing A , then $A \subset \tau_1 \tau_2 - clii(A)$, So $B \subset \tau_1 \tau_2 - clii(B)$. We get $\tau_1 \tau_2 - clii(A) \subset \tau_1 \tau_2 - clii(B)$.
6. $X \in X - \tau_1 \tau_2 - clii(A)$ if $x \notin X - \tau_1 \tau_2 - clii(A)$, then for all $G \in \tau_1 \tau_2 - iio(X)$ containing x , then $A \cap G \neq \emptyset$ if and only if $x \in G \subset X - A$ if and only if $x \in \tau_1 \tau_2 - intii(X - A)$.
7. In a similar way to proof (6).
8. Let $x \in \tau_1 \tau_2 - clii(A)$ we shall prove that $A \cap G \neq \emptyset$ for all ii -open set G containing x . Let there exists set G is $\tau_1 \tau_2 - ii - open$ containing x such that $A \cap G = \emptyset$, so $A \subseteq G^c$ and G^c is $\tau_1 \tau_2 - ii - closed$ set, therefore $\tau_1 \tau_2 - clii(A) \subset \tau_1 \tau_2 - clii(G^c)$. Since $x \in \tau_1 \tau_2 - clii(A)$ this implies that $x \in \tau_1 \tau_2 - clii(G^c)$. Since G^c is $\tau_1 \tau_2 - ii - closed$ set by theorem(2.2)(3) $x \in G^c$ means that $x \notin G$. Therefore $A \cap G \neq \emptyset$ for all $\tau_1 \tau_2 - open$ sets G containing x .

Converse: Let $A \cap G \neq \emptyset$ for all G is $\tau_1 \tau_2 - ii - open$ set containing x , we shall prove that $x \in \tau_1 \tau_2 - clii(A)$. Let $x \notin \tau_1 \tau_2 - clii(A)$ by definition (2.1), there exists $\tau_1 \tau_2 - ii - closed$ set F containing A such that $x \notin F$, so $x \in F^c$ and F^c is $\tau_1 \tau_2 - ii - open$ set. Therefore $A \cap F^c = \emptyset$. This implies a contradiction. Therefore $x \in \tau_1 \tau_2 - clii(A)$. ■

Theorem 2.3: Every open set in bitopological space (X, τ_1, τ_2) is $\tau_1 \tau_2 - ii - open$

Proof: Let G be open set in bitopological space (X, τ_1, τ_2) :

1. If $G \in \tau_1$ we shall prove that G is $\tau_1 \tau_2 - ii - open$. Put $U=G$, we get
 - i. $G \subseteq CL^1(G \cap G) \subseteq CL^1(G)$
 - ii. $G = int^1(G)$ because $G \in \tau_1$
2. If $G \in \tau_2$, put $V=G$, we get
 - iii. $G \subseteq CL^2(G \cap G) \subseteq CL^2(G)$
 - iv. $G = int^2(G)$ because $G \in \tau_2$

By using the definition (1.2) we get in both cases G is $\tau_1 \tau_2 - ii - open$ set. The converse of the above theorem is not true in general as shown in the following example:

Example 2.4: Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{b\}, \{a, c\}\}$, $\tau_2 = \{\emptyset, X, \{c\}, \{a, b\}\}$

$$\tau_1 \cup \tau_2 = \{\emptyset, X, \{b\}, \{c\}, \{a, c\}, \{a, b\}\}$$

$$\tau_1 - closed \text{ sets are: } \{\emptyset, X, \{a, c\}, \{b\}\}$$

$\tau_2 - closed \text{ sets are: } \{\emptyset, X, \{c\}, \{a, b\}\}$. By using the definition (1.2) we get the $\tau_1 \tau_2 - ii - open \text{ sets are: } \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{a, b\}\}$.

We see that $A = \{a\}$ is $\tau_1 \tau_2 - ii - open \text{ set}$ but A is not $\tau_1 \tau_2 - open \text{ set}$.

Remark 2.5: From the definition (1.2) we get every $\tau_1 \tau_2 - ii - open$ set in bitopological space is $\tau_1 \tau_2 - i - open$ set. The converse of the above remark is not true in general as shown in the following example:

Example 2.6: Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$, $\tau_2 = \{\emptyset, X, \{a\}, \{a, b\}\}$

$$\tau_1 \cup \tau_2 = \{\emptyset, X, \{a\}, \{a, b\}\}$$

$$\tau_1 - closed \text{ sets are: } \{\emptyset, X, \{b, c\}\}$$

$$\tau_2 - closed \text{ sets are: } \{\emptyset, X, \{b, c\}, \{c\}\}$$

$$\tau_1 \tau_2 - i - open \text{ sets are: } \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$$

$$\tau_1 \tau_2 - ii - open \text{ sets are: } \{\emptyset, X, \{a\}, \{a, b\}\}$$

$A = \{a, c\}$ is $\tau_1 \tau_2 - i - open \text{ set}$ but A is not $\tau_1 \tau_2 - ii - open \text{ set}$.

