Free Convection Flow of Viscous Dissipative Fluid in a Rectangular Cavity

Ahmed M. Jassim **Tagread H. Shuker**
College of Computer Sciences and Mathematics *College of Education*
University of Mosul, Iraq

Received on: 18/12/2005 **Accepted on: 05/03/2007**

ABSTRACT

Free convection flow of incompressible viscous fluid with dissipation in a rectangular cavity has been studied, a finite difference technique based on **ADI** scheme is adopted in the solution of the problem. The effect of dissipation parameter (ε), which usually appears as a term in the energy equation, has been taken into account.The results indicated that the effect of dissipation number ($\varepsilon = \mp 0.004$) was very small which is accepted with the fact of neglecting the dissipation function in the energy equation of most convection problems.

Keywords: convection problems, flow, viscous fluid, finite difference technique, energy equation.

الحمل الحراري الحر في تجويف مستطيل لمائع لزج قابل للتبدد

أحمد جاسم **تغريد حمدون شكر**
كلية علوم الحاسوب والرياضيات، جامعة الموصل *كلية التربية، جامعة الموصل*

تاريخ الاستلام: 2005/12/18 **تاريخ القبول: 2007/03/05**

الملخص

يتناول هذا البحث دراسة الحمل الحراري الحر في تجويف مستطيل لمائع لزج قابل للتبدد وقد تم استخدام طريقة الفروقات المنتهية (ADI) في حل المعادلات التي تغطي المسألة، إن عامل التشتت ε الذي هو احد العوامل التي تظهر في معادلة الطاقة قد تم أخذه بنظر الاعتبار ولقد أظهرت النتائج بأن تأثير معامل التشتت الذي تم التعبير عنه (بعدد التشتت)، طفيف جداً وهذا لا يتعارض مع حقيقة إهمال هذا العامل من معادلة الطاقة في معظم مسائل الحمل الحراري.

الكلمات المفتاحية: مسائل الحمل الحراري، الجريان، مائع لزج، طريقة الفروقات المنتهية، معادلة الطاقة.

1-Introduction:

Convection heat transfer is considered one of the most important way of heat transfer especially heat transfer by free convection, because of its application in technological fields, and the study of free convection flow of viscous incompressible fluids has received the attention of many researchers. The dissipation function can be found in the general form of energy equation as follows, [2]:

$$\rho C_V \frac{DT}{Dt} = -P\nabla \cdot V + k\nabla^2 T - \nabla \cdot q + \Phi + q''' \qquad ..(1.1)$$

where $\rho, C_V, P, k, T, \Phi, q'''$, are the density, specific heat at constant volume, pressure, thermal conductivity, temperature, dissipation function and internal heat generation rate, respectively.

The effect of viscous dissipation represented by Φ has not been studied till 1999, when Soundalgekar [7] has presented a transient free convection flow of viscous incompressible fluid past a semi-infinite vertical plate by taking into consideration viscous dissipative heat and he solved the governing non-linear equations by using the implicit finite-difference method of Crank-Nicolson type. In this paper we study the effect of viscous dissipative heat convection and we solve the problem by using ADI scheme.

2-Mathematical Model:

A two-dimensional unsteady flow of a viscous, incompressible fluid in a rectangular cavity is considered, the flow under the usual Boussinesq's approximation can be shown to be governed by the following boundary layer equations,