Remark 2.7: We see that every $\tau_1 \tau_2 - \alpha - open\ set$ in bitopological space is not necessary $\tau_1 \tau_2 - ii - open\ set$, but this relation is true in topological spaces, that is, every $\alpha - open\ set$ in topological space is $ii - open\ set$ Theorem(2.12)[1]. Also, we see that every $\tau_1 \tau_2 - ii - open\ set$ in bitopological space is not necessary $\tau_1 \tau_2 - \alpha - open\ set$, it means that:

$$\tau_1 \tau_2 - \alpha - open\ set \begin{array}{c} \xrightarrow{\text{---}} \\ \xleftarrow{\text{---}} \end{array} \tau_1 \tau_2 - ii - open\ set$$

As in the following examples:

Example 2.8:

Let $X = \{a, b, c, d\}$, $\tau_1 = \{\emptyset, X, \{a\}, \{b, c, d\}\}$, $\tau_2 = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$

$\tau_1 \cup \tau_2 = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, c, d\}\}$

$\tau_1 - closed\ sets\ are: \{\emptyset, X, \{b, c, d\}, \{a\}\}$

$\tau_2 - closed\ sets\ are: \{\emptyset, X, \{b, c, d\}, \{a, b, d\}, \{b, d\}\}$

$\tau_1 \tau_2 - ii - open\ sets\ are: \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, c, d\}\}$

$\tau_1 \tau_2 - \alpha - open\ sets\ are: \{\emptyset, X, \{a\}, \{b, c, d\}\}$

$A=\{c\}$ is $\tau_1 \tau_2 - ii - open\ set$ but A is not $\tau_1 \tau_2 - \alpha - open\ set$

Example 2.9: Let $X = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$, $\tau_2 = \{\emptyset, X, \{a\}, \{a, b\}\}$

$\tau_1 \cup \tau_2 = \{\emptyset, X, \{a\}, \{a, b\}\}$

$\tau_1 - closed\ sets\ are: \{\emptyset, X, \{b, c\}\}$

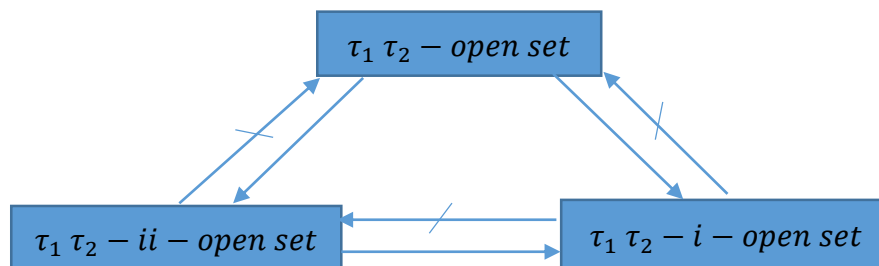
$\tau_2 - closed\ sets\ are: \{\emptyset, X, \{b, c\}, \{c\}\}$

$\tau_1 \tau_2 - ii - open\ sets\ are: \{\emptyset, X, \{a\}, \{a, b\}\}$

$\tau_1 \tau_2 - \alpha - open\ sets\ are: \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$

$A=\{a, c\}$ is $\tau_1 \tau_2 - \alpha - open\ set$ but A is not $\tau_1 \tau_2 - ii - open\ set$.

Remark 2.10: From the above results we get the following diagram.



3. ii- Continuity on Bitopological Spaces.

In this section, we define *ii*-continuous mapping from (X, τ_1, τ_2) in (Y, δ_1, δ_2) by using the definition of *ii*-open sets in topological space (X, τ) see[1],[2]. Also, we study the relation between *ii*-continuous mapping on bitopological spaces with continuous and *i*-continuous mapping [3].

Definition 3.1: A mapping $f: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ is called

1. Continuous if $f^{-1}(G)$ is $\tau_1 \tau_2 - open\ set$ in (X, τ_1, τ_2) for any open set G in (Y, δ_1, δ_2) .
2. *i*-Continuous if $f^{-1}(G)$ is $\tau_1 \tau_2 - i - open\ set$ in (X, τ_1, τ_2) for any open set G in (Y, δ_1, δ_2) .