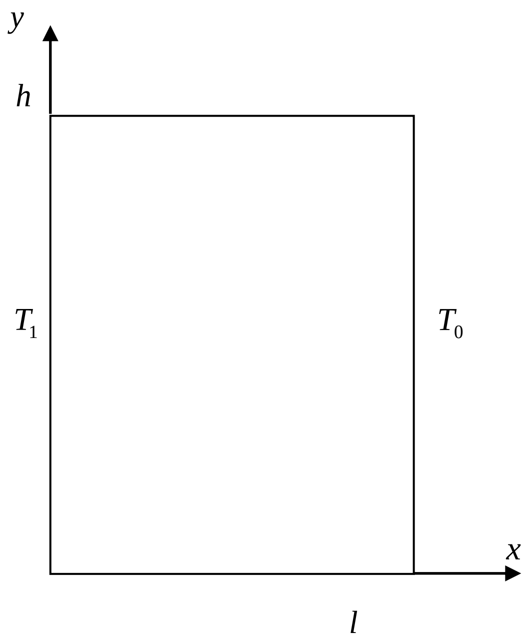

Figure (2-1)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad ...(2.1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\nu\nabla^2 u \qquad ...(2.2a)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\nu\nabla^2 v+g\beta(T-T_0) \qquad ...(2\text{-}2b)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\nabla^2 T+\frac{\mu}{\rho C_p}\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2\right] \qquad ...(2.3)$$

eliminating the pressure terms from equations (2-2a)-(2-2b), the set of governing equations becomes:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad ...(2.3)$$

$$\frac{\partial}{\partial t}\left[\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right]+\left[\frac{\partial}{\partial x}\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)-\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)\right]=\nu\nabla^2\left[\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right]+g\beta\frac{\partial T}{\partial y} \qquad ...(2.4)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\nabla^2 T+\frac{\mu}{\rho C_p}\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2\right] \qquad ...(2.5)$$

with the following boundary conditions:

$$\left.\begin{array}{lll} u=0 & at & x=0,l \\ v=0 & at & y=0,h \\ \dfrac{\partial T}{\partial y}=0 & at & y=0,h \\ T=T_1,T_0 & at & x=0,l \end{array}\right\} \qquad ...(2.6)$$

3- Non-dimensional form:

In order to solve the governing equations (2-1)-(2-3) with the boundary conditions (2-4), we have to convert these equations to the non-dimensional form and this will occur by introducing the non-dimensional quantities [10],

$$\left.\begin{aligned} &X=\frac{x}{L}, \qquad Y=\frac{yGr^{1/4}}{L} \\ &U=\frac{uLGr^{-1/2}}{\vartheta}, \quad V=\frac{vLGr^{-1/4}}{\vartheta} \\ &Gr=\frac{g\beta L^{3}\Delta T}{\vartheta^{2}} \\ &\tau=\frac{t\vartheta Gr^{1/2}}{L^{2}} \\ &pr=\frac{\vartheta}{\alpha} \\ &\theta=\frac{T-T_{0}}{\Delta T} \end{aligned}\right\} \qquad ...(3.1)$$

The governing equations under these non-dimensional quantities become,

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \qquad ...(3.2)$$

$$\frac{\partial \xi}{\partial \tau}+U\frac{\partial \xi}{\partial X}+V\frac{\partial \xi}{\partial Y}=pr\nabla^{2}\xi+prRa\frac{\partial \theta}{\partial Y} \qquad ...(3.3)$$

$$\frac{\partial \theta}{\partial \tau}+U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\nabla^{2}\theta+\varepsilon\left[\left(\frac{\partial U}{\partial Y}\right)^{2}+\left(\frac{\partial V}{\partial X}\right)^{2}\right] \qquad ...(3.4)$$

where

$$\xi=-\nabla^{2}\psi$$

$$U=\frac{\partial \psi}{\partial y},\ V=-\frac{\partial \psi}{\partial X},\ \varepsilon=\frac{g\beta L}{C_{p}}$$

and the boundary conditions (2.4) in the non-dimensional form become,

$$\left.\begin{aligned} &U=V=0 && at\ X=0,1 \\ &U=V=0 && at\ Y=0,1 \\ &\theta=0 && at\ X=0 \\ &\theta=1 && at\ X=1 \\ &\frac{\partial \theta}{\partial Y}=0 && at\ Y=0,1 \end{aligned}\right\} \qquad ...(3.5)$$

4- Method of solution:

We shall use the ADI finite difference method in the solution of the heat equation (3.4) as follows:, see [3,4,5,7]

$$\frac{\theta^{*}_{i,j}-\theta^{n}_{i,j}}{\Delta\tau/2}+U^{n}_{i,j}\frac{\theta^{*}_{i+1,j}-\theta^{*}_{i-1,j}}{2\Delta X}+V^{n}_{i,j}\frac{\theta^{n}_{i,j+1}-\theta^{n}_{i,j-1}}{2\Delta Y}=\delta^{2}_{x}\theta^{*}_{i,j}+\delta^{2}_{y}\theta^{n}_{i,j}+$$
$$+\varepsilon\left[\left(\frac{U^{n}_{i,j+1}-U^{n}_{i,j-1}}{2\Delta Y}\right)^{2}+\left(\frac{V^{n}_{i+1,j}-V^{n}_{i-1,j}}{2\Delta X}\right)^{2}\right] \qquad ...(4.1)$$

$$\frac{\theta^{n+1}_{i,j}-\theta^{*}_{i,j}}{\Delta\tau/2}+U^{n}_{i,j}\frac{\theta^{*}_{i+1,j}-\theta^{*}_{i-1,j}}{2\Delta X}+V^{n}_{i,j}\frac{\theta^{n+1}_{i,j+1}-\theta^{n+1}_{i,j-1}}{2\Delta Y}=\delta^{2}_{x}\theta^{*}_{i,j}+\delta^{2}_{y}\theta^{n+1}_{i,j}+$$
$$+\varepsilon\left[\left(\frac{U^{n}_{i,j+1}-U^{n}_{i,j-1}}{2\Delta Y}\right)^{2}+\left(\frac{V^{n}_{i+1,j}-V^{n}_{i-1,j}}{2\Delta X}\right)^{2}\right] \qquad ...(4.2)$$

with boundary conditions,

$$\left.\begin{array}{lll} U^{0}_{i,j}=constant=1.0, & U^{n}_{0,j}=0, & U^{n}_{i,0}=0 \\ V^{0}_{i,j}=constant=1.0, & V^{n}_{0,j}=0, & V^{n}_{i,0}=0 \\ \theta^{0}_{i,j}=0, & \theta^{n}_{0,j}=0 & \theta^{n}_{i,N}=1 \\ \theta^{n}_{i,1}=\theta^{n}_{i,-1}, & \theta^{n}_{i,N+1}=\theta^{n}_{i,N-1} & \end{array}\right\} \qquad ...(4.3)$$

Equations (4.1) and (4.2) can be reduced to give,

$$A_{1}(I)\theta^{*}_{i-1,j}+B_{1}(I)\theta^{*}_{i,j}+C_{1}(I)\theta^{*}_{i+1,j}=D_{1}(I), \quad I=0,1,2,...N \qquad ...(4.4)$$

where

$$\left.\begin{array}{l} A_{1}(I)=-\left(U^{n}_{i,j}\frac{\Delta\tau}{2\Delta X}+\frac{\Delta\tau}{(\Delta X)^{2}}\right) \\ B_{1}(I)=2\left(1+\frac{\Delta\tau}{(\Delta X)^{2}}\right) \\ C_{1}(I)=\frac{U^{n}_{i,j}\Delta\tau}{2\Delta X}-\frac{\Delta\tau}{(\Delta X)^{2}} \\ D_{1}(I)=\left(\frac{V^{n}_{i,j}\Delta\tau}{2\Delta y}+\frac{\Delta\tau}{(\Delta X)^{2}}\right)\theta^{n}_{i,j-1}+2\left(1+\frac{\Delta\tau}{(\Delta Y)^{2}}\right)\theta^{n}_{i,j}+\left(\frac{\Delta\tau}{(\Delta Y)^{2}}-\frac{V^{n}_{i,j}\Delta\tau}{2\Delta Y}\right)\theta^{n}_{i,j+1}+ \\ \qquad +\varepsilon\left[\left(\frac{U^{n}_{i,j+1}-U^{n}_{i,j-1}}{2\Delta Y}\right)^{2}+\left(\frac{V^{n}_{i+1,j}-V^{n}_{i-1,j}}{2\Delta X}\right)^{2}\right] \end{array}\right\} \qquad ...(4.5)$$