Definition 3.2: A mapping $f: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ is called *ii*-continuous if $f^{-1}(G)$ is $\tau_1 \tau_2 - ii - open$ set in (X, τ_1, τ_2) for any open set G in (Y, δ_1, δ_2) .

Theorem 3.3: Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ be mapping then:

1. Every continuous mapping is an *ii*-continuous,
2. Every *ii*-continuous mapping is an *i*-continuous.

Proof: (1) Let G be an open set in (Y, δ_1, δ_2) . Since f is continuous, it follows that $f^{-1}(G)$ is $\tau_1 \tau_2 - open$ set in (X, τ_1, τ_2) . By Theorem (2.3). Hence $f^{-1}(G)$ is $\tau_1 \tau_2 - ii - open$ set in (X, τ_1, τ_2) . Thus f is *ii*-continuous.

(2) Let G be open set in (Y, δ_1, δ_2) . Since f is *ii*-continuous, it follows that $f^{-1}(G)$ is $\tau_1 \tau_2 - ii - open$ set in (X, τ_1, τ_2) . By remark (2.5). Hence $f^{-1}(G)$ is $\tau_1 \tau_2 - i - open$ set in (X, τ_1, τ_2) . Thus f is *i*-continuous. ■

The converse of the above theorem is not true in general as shown in the following example:

Example 3.4: Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\emptyset, X, \{a\}$, $\tau_2 = \{\emptyset, X, \{a\}, \{a, b\}\}$

$\delta_1 = \{\emptyset, Y, \{a\}, \{a, c\}\}$, $\delta_2 = \{\emptyset, Y, \{a, c\}\}$

$\tau_1 \cup \tau_2 = \{\emptyset, X, \{a\}, \{a, b\}\}$

$\tau_1 - closed$ sets are: $\{\emptyset, X, \{b, c\}\}$

$\tau_2 - closed$ sets are: $\{\emptyset, X, \{b, c\}, \{c\}\}$

$\tau_1 \tau_2 - ii - open$ sets are: $\{\emptyset, X, \{a\}, \{a, b\}\}$

$\tau_1 \tau_2 - i - open$ sets are: $\{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$

Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \delta_1, \delta_2)$ be the identity mapping then $f^{-1}(\{a\}) = \{a\}$, $f^{-1}(\{b\}) = \{b\}$, $f^{-1}(\{c\}) = \{c\}$. Then f is *i*-continuous on bitopological spaces but f is not *ii*-continuous, Since $\{a, c\}$ is open set in (Y, δ_1, δ_2) $f^{-1}(\{a, c\}) = \{a, c\}$ is not $\tau_1 \tau_2 - ii - open$ set in (X, τ_1, τ_2) .

Conclusion 3.5:

For future, also we can study the concept of $\tau_1 \tau_2 - ii - open$ mapping and $\tau_1 \tau_2 - ii - homeomorphisem$ and other topological concepts between bitopological spaces (X, τ_1, τ_2) and (Y, δ_1, δ_2) .

REFERENCES

- [1] Abdullah.B.S. and Mohammed A.A., (2019), ii- Open Sets in Topological Space, Internatinal Mathematical Forum, Vol.14, No.1, 41-48.
[https:// doi.org/10.12988/imf.2019.913](https://doi.org/10.12988/imf.2019.913)
- [2] Abdullah.B.S. and Mohammed A.A., (2019),On Standard Concept Using ii-Open Sets, Open Access Journal, Vol 6, July 15.
[http:// dx.doi.org/10.4236/****2019****](http://dx.doi.org/10.4236/****2019****)
- [3] Askander.S.W. and Mohammed A.A., (2018), i-Open Sets in Topological Spaces, Al-Rafidain Journal of Computer Sciences and Math, Vol 12, No.1,13-23.
- [4] Mahdi,Y.K.,(2007), Semi-Open and Semi-Closed Sets in Bitopological Spaces, Accepted in The First Science Conference of Education College, Babylon Unive,February 18-19.
- [5] Kelly.j.c, (1963), Bitopological Spaces, London Math. Soc., Vol. 13, 71-89.
- [6] Lellis Thivagav,M. and B.Meera.Devi, (2010), Bitopological B-Open Sets, International Journal of Algorithms, Computing and Math, Vol. 3,No. 3
- [7] Lellis Thivagav,M., (1991), Generalization of Pairwise α -Continuous Functions, Pure and Appl. Mathematica sci, Vol.28, 55-63.