followed by,

$$A_2(J)\theta_{i,j-1}^{n+1} + B_2(J)\theta_{i,j}^{n+1} + C_2(J)\theta_{i,j+1}^{n+1} = D_2(J), \quad J = 0,1,2,...N \qquad ...(4.6)$$

where

$$\left.\begin{aligned}
A_2(J) &= -\left(V_{i,j}^n \frac{\Delta\tau}{2\Delta Y} + \frac{\Delta\tau}{(\Delta Y)^2}\right) \\
B_2(J) &= 2\left(1 + \frac{\Delta\tau}{(\Delta Y)^2}\right) \\
C_2(J) &= \frac{V_{i,j}^n \Delta\tau}{2\Delta Y} - \frac{\Delta\tau}{(\Delta Y)^2} \\
D_2(J) &= \left(\frac{U_{i,j}^n \Delta\tau}{2\Delta X} + \frac{\Delta\tau}{(\Delta X)^2}\right)\theta_{i-1,j}^* + 2\left(1 + \frac{\Delta\tau}{(\Delta X)^2}\right)\theta_{i,j}^* + \left(\frac{\Delta\tau}{(\Delta X)^2} - \frac{U_{i,j}^n \Delta\tau}{2\Delta X}\right)\theta_{i+1,j}^* + \\
&\quad + \varepsilon\left[\left(\frac{U_{i,j+1}^n - U_{i,j-1}^n}{2\Delta Y}\right)^2 + \left(\frac{V_{i+1,j}^n - V_{i-1,j}^n}{2\Delta X}\right)^2\right]
\end{aligned}\right\} \qquad ...(4.7)$$

The coefficients $U_{i,j}^n, V_{i,j}^n$ are treated as constants during any one time-step of the computation, each of the equations (4.4) and (4.6) are creating a tridiagonal system which are solved by using Gauss elimination method, [1].

Table-1:The temperature distribution at the left side of the cavity

dissipation Number / Temperature	$\varepsilon = 0.004$	$\varepsilon = 0.02$	$\varepsilon = 0.0$	$\varepsilon = -0.02$	$\varepsilon = -0.004$
$\theta(1,1)$	0.7256449	1.3154280	0.5781990	-0.1590302	0.4307532
$\theta(1,2)$	0.7243975	1.3091820	0.5782012	-0.1527798	0.4320050
$\theta(1,3)$	0.7206617	1.2903330	0.5782440	-0.1338446	0.4358262
$\theta(1,4)$	0.7144623	1.2585680	0.5784358	-0.1016963	0.4424094
$\theta(1,5)$	0.7058397	1.2133760	0.5789555	-0.0554648	0.4520715
$\theta(1,6)$	0.6948394	1.1540330	0.5800409	0.0060489	0.4652425
$\theta(1,7)$	0.6814814	1.0795790	0.5819570	0.0843353	0.4824326
$\theta(1,8)$	0.6657020	0.9887682	0.5849354	0.1811025	0.5041687
$\theta(1,9)$	0.6472605	0.8799969	0.5890763	0.2981557	0.5308922
$\theta(1,10)$	0.6256498	0.7512406	0.5942520	0.4372635	0.5628543
$\theta(1,11)$	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000

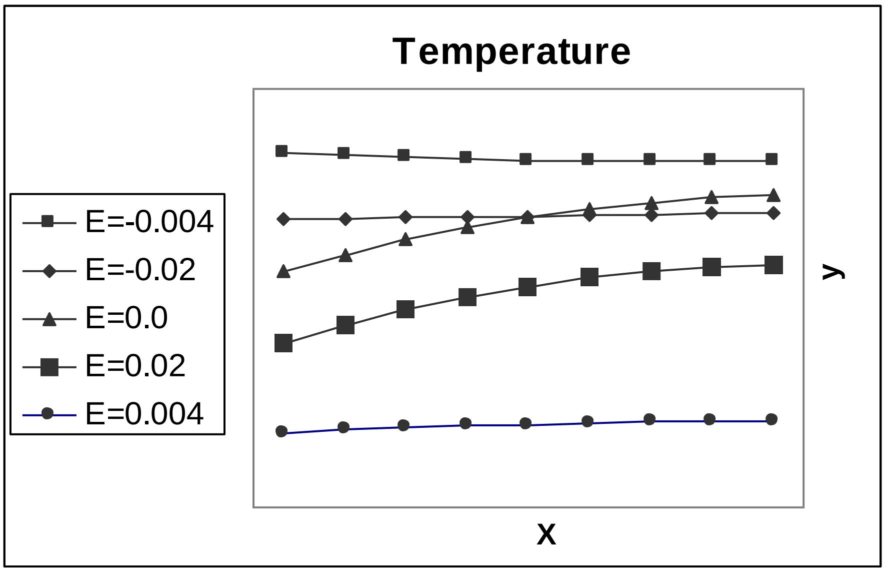

Figure -1: The non-dimensional temperature θ on the right side of cavity for different values of dissipation number ε

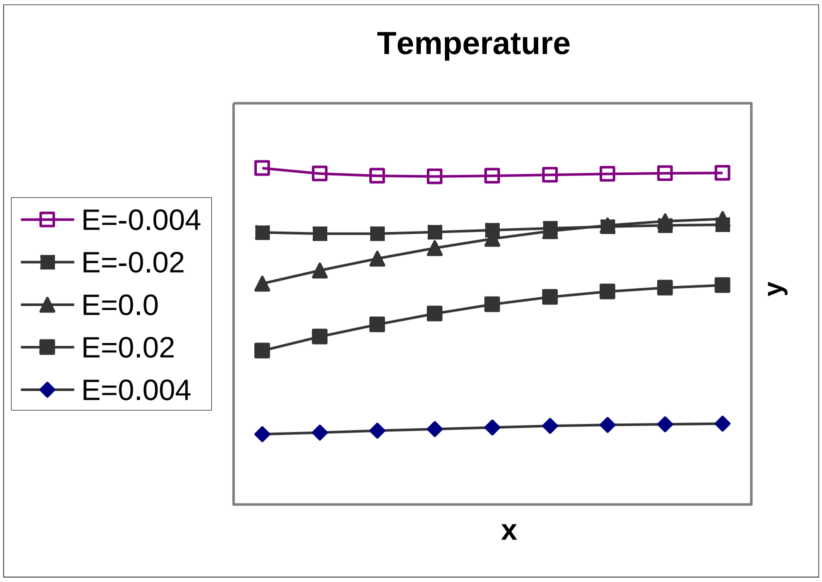

Figure-2: The non-dimensional temperature θ in the core of the cavity for different values of dissipation number ε

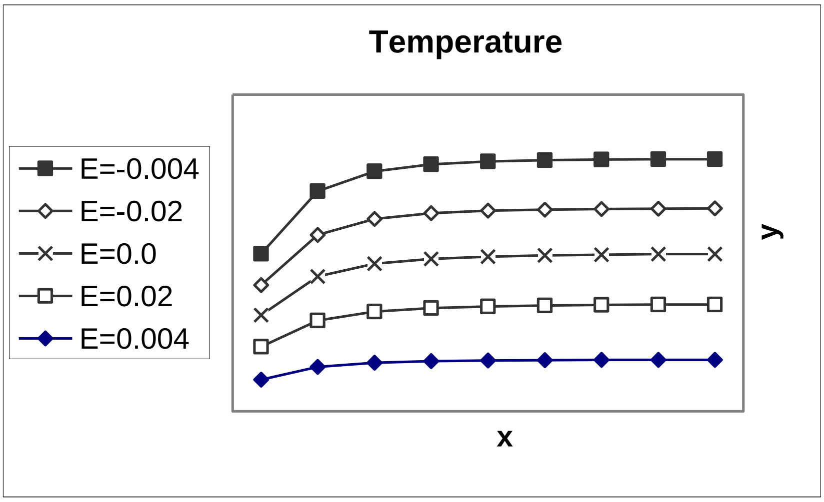

Figure-3: The non-dimensional temperature θ on the left side of cavity with different values of dissipation number ε

5- Results and Conclusions:

We investigate in this work the effect of dissipation number ε into the non-dimensional temperature θ, so that we have concentrated our effort only on heat equation .It is necessary to say that Rayleigh number Ra play a very important rule in the solution of motion equation, refer to [6,8,9], but in this paper we have presented a numerical solution of a non-linear equation (3.4) by using ADI finite difference method which leads to conclude the following remarks:

1- In order to implement the ADI in the solution of heat equation as it has appeared in the model under discussion, we assumed that the velocity components U,V are constants with small quantities [10], for this reason we don't need to discuss the Rayleigh number Ra .
2- We have presented some results for the values of the temperature θ in table-1, it shows the non-dimensional temperature θ at the left side of the cavity for different values of dissipation number ε, figures 1 to 3 represent the curve of temperature θ at different positions of the cavity, the first one is showing the effect of the parameter ε into the curve of the temperature near the left side of the enclosure. It seems that the effect of ε is very small, same remark has been noticed for figure -2 which represents the temperature at the core of the enclosure. Finally, figure -3 shows the effect of dissipation parameter ε into the temperature near the right side of the region.
3- There is no contradiction between the results which we have obtained

and the fact of neglecting the dissipation function of the heat equation, which means that the effect of number ε is so small that it can be neglected in most convection problems.

REFERENCES

[1] Brice Carnhan , H.A. Luther and James O. Wilkes (1969) **Applied Numerical Methods** , John Wiley & Sons, INC. New York.

[2] Brighton, J. A. and Hughes, W. F. (1967) **Fluid Dynamics**, McGraw-Hill.

[3] Jim, Douglas (2003) "On Accuracy of Alternating Direction Implicit Methods for Parabolic Equation", Department of Mathematics , Purdue University, W. Lafayette, IN 47097 USA .

[4] Kwak,. D. (1997) "A norm estimate for the ADI method for nonsymmetric problems" , Linear Algebra and its Applications, 266, PP. 127-141.

[5] Mictchell, R. A. and Griffiths F. D. (1980) **The Finite Difference Method In Partial Differential Equations**, John Willey & sons.

[6] Morn, V. and Dormy E. (2006) "Dissipation Mechnisims for convection in rapidly rotating sphers and the formation of bounded structures", physiscs of fluid 18, pp. 1-4.

[7] Namiki, T. (2000) "3-D ADI-FDTD method – Unconditionally stable time-domain algorithm for solving full vector Maxwell's equations", IEEE Transactions on Microwave Theory and Techniques,Vol.48, no.10 PP.1743-1748.

[8] Rogers, J. I.; Schatz M. F.; Bougie J. L. and Swift J. B. (2000) "Rayleigh-Benard convection in vertically oscillated fluid layer", physics review letters, Vol. 84,pp. 87-90.

[9] Solomator, V. S. and Barr A. C. (2005) "Onset of convection in fluids with strongly temperature-dependent power-law viscosity", physics of the earth and planery interiors, 155, pp. 140-145.

[10] Soundalgekar, V.M. (1999) "Transient free convection flow of viscous dissipative fluid past a semi-infinite vertical plate", Applied Mechanics and engineering ,Vol.4, No.2, pp. 203-218